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On the Past, Present, and Future of the Diebold-Yilmaz Approach to Dynamic Network Connectedness

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Abstract: We offer retrospective and prospective assessments of the Diebold-Yilmaz connectedness research program, combined with personal recollections of its development. Its centerpiece in many respects is Diebold and Yilmaz (2014), around which our discussion is organized.

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It is a pleasure to help celebrate the fiftieth birthday of the *Journal of Econometrics*, and a great honor to have Diebold and Yilmaz (2014) selected as one of the five papers included in this Jubilee Issue. In this note we offer retrospective and prospective assessments of the Diebold-Yilmaz connectedness research program, combined with personal recollections of its development.

1 Origins in the Asian Contagion

It all began more than thirty years ago, when FD was an Economist at the Federal Reserve Board in Washington, DC, and KY was a Ph.D. student at the University of Maryland. In the late 1980s John Haltiwanger had kindly invited FD to teach an advanced Ph.D. macro-econometrics course at Maryland, and KY was a student in the course. We had many stimulating discussions, some of which eventually produced, for example, Diebold et al. (1994). But our paths diverged when FD went to the University of Pennsylvania as an assistant professor in 1989, and KY went to Koç University as an assistant professor in 1994, after working for two years as an Economist at the World Bank. The story might naturally have ended there, but no.

There was indeed a long gap, but then KY took a sabbatical at Penn during the 2003-2004 academic year. There was no real goal other than general intellectual stimulation for each of us. In particular, we had no joint work in progress, and indeed no real plans for joint work, when KY and his family arrived in Philadelphia in the summer of 2003. But of course we subsequently had many discussions on various issues and ideas.

One thing that intrigued us both was financial market contagion. In particular, the infamous 1997 sequential collapse of several far-eastern currencies – the "Asian Contagion" – was still fresh in the professional consciousness, and hence in ours. How to define "contagion"? How to measure it? Did it really exist? We had many conversations, typically on the ninth floor of the Wharton School's Huntsman Hall, after visiting the adjacent coffee room, looking out over the Philadelphia skyline.

2 Variance Decompositions and Network Connectedness

The Diebold-Yilmaz dynamic connectedness measurement framework grew from those Huntsman Hall conversations. It seemed natural to define h-step connectedness in terms of variance decompositions. We liked the fact that variance decompositions naturally promote uniform measurement across variables via their "fraction of h-step-ahead forecast error variance" perspective, whereas other possible approaches based for example on impulse-response functions did not, so we simply proposed the variance decomposition approach, directly.

That is, we proposed measuring what we would later call "h-step pairwise directional connectedness from j to i" as the fraction of h-step forecast error variance of variable i arising from shocks to variable j. Allowing for time-varying parameters in the model from which the variance decompositions were calculated, moreover, allowed for the important possibility of time-varying connectedness.

Our goal was always the empirical description of connectedness and its evolution, "getting the facts straight" with minimal assumptions. In particular, we didn't want to have to take a stand on the deep underlying structural mechanics, and indeed our intentionally reduced-form measurements are consistent with a wide variety of possible underlying structures. This is important in economic and financial environments, where connectedness and its evolution are typically incompletely understood at best.

Initially we didn't use the term "connectedness". Instead we used more traditional and perhaps more exciting, if nevertheless only vaguely-defined, terms like "contagion" or "spillovers". It soon became clear, however, that different research tribes had different, strongly-held, and sometimes conflicting views about the meaning of such terms. We wanted an uncontroversial term that simultaneously conveyed the essence of our measure, and we eventually settled on "connectedness".

Diebold and Yilmaz (2009) contains the embryonic idea. We use a VAR approximating model, featuring Cholesky factor identification, small data, and no awareness of network theory or graphics, to obtain the variance decomposition.[1] The empirical work focuses on volatility connectedness in equity markets, although the term "connectedness" is not used.

Diebold and Yilmaz (2012) takes a significant step forward, moving to the generalized identification of Pesaran and Shin (1998), which builds on Koop et al. (1996). In the generalized identification environment, unlike with Cholesky factor identification, there is no issue of variable ordering. Equally importantly, Diebold and Yilmaz (2012) also considers not only "total" but also "directional" aspects, so that $i \to j$ connectedness need not match $j \to i$ connectedness. The term "connectedness" is still never used, and network perspectives are absent. The empirical work again involves only small data, focusing on daily U.S. asset classes (stocks, bonds, commodities, and foreign exchange).

[1] For colorful background on "small data" vs. "Big Data" definitions and history, see Diebold (2021).

Table 1: Connectedness Table / Network Adjacency Matrix, D

	x_1	x_2	...	x_N	From Others to i (In-Degrees)
x_1	d_{11}^H	d_{12}^H	$\cdots$	d_{1N}^H	$\Sigma_{j=2}^N d_{1j}^H$
x_2	d_{21}^H	d_{22}^H	$\cdots$	d_{2N}^H	$\Sigma_{j=1, j\neq 2}^N d_{2j}^H$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
x_N	d_{N1}^H	d_{N2}^H	$\cdots$	d_{NN}^H	$\Sigma_{j=1}^{N-1} d_{Nj}^H$
To Others From j (Out-Degrees)	$\Sigma_{i=2}^N d_{i1}^H$	$\Sigma_{i=1, i\neq 2}^N d_{i2}^H$	$\cdots$	$\Sigma_{i=1}^{N-1} d_{iN}^H$	$\Sigma_{i,j=1; i\neq j}^N d_{ij}^H$

Notes: We show an illustrative connectedness table (weighted, directed network adjacency matrix) based on an N-variable H-step variance decomposition, D. The fraction of variable i's H-step-ahead forecast error variance due to shocks in variable j is d_{ij}^H. Equivalently, H-step pairwise directional connectedness from j to i, $C_{i\leftarrow j}^H$, is d_{ij}^H. Total directional connectedness from others to i (the in-degree, or from-degree of network node i) is given by $C_{i\leftarrow \bullet}^H = \sum_{j\neq i} d_{ij}^H$, and total directional connectedness to others from j (the out-degree, or to-degree of network node j) is $C_{\bullet\leftarrow j}^H = \sum_{i\neq j} d_{ij}^H$. Total system-wide connectedness is the grand sum of all off-diagonal elements, $C^H = \sum_{i,j; i\neq j} d_{ij}^H$.

Diebold and Yilmaz (2014) is the real breakthrough, and marvelously, it also happens to be the paper included in this issue of *Journal of Econometrics*. We of course maintain generalized identification and directional perspective, but crucially, we also couch the analysis in a network framework for the first time, and we introduce some crude network graphics. We also use the "connectedness" language throughout.

Let us provide a little background color on Diebold and Yilmaz (2014). Around that time it was hard to go through a week in a department of economics, statistics, computer science, etc., without encountering numerous seminars and conversations about networks – network formation, network structure, etc. The excitement was palpable. Surely, we thought, our work on connectedness (as we had by then begun to call it) must be somehow related to the characterization of network structure. Unfortunately, however, we knew very little about the characterization of network structure!

So we took the plunge, starting with Newman (2010) and then working forward and backward, and our eyes were opened. What a playground: weighted, directed networks and their node degrees; in-degrees and out-degrees; degree distributions; numerous so-called "centrality" measures; and on and on. The key insight – the real ah-ha! moment – came when we realized that the variance decomposition matrix, or "connectedness table", at the center of the Diebold-Yilmaz research program could be viewed as the adjacency matrix of a weighted, directed network, as illustrated in Table 1. Hence the powerful toolkit of methods for network measurement and characterization immediately applies to connectedness. Indeed several of the connectedness measures that we had independently proposed are prominent network statistics (e.g., in-degrees, out-degrees).

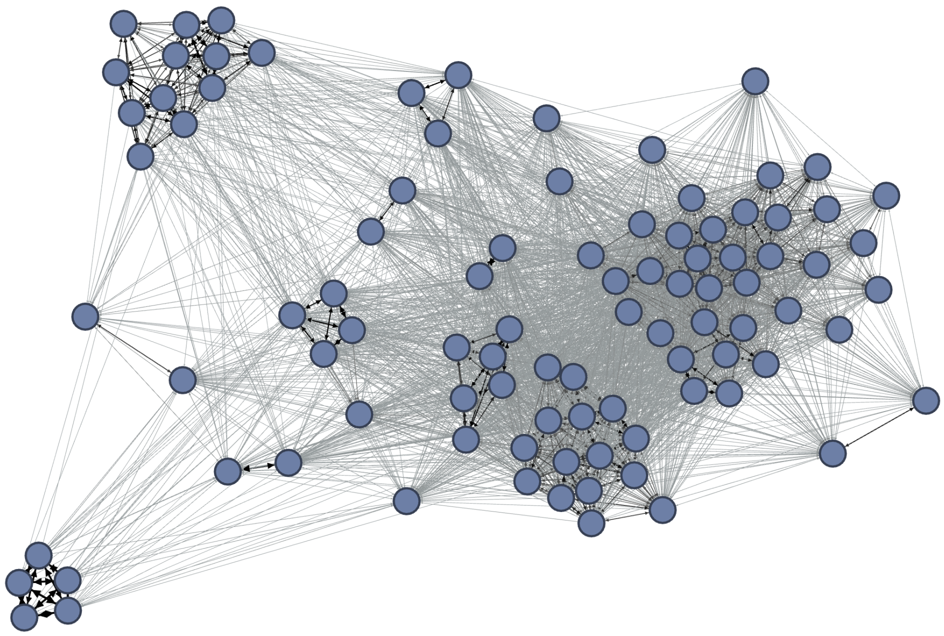
Now let us move ahead a few years. Demirer et al. (2018) take the connectedness framework close to completion, moving to Big Data environments with regularized estimation and sophisticated network graphics that scale effortlessly with network dimension. Consider estimation first. In contrast to a typical small-data VAR circa 1985 (perhaps a 5-variable VAR(3), which can be estimated by OLS), regularization is *essential* in Big Data environments circa 2023 (perhaps a 500-variable VAR(3)).[2] Demirer et al. (2018) use the LASSO (Tibshirani, 1996), which selects and shrinks.

Now consider *understanding* the estimated VAR. Even in the small-data 5-variable VAR(3) case, we can't stare productively at nearly 100 estimated coefficients and hope to learn much, but we can certainly stare productively at a 5×5 connectedness table, as per Table 1. That is, variance decompositions save the day, rendering dozens of uninterpretable estimated coefficients economically interpretable by viewing them through a particular lens, which is a key part of the message of Sims (1980). But in the Big Data 500-variable VAR(3), in contrast, we are inescapably in trouble. Of course we can't stare productively at coefficients – now in the hundreds of thousands – but we also can't stare productively at a 500x500 connectedness table! Tabular and conventional graphical presentations of variance decompositions, which save the day in small-data environments, are themselves unworkable in high-dimensional environments.

But network tools come to the rescue throughout. First, they are the key to connectedness-based summaries of variance decompositions, through various statistics based on the in- and out-degree distributions (i.e., all of the connectedness statistics described in Table 1). Second, and crucially, they are the key to highly-revealing and readily-scalable visualizations of variance decompositions, as in the illustrative "spring graph" shown in Figure 1, which is immediately interpretable, whether for a network of dimension 5 or 500 or 5000. In particular, network graphics are far from pretty afterthoughts; rather, they are central to the

[2]Interestingly, even in small-data environments some regularization in the form of Bayesian shrinkage toward random walk dynamics has long been recognized as helpful; see Doan et al. (1984).

Figure 1: Illustrative Network "Spring Graph"



Notes: The underlying algorithm (Jacomy et al., 2014) finds a steady state in which repelling and attracting forces exactly balance, where nodes repel each other, and links, like springs, attract their nodes with force proportional to average pairwise directional connectedness "to" and "from."

connectedness framework, facilitating the understanding of estimated approximating models in high dimensions by *visualizing* their variance decompositions. Even without knowing the details of how Figure 1 is constructed, for example, a viewer can immediately see two small and very tightly-connected clusters in the upper-left and lower-left, and two large and somewhat less-tightly connected clusters in the upper- and lower-right.[3,4]

Finally, as mentioned earlier, an important part of the Diebold-Yilmaz framework is allowance for time-varying coefficients in approximating models (whether by rolling the estimation or by modeling explicitly time-varying coefficients), and hence allowing for time-varying variance decompositions that translate into time-varying network graphs. There will then be a *different* Figure 1 each period as the dynamics evolve, and one could watch an animation over time.[5]

[3]For details of spring graph construction, see the notes to Figure 1.

[4]For readers curious as to the actual network represented by Figure 1, see Demirer et al. (2018).

[5]Some details remain to be worked out, such as how to "anchor" the set of graphs, and we look forward to someone doing so.

3 Parallels in the Measurement of International Trade and Systemic Financial Risk

We have already discussed the deep connections between our connectedness measures and traditional tools for characterizing network structure. Here we briefly discuss some other deep connections, first to the measurement of international trade and then to the measurement of systemic financial risk.

3.1 International Trade

An "import/export" perspective provides immediate insight on our connectedness measures, at different levels of aggregation from the most granular to the most aggregative. In an obvious notation, pairwise directional connectedness $C^H_{i \leftarrow j} = d^H_{ij}$ is "i's imports from j". On net, $C^H_{ij} = C^H_{j \leftarrow i} - C^H_{i \leftarrow j}$ is the "ij bilateral trade balance". Total directional connectedness from others to i, $C^H_{i \leftarrow \bullet} = \sum_{j \neq i} d^H_{ij}$ is "i's total imports", and total directional connectedness to others from j, $C^H_{\bullet \leftarrow j} = \sum_{i \neq j} d^H_{ij}$ is "j's total exports". On net, $C^H_i = C^H_{\bullet \leftarrow i} - C^H_{i \leftarrow \bullet}$ is "i's multilateral trade balance". System-wide connectedness, $C^H = \sum_{i \neq j} d^H_{ij}$ is "total world exports" (or imports, since they must agree at the world level).

3.2 Systemic Financial Risk

Marginal expected shortfall, MES, is an important measure of systemic financial risk; see Acharya et al. (2012) and the references therein. It is essentially a measure of the sensitivity of an individual firm's return to extreme market-wide returns. In particular,

$$MES^{j|mkt} = E(r_j \,|\, \mathbb{C}(r_{mkt})),$$

where r_j denotes firm j's return and $\mathbb{C}(r_{mkt})$ an extreme market return event. MES is effectively a market-based "stress test" of firm j's fragility during systemic events. It is also clearly and intimately related to the Diebold-Yilmaz measure of "total directional connectedness from others to j" (j's from-degree).

Another important systemic financial risk measure is $CoVaR$, the value at risk (VaR) of an institution j, or of the entire the financial system, conditional on an individual institution i being in distress; see Adrian and Brunnermeier (2016). The p-percent $CoVaR$ from firm i

to firm j is defined by

$$CoVaR^{p,j|i}\colon p = P\left(r_j < -CoVaR^{p,j|i} \mid \mathbb{C}\left(r_i\right)\right),$$

and the p-percent $CoVaR$ from firm i to the market is defined by

$$CoVaR^{p,mkt|i}\colon p = P\left(r_{mkt} < -CoVaR^{p,mkt|i} \mid \mathbb{C}\left(r_i\right)\right).$$

The leading choice of $\mathbb{C}\left(r_i\right)$ is a VaR breach, in which case $CoVaR$ measures VaR linkages (hence its name). The $CoVaR$ directionality, however, is the opposite of MES. It is clearly and intimately related to Diebold-Yilmaz "total directional connectedness to others from i" (i's to-degree).

4 Applications

Many applications followed from the methodology developed in the above-discussed papers.[6] Some were implemented by us. For example, in Diebold and Yilmaz (2009) we obtained "a total spillover index" (total system-wide connectedness in our current jargon) and showed how global stock return and volatility spillovers behave during major financial crises, such as the East Asian crisis of 1997. In Diebold and Yilmaz (2012) we extended our approach to study directional spillovers and applied the new framework to four financial markets, namely, the U.S. stock, bond, and foreign exchange markets, and the global commodity market. Going from low-dimensional to high-dimensional environments, Diebold and Yilmaz (2014), Diebold and Yilmaz (2016), and Demirer et al. (2018) studied connectedness of financial institutions within a country (the U.S.), across the Atlantic, and across the globe, respectively. These papers put our framework at the center of the literature studying systemic risk in the banking industry, and indeed it has been widely used in policy and industry to assess systemic risk in financial markets.[7]

[6]Software implementations that facilitate applications include, among others, the EViews add-in "Diebold-Yilmaz Index"; RATS programs to replicate Diebold and Yilmaz (2009) (`https://estima.com/ratshelp/index.html?dieboldyilmazspilloverpapers.html`) and Diebold and Yilmaz (2012) (`https://estima.com/ratshelp/index.html?dieboldyilmazijf2012.html`); and R code at `https://rdrr.io/cran/Spillover/`, `https://github.com/tomaskrehlik/frequencyConnectedness`. There is also a website, `www.financialconnnectedness.org`, with regularly updated dynamic connectedness graphs for major global stock, foreign exchange, sovereign bond, and CDS markets.

[7]Policy examples include numerous IMF, European Central Bank, and country central bank policy and research reports.

5 Concluding Remarks

With hindsight it is easy to understand the popularity of Diebold-Yilmaz connectedness measurement. The methodology is simple and appealing, bridging traditional "econometric modeling thinking" on the one hand, and modern "network and Big Data thinking" on the other, assembling the pieces to go to very new places. It is based on variance decompositions, which are familiar and comfortable, and it rests on a novel connection between the seemingly-distinct variance-decomposition and network literatures, namely the insight that "a variance decomposition is a network". It follows that network tools, which scale effortlessly to high dimensions, provide powerful help in summarizing and visualizing connectedness as defined by variance decompositions.

Interesting recently-published work pushing the frontier outward includes Baruník and Křehlík (2018), Barigozzi and Brownlees (2019), and Bykhovskaya (2023). Some new unpublished work, moreover, is particularly novel. In one new development, Barigozzi et al. (2022) develop methods for connectedness in dynamic "multilayer" networks, reflecting the insight that rich networks may have many kinds of connections, each governed by its own adjacency matrix. Parsimonious modeling then becomes absolutely crucial, and in that regard Barigozzi et al. (2022) effectively propose a modeling framework with a "factor structure" for the set of adjacency matrices.

In a second new development, Mlikota (2023) explores the flip-side of Diebold-Yilmaz thinking. In particular, while Diebold and Yilmaz "turn VARs into networks", Mlikota "turns networks into VARs". More precisely, Diebold and Yilmaz map approximating model variance decompositions into weighted, directed, time-varying networks, which they use to understand dynamic connectedness, whereas Mlikota goes in the opposite direction, modeling approximating model conditional mean functions but exploiting the restrictions implied by network structure to achieve regularization. Those restrictions take the form of chains, providing a natural underpinning for rich patterns of multi-step causality (Dufour and Renault, 1998).

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