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Economic Value of Modeling the Joint Distribution of Returns and Volatility: Leverage Timing

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Abstract

We propose a joint modeling strategy for timing the joint distribution of the returns and their volatility. We do this by incorporating the potentially asymmetric links into the system of 'independent' predictive regressions of returns and volatility, allowing for asymmetric cross-correlations, denoted as instantaneous leverage effects, in addition to cross-autocorrelations between returns and volatility, denoted as intertemporal leverage effects. We show that while the conventional intertemporal leverage effects bear little economic value, our results point to the sizeable value of exploiting the contemporaneous asymmetric link between returns and volatility. Specifically, a mean-variance investor would be willing to pay several hundred basis points to switch from the strategies based on conventional predictive regressions of mean and volatility in isolation of each other to the joint models of returns and its volatility, taking the link between these two moments into account. Moreover, our findings are robust to various effects documented in the literature.

Keywords: Economic value, system of equations, leverage timing, market timing, volatility timing
JEL Classification: C30, C52, C53, C58, G11

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1 Introduction

Predictability of the distribution of asset returns remains an area of intense debate for many decades. Main findings typically point to the lack of predictability in the mean of the returns, especially at a higher frequency such as daily returns. In contrast, when we consider the second moment of the distribution, there is ample evidence documenting predictability in the volatility of the returns, which potentially translates into profitable investment strategies. While economic values of the mean/market and volatility timing are well documented, there has been a dearth of analysis on the economic value of the timing the 'joint' distribution of the returns and volatility, or put differently, the leverage effect referring to the link between the returns and volatility. The economic value of this link is the focus of this paper.

Conventionally, the leverage effect of returns on volatility is captured by including the past return innovation among the volatility predictors. Typically, negative returns have much more significant leverage on the volatility compared to positive returns. This asymmetry leads to the lag of the absolute value of returns to be added among the predictors disentangling the leverage effects of the negative realizations from the positive ones. Still, the link between the returns and their volatility might be much richer than this intertemporal link. Giraitis et al. (2004); Bollerslev et al. (2006), among others, for example, document a sizable instantaneous leverage effect in addition to the intertemporal dynamics between the returns and volatility. Accounting for these links requires setting up a joint model of mean and the volatility where the contemporaneous correlation between the return and volatility innovations are explicitly considered.

This paper proposes a joint modeling strategy that extends the conventional predictive regressions of returns and their volatility typically independent of each other. Specifically, we consider both the cross-correlations and the cross-autocorrelations for capturing the 'instantaneous' and the 'intertemporal' leverage effects. We further incorporate the asymmetric nature of the leverage effect for the cross-correlations and the cross-autocorrelations into the modeling framework. Our results point to the sizeable economic value of exploiting the contemporaneous link between returns and volatility explicitly. Specifically, a mean-variance investor would be willing to pay several hundred basis points to switch from the strategies based on conventional predictive regressions of mean and volatility in isolation to the joint models of returns and its volatility, taking the link between these two moments into account.

Considering the predictability of the returns, i.e., market timing, there is a consensus that the extent of the predictability of returns increases with lower frequencies. Therefore, the predictability remains absent at high frequencies such as daily returns. However, the dispute is mainly on the predictability of the returns at lower frequencies such as monthly or quarterly returns as these are closely related to the accounting ratios and macroeconomic fundamentals, see Welch and Goyal (2007) among others, for example on the non-predictability and Pesaran and Timmermann (1995); Cremers (2002); Campbell and Thompson (2008); Rapach et al. (2010), on the evidence of profitable strategies deduced from predictive models of market timing.

Considering the predictability of returns' volatility, i.e., volatility timing, there is a consensus on the predictability of the volatility mostly stemming from the persistence in the second moment of returns. Fleming et al. (2001, 2003) show that the statistical significance of predictability of volatility could indeed be translated into profitable strategies. Specifically, the latter study analyzed the economic value of realized volatility, often considered as the observed value of volatility, see Barndorff-Nielsen and Shephard (2002); Andersen et al. (2003); Hansen and Lunde (2006) for example. Many studies confirm these findings of economic value in timing the volatility with further modifications, taking various stylized facts such as persistence and potential jumps in intraday equity prices.

First, volatility is quite persistent and exhibits long-memory as modeled explicitly using the fractionally integrated GARCH model of Baillie et al. (1996). For the specific case of realized volatility, Corsi (2009); Corsi et al. (2012) propose Heterogeneous Autoregressive model of realized volatility (HAR) and demonstrates that it can successfully capture the long memory behavior of the volatility. Second, we observe a leverage effect of returns, most notably of sizable negative returns, elevating the volatility of the returns. The asymmetric link between returns and volatility is well documented in many studies, see Nelson (1991); Glosten et al. (1993); Engle and Ng (1993); Engle and Siriwardane (2017) for example, in terms of the GARCH framework and Andersen et al. (2001); Bollerslev et al. (2006); Scharth and Medeiros (2009); Jin and Maheu (2012); Aït-Sahalia et al. (2013); Jensen and Maheu (2014); Anatolyev and Kobotaev (2018) in terms of realized measures. A common feature of these studies is defining the leverage effect as the relation between past (negative) returns and the current volatility. A key driving source of this phenomenon might be the increased risk perception due to increasing financial leverage (trough the changing debt-to-equity ratio) by the sizable drop in the asset price.[1] Bollerslev et al. (2006) analyze the leverage effect focusing on the correlations between the (realized) volatility and the lagged returns as well as current returns. They conclude that, at the daily frequency, the leverage effects are present not only between current volatility and lagged returns but between current volatility and current returns predominantly. Moreover, a model allowing for asymmetry for the contemporaneous relation between returns and volatility and the intertemporal relation can account for the joint distribution of the returns and volatility much better than other models where this asymmetric link is missing.

We evaluate the robustness of our findings on the sizeable economic value of our joint modeling strategy using various extensions. First, several studies document the presence of jumps in the intraday prices that may obscure the measurement of volatility using the realized volatility measure, see for example Barndorff-Nielsen and Shephard (2005). Indeed, Nolte and Xu (2015) document the economic gains of modeling volatility and jumps separately using the bipower variation for the jump robust measure of volatility, see Barndorff-Nielsen and Shephard (2004). We show that the economic value of the proposed joint modeling strategy remains prevalent in case we explicitly model jumps and volatility together with returns for capturing the joint distribution of these components. Second, as the leverage effects are driven chiefly by significant negative returns, the timing for the downside risk by modeling the semivariance for negative and positive returns disjointly might capture a bulk of the documented economic gains. Our results are robust when we consider the joint distribution of returns together with negative and positive semivariances. Finally, we consider principal determinants of the volatility documented in earlier research, see for example Christiansen et al. (2012) for a comprehensive analysis, to scrutinize whether these sources of volatility might explain part of the leverage effects. Our results verify the robustness of the findings in the presence of various determinants of volatility as well.

[1] An alternative explanation relies on time-varying risk premium. This variation reflects the idea of volatility feedback effects implying the decline in prices due to an anticipated increase in volatility. While the leverage effect postulates that a negative return leads to higher volatility, the volatility feedback effect refers to the case that an anticipated increase in volatility may result in negative returns.

Our econometric inference relies on a simple recursive estimation strategy similar to a seemingly unrelated regression framework using observed returns and (realized) volatility. While this facilitates the inference substantially, it also enables to mimic an investor who re-balances her portfolio by daily econometric inference. Alternatively, we could also utilize a more intensive econometric framework using a stochastic volatility model that would require only the observed returns for extraction of volatility and joint modeling of this with returns in a state-space framework. Note that, in this case, the presence of two sources of leverage effects as instantaneous and intertemporal might plague econometric inference and pose challenges on the identification of volatility. Furthermore, this strategy would also require an intensive computation involving a particle filter or an approximation using mixtures of Normals, see Omori et al. (2007), that might not be feasible for a recursive analysis as ours.

The remainder of this paper is as follows. Section 2 displays the data and discusses various stylized facts on the realized volatility. Section 3 presents model specifications and the method for economic evaluation. Section 4 evaluates the empirical results and provides a detailed robustness check. Finally, Section 6 concludes.

2 Data and stylized facts

In our analysis, we use the daily S&P 500 market index as the primary financial asset. This choice of the dataset facilitates the comparison of our findings with the previous analysis. The sample consists of daily observations from January 2000 to December 2019. We exclude the (hopefully) unique and extraordinary periods of the Covid-19 pandemic to uncover our modeling strategy's potential without the effects of the pandemic. We use returns on excess to the risk-free rate, where for the risk-free rate, we use the 3-month US T-bill rate. For the dates when the stock market is open, but the bond market is closed, we used the value of the last trading day in the bond market. Realized measures used in the analysis are obtained from the Oxford-Man Institute's realized library, see Heber et al. (2009). We provide details on these measures in the next section.

2.1 Realized volatility

The use of realized volatility (RV) for the *ex-post* measure of variance relies on the theory of quadratic variation. Specifically, let p_t be the (logarithm) of the asset price that follows a diffusion process as

$$p_t = \int_0^t \mu(s)ds + \sigma(s)dW(s) \tag{1}$$

where $\mu(s), \sigma(s)$ are drift and diffusion parameters, and $dW(s)$ follows a standard Brownian motion. Given the diffusion process, the integrated variance is computed as

$$dp_t = \int_{t-1}^t \sigma^2(s)d(s) \tag{2}$$

which can be approximated by intraday returns to construct a daily measure of volatility. Consider the intraday returns as $r_{t,j} = p_{(t-1)h+h\frac{j}{M}} - p_{(t-1)h+h\frac{j-1}{M}}$ observed in day t, where M is the number of intraday returns during the day, j is the specific time in day t, and h is the sampling interval. Realized volatility (RV) measured for the day t can be computed as

$$RV_{t,M} \quad = \quad \textstyle\sum_{j=1}^{M} r_{t,j}^2 \tag{3}$$

Given the diffusion process as in (1) the integrated variance can be approximated by the realized volatility as $M \to \infty$, see Andersen et al. (2001); Barndorff-Nielsen and Shephard (2002), for further details. The choice of the intraday frequency is crucial since there is a trade-off in choosing the right intraday frequency. Increasing the frequency often contaminates the estimator due to market microstructure noise-related biases, but, in return, it leads to a larger number of observations for estimation, and thus, to efficiency gains. Following the common practice, we use the measures computed using the 5-minutes intraday returns. Furthermore, we use the estimator averaged across the estimators, each of which is computed using different subsets constructed by subsampling, to mitigate the effects of market microstructure noise, see Zhang et al. (2005); Shephard and Sheppard (2010) for details. Finally, we use the logarithm of volatility in our predictive regressions as this is approximated nicely by a normal distribution, see Andersen et al. (2001). In Figure 1, we display the returns and (logarithm of) realized volatilities in our sample period.

[Insert Figure 1 about here]

As can be seen from Figure 1, considerable variation in returns is accompanied by the increasing values of realized volatility. We see this link most precisely during the collapse of Lehman Brothers amid the recent recession of 2008-9 highlighted by the grey shaded area.

2.2 Leverage effects

The intertemporal and contemporaneous impact of returns on volatility, denoted as the leverage effect, has long been documented in many studies; see Anatolyev and Kobotaev (2018) for a recent analysis. We first display the (conditional) correlations between volatility and contemporaneous and lagged returns in Figure 2.

[Insert Figure 2 about here]

The top panel shows the cross-autocorrelations for the entire sample of the returns. For a refiner snapshot of the correlation structure between returns and volatility, the middle (bottom) panel shows these for the subsample of only the negative (positive) return realizations. While we discuss the regression specifications for computing the conditional correlations in the next section broadly, these results on conditional correlation structure do not change for other specifications as well. In line with the earlier findings, we observe that the intertemporal leverage effect, as the correlation between the volatility and first lag of returns, is largest when considering the whole sample. Additionally, the instantaneous leverage effect is of similar magnitude as the intertemporal leverage effect, as noted in Bollerslev et al. (2006, 2009). This picture becomes more eminent when we split the sample as positive and negative returns. When we consider negative returns in the middle panel, while we still can observe intertemporal leverage dynamics, we observe a more significant instantaneous leverage effect. We can also observe a similar pattern for positive returns, albeit with reversing signs. In this case, the contemporaneous correlation turns out to be positive for the positive returns, while the intertemporal leverage effect is still negative. It seems that these reversing signs are one of the driving forces of lower values of instantaneous leverage effects compared to the intertemporal leverage effect. Therefore, asymmetry in contemporaneous leverage effects is considerably more pronounced compared to asymmetry in intertemporal leverage effects. Next, we explore whether specific observations drive these asymmetric contemporaneous leverage effects or these effects prevail over the whole sample period. Therefore, we compute these correlations recursively over time, starting from the first 500 observations as the estimation sample and each time adding an observation. Figure 3 displays the evolution of these asymmetric contemporaneous leverage effects throughout our sample period.

[Insert Figure 3 about here]

The correlations in Figure 3 show that overall correlations between all returns and (log-)realized volatility are steady around the values -0.2. Disentangling these correlations for negative and positive returns, we see correlations between positive returns and (log-)realized volatility to increase from values around 0.1 at the beginning of the 2000s to values around 0.2 at the end of our sample period. In contrast, correlations between negative returns and (log-)realized volatility increases to -0.3 in 2019 from values around -0.2 at the onset of our sample. Hence, distinct dynamics of the contemporaneous leverage effects become even more noticeable towards the end of our sample period. Figure 2 and 3 reveal the potential for timing the joint distribution of the returns and its volatility.

3 Model and Economic Evaluation

3.1 Predictive Regressions

We consider one-period ahead prediction of daily returns and volatility based on the linear regression model using the following predictive regressions,

$$r_t^e = \mu + \epsilon_t \tag{4}$$

$$\log(RV_t) = X_t'\beta + v_t \tag{5}$$

where r_t^e is the daily excess return in period t, i.e., $r_t - r_t^f$, $\log(RV_t)$ is the logarithm of daily realized volatility, and X_t is the set of predictors that are available in real-time. For the return prediction, following existing studies, we use the historical mean as predictability of returns is evident more at lower frequencies such as monthly or annual returns, see for example Welch and Goyal (2007); Çakmaklı and van Dijk (2016).

For the timing of the volatility, the set of predictors X_t is selected based on the stylized facts provided in the literature. First, for modeling the long memory of the volatility process, see for example Baillie et al. (1996), we use the Heterogenous AutoRegressive (HAR) model introduced by Corsi (2009) similar to Bollerslev et al. (2009); Maheu and McCurdy (2011); Nolte and Xu (2015). The resulting predictive regression is as follows,

$$\log RV_t = \beta_0 + \beta_1 \log RV_{t-1} + \beta_2 \log RV_{t-1,t-5} + \beta_3 \log RV_{t-1,t-22} + v_t \tag{6}$$

where $RV_{t,t+h} = [RV_t + RV_{t+1} + \cdots + RV_{t+h}]$, see Andersen et al. (2007) for example. Therefore, $RV_{t-1,t-5}$ and $RV_{t-1,t-22}$ are the past weekly and monthly volatility capturing the long memory in the volatility process. This model, together with (4) is denoted as the HAR model, where both the return and volatility process is evaluated independently of each other. Many of the existing studies measuring the economic value of volatility timing fall in this category see, for example, Fleming et al. (2003); Marquering and Verbeek (2004); Nolte and Xu (2015) among others, and it serves as our primary benchmark model.

We enhance the model with various forms of leverage effects. First, taking the instantaneous leverage effects into account, we proceed with a joint estimation considering the correlation between the return and volatility innovations as $\text{Corr}(v_t, \epsilon_t) = \rho$, where Corr() implies the correlation coefficient between the two variables shown inside the parenthesis. By doing so, we extend the models of measuring the economic value of market timing, see Pesaran and Timmermann (1995), and volatility timing, see Fleming et al. (2003), by accounting for the joint distribution of returns and volatility rather than considering the first and second moments independently as in Marquering and Verbeek (2004). This joint distribution perspective implies a system estimation of a Seemingly Unrelated Regressions (SUR) model taking the cross-correlation into account. This model is denoted as the JHAR-I model.

The middle and bottom panels in Figure 2 reveal the distinct patterns of contemporaneous correlations for positive and negative returns. Accordingly, we extend the SUR system of (4)-(6)

allowing for distinct correlations for negative and positive returns as follows,

$$\begin{aligned} \text{Corr}(v_t, \epsilon_t) &= \rho^- \quad \text{if} \quad r_t^e \leq 0 \\ \text{Corr}(v_t, \epsilon_t) &= \rho^+ \quad \text{if} \quad r_t^e > 0. \end{aligned} \tag{7}$$

This Asymmetric Joint model is denoted as the AJHAR-I model. Comparing the JHAR-I model with the HAR model would reveal the economic value of the instantaneous leverage effect. A further comparison of the AJHAR-I model with the JHAR-I model would measure the economic value of taking the asymmetric nature of the instantaneous leverage effect into account.

Next, we consider the intertemporal leverage effect by extending the specifications with the first lag of returns. Notice that the intertemporal leverage effect is often considered as the sole source of leverage; see the discussion for example in Bollerslev et al. (2006), see Christiansen et al. (2012) for a forecasting exercise. The resulting volatility equation is as follows,

$$\begin{aligned} \log(RV_t) &= \beta_0 + \beta_1 \log(RV_{t-1}) + \beta_2 \log(RV_{t-1,t-5}) + \beta_3 \log(RV_{t-1,t-22}) \\ &+ \beta_4 r_{t-1}^e + v_t. \end{aligned} \tag{8}$$

We denote the system equations together with (8) as JHAR-II and AJAR-II for both cases of joint modeling and returns and their volatility. Finally, considering the asymmetry in the intertemporal leverage effects, the last specifications involve the absolute returns in the spirit of the E-GARCH model of Nelson (1991) as follows,

$$\begin{aligned} \log(RV_t) &= \beta_0 + \beta_1 \log(RV_{t-1}) + \beta_2 \log(RV_{t-1,t-5}) + \beta_3 \log(RV_{t-1,t-22}) \\ &+ \beta_4 r_{t-1}^e + \beta_5 |r_{t-1}^e| + v_t. \end{aligned} \tag{9}$$

We denote the system equations together with (9) as JHAR-III and AJAR-III. On the one hand, comparing (.)HAR-II systems with (.)HAR-I counterparts illustrates the economic value of the intertemporal leverage effect. On the other hand, comparing (.)HAR-III systems with (.)HAR-II counterparts would further uncover the value-added of modeling asymmetry in the intertemporal leverage effect.

3.2 Economic Evaluation

The models described in the previous section are estimated on the day t using the available information to form the predictions of stock return and its realized volatility for the next day, $t+1$. To evaluate the economic value of competing models, we follow the framework in Fleming et al. (2003); Marquering and Verbeek (2004) that draws on West et al. (1993) for portfolio allocation using these predictions performed each day by adding the new observations recursively. Briefly, we assume a representative risk-averse investor with mean-variance preferences, managing a portfolio consisting of stocks and risk-free T-bills. Each day the investor decides upon the fraction of wealth invested in stocks using the objective function

$$\max_{w_{t+1}} \quad \mathsf{E}_t(r_{p,t+1}) - \frac{1}{2}\gamma \mathsf{Var}_t(r_{p,t+1}) \tag{10}$$

where $\mathsf{E}_t(r_{p,t+1})$ and $\mathsf{Var}_t(r_{p,t+1})$ are the expected value and the variance of the portfolio return in period $t+1$ conditional on the information in period t. The portfolio return is given by

$$r_{p,t+1} = r_{f,t+1} + w_{t+1} r_{e,t+1}, \tag{11}$$

where $r_{f,t+1}$ denotes the risk-free return, $r_{e,t+1}$ is the excess stock return and w_{t+1} is the fraction of wealth allocated to stocks in period $t+1$. Optimizing her utility the investor solves the maximization problem in (10) at the end of the day in order to determine the portfolio weights for the next day. Assuming the risk-free rate $r_{f,t+1}$ is fixed, the optimal portfolio weight for stocks is given by

$$w^*_{t+1} = \frac{E_t(r^e_{t+1})}{\gamma \mathsf{Var}_t(r_{t+1})}, \tag{12}$$

where the investor uses predictions from competing models to form the expectations in (12). The investor further imposes restrictions on the portfolio weights. In the first scenario, the investor restricts the portfolio weights as $w^*_{t+1} \in [-1, 2]$, thus allowing for limited short-sales. In the second scenario, short-sales are not allowed at all, thus $w^*_{t+1} \in [0, 1]$.

For measuring the economic performance of competing models, we can use the average realized utility to estimate the expected utility consistently as,

$$\frac{1}{T}\sum_{t=0}^{T-1}\left(r_{p,t+1}-\frac{\gamma}{2}w_{t+1}^{*2}RV_{t+1}\right) \tag{13}$$

where the initial wealth is normalized to be unity.[2]

We assess the economic value of predicting the joint distribution of returns and volatility by computing a 'performance fee' that an investor should be willing to pay for switching from one investment strategy to another. For example, suppose that a dynamic portfolio constructed using one of the benchmark models yields the same average utility as a dynamic portfolio constructed using the predictive model of joint distribution subject to annual expenses, Δ, expressed as a fraction of wealth invested. Because the investor would be indifferent between the two strategies, we can interpret Δ as the maximum fee an investor should be willing to pay to switch from the benchmark strategy to the strategy based on the joint distribution of returns and volatility. The fee is given by the value of Δ that satisfies

$$\sum_{t=0}^{n-1}\left((r_{p,t+1}^{a}-\Delta_{a,b})-\frac{\gamma}{2}w_{a,t+1}^{2}RV_{t+1}^{a}\right)=\sum_{t=0}^{n-1}\left(r_{p,t+1}^{b}-\frac{\gamma}{2}w_{b,t+1}^{2}RV_{t+1}^{b}\right), \tag{14}$$

where the superscripts a and b denote the alternative strategies.

We compare the economic value of the predictive models of the joint distribution of returns and volatility relative to the HAR model, where we predict returns and volatility independent of each other. Additionally, we also consider three buy-and-hold strategies. These strategies assume that the investor invests a constant fraction of 100% (0%), 50% (50%), or 0% (100%) of her wealth in stocks (risk-free T-bills), respectively. The strategy with 50% weights provides an appealing competitor because it provides a simple but quite realistic strategy pursued by investors. Finally, in line with conventional investment performance comparison metrics', we also compute average returns, standard deviations, and Sharpe ratios for each portfolio.

[2]Alternatively, we could use the portfolio variance as $Var_t(r_{p,t+1})$. Results are similar in both cases and available upon request by the authors.

4 Empirical Results

Table 1 displays the economic performance of competing models in the absence of transaction costs together with the relative risk aversion γ equal to 6. We consider alternative risk aversion levels and the presence of transactions costs in the next section of robustness checks. The models include passive buy-and-hold strategies and active strategies that exploit predictions of the return and volatility distribution independently or jointly. The investor uses the first 500 observations as the training sample and uses the predictions for the next day to compute the optimal portfolio weight in (12). These predictions are repeated each day, adding the past realized returns and volatilities to the training sample in a recursive manner.

[Insert Table 1 about here]

The first three rows of Table 1 provide the (annualized) mean return and volatility of the static buy-and-hold strategies together with their resulting Sharpe ratios. As expected, the portfolio with equal weights for both risky and risk-free asset provides a trade-off between a reduced mean return compared to the market return in response to the reduced risk resulting from lowering volatility. When we consider the first scenario where limited short sales are allowed in Panel A of Table 1, the HAR model, predicting return and volatility independently, does not provide many gains. Although the volatility of this portfolio is reduced considerably due to the volatility timing ability of the predictive model, the mean return of the portfolio remains limited, leading to a lower Sharpe ratio compared to static strategies. Still, reduced risk due to volatility timing translates into a favorable performance fee, computed as in (14), when we compare the HAR model's performance to that of the static 100% market portfolio. However, these performance fees become negative or negligible when comparing the HAR model with the less risky static portfolios of 50% market and 0% market portfolios. In this case, volatility timing does not contribute much as these static portfolios already bear low risk.

When we consider the system allowing for instantaneous leverage between returns and realized volatility, we observe only a minor improvement on top of the case with the assumption of independence between returns and volatility. When we compare the J-HAR-I model with the HAR model, which is in the last column of Table 1, we see that the performance fee is almost zero with a value of -0.01 annualized basis points, implying essentially no difference in the economic value of predictions. Summating the intertemporal leverage effect using the J-HAR-II model, we observe a slight improvement in the performance fee for switching from the HAR model to the J-HAR-II model with a 12.01 basis points performance fee. When we further model the asymmetric nature of the intertemporal leverage effect using the J-HAR-III model, displayed in (9), the performance fee increases to only 15.66 basis points. These findings indicate that the intertemporal leverage effect, regardless of whether the asymmetric nature is captured or not, does not provide any significant economic value on top of the predictive regression of independent market timing and volatility timing. Instantaneous leverage effects also do not provide any additional economic value. However, notice that the J-HAR(.) type of models still does not take the asymmetry in instantaneous leverage effects into account.

When we focus on the models that take the asymmetric nature of the instantaneous leverage effect, we observe pretty striking results. The performance fees for switching from the HAR model assuming independent market and volatility timing, to the AJ-HAR models allowing for asymmetric cross-correlations, soar to as high as 270 basis points. This increase is regardless of whether the asymmetry in the intertemporal leverage effects is dealt with or not. This superior performance of the AJ-HAR models stems from the fact that the mean return of these portfolios improves substantially compared to the mean return obtained using the predictions of the HAR model or JHAR models. The volatility of these portfolios also increases, albeit more limited compared to the increase in mean returns. In response, Sharpe ratios rise to values around 0.53 which are larger than those of the static buy-a-hold strategies and the HAR and J-HAR models. These results show that instantaneous leverage effects bear considerable economic value when adequately modeled, taking the asymmetry into account, but the same does not apply to intertemporal leverage effects. The superior performance of the AJ-HAR models endures against the static buy-and-hold strategies as well. In this case, the performance fees rise to as high as 350 basis points for switching from the 100% market portfolio to the AJ-HAR models and as high as 270 basis points for switching from the 0% market portfolio to the AJ-HAR models. Finally, we can also trace favorable performance fees for switching from the 50% market portfolio to the AJ-HAR models, albeit relatively limited. The performance fees for this case reduces to around 120 basis points, which is still highly sizeable.

Panel B of Table 1 displays the results when the portfolio weights are restricted between 0 and 1 for not allowing for short-sales. Notice that such economic restrictions on the investment strategies also assist in reducing the effects of significant forecasting errors, see for example Barberis (2000); Campbell and Thompson (2008) with a trade-off of not fully exploiting successful predictions. This trade-off is also reflected in the Sharpe ratios and performance fees. In this case, volatility timing solely suffices to improve the portfolio performance because the Sharpe ratio of the HAR model increases to 0.35 from 0.18 in the previous case when limited short sales are allowed. This increase is mainly due to the considerable reduction in the portfolio standard deviations. We also see a similar effect when we focus on performance fees. The performance fee for switching from the 100% market portfolio to the HAR model is boosted to around 200 basis points. Additionally, the small performance fees for switching from the 0% market portfolio turn out to be almost 120 basis points. However, the 50% market portfolio still outperforms the portfolio constructed using the predictions of the HAR model, albeit with a pretty poor performance fee around 30 basis points, unlike the previous case.

As in the previous case, there is no improvement when the intertemporal leverage effect and its asymmetric nature are utilized together with the direct instantaneous leverage effects, as shown in the last column of Panel B. When the portfolio weights are restricted to the interval between 0 and 1, the performance fees for switching from the HAR model to J-HAR type models are small and mostly negative. On the contrary, the superior performance of the AJ-HAR models, where we model the asymmetric nature of the instantaneous leverage effects, prevails when the portfolio weights are restricted to be between 0 and 1. The performance fees for switching from the predictions produced by the HAR model to those of the AJ-HAR models are around 200 basis points, which is considerably sizeable, although it implies an around 70 basis points reduction compared to the models when short-sales are allowed. This reduction is because the efficient predictive performance of the AJ-HAR models is not fully exploited in the case where we do not allow for short-sales at all. Nevertheless, this restriction on the portfolio weights pays off when AJ-HAR models' economic performance is compared to the static buy-and-hold strategies. In this case, a mean-variance investor is willing to pay 170 basis points for switching from the 50% market portfolio to the portfolio constructed using the predictions of the AJ-HAR models. The performance fees increase to 400 and 320 basis points when we consider 100% and 0% static market portfolios. Overall, the results in Table 1 show that the timing for asymmetric instantaneous leverage effects provides considerable economic gains to an investor regardless of whether short-sales are allowed or not and regardless of whether the competing strategy involves conventional dynamic strategies or static buy-and-hold strategies.

4.1 Robustness checks

4.1.1 Value of risk aversion parameter

The results displayed in Table 1 are computed for a representative risk-averse agent with a risk aversion parameter, $\gamma = 6$. This value often reflects the risk aversion degree of a typical investor; see, for example, Marquering and Verbeek (2004). This section displays our findings for a mildly and excessively risk-averse agent for $\gamma = 2$ and 10, respectively, in Table 2.

[Insert Table 2 about here]

The results reveal the impact of the risk aversion parameter value. For a mildly risk-averse investor with $\gamma = 2$, excess volatility is hardly penalized. Therefore, for the models with volatility timing either independent of the return process or by just considering the direct contemporaneous and intertemporal leverage effects, the performance fees compared to the static 100% market portfolio are negative and sizable when short-sales are allowed. Notice that the static 100% market portfolio has a high return, but it also has the highest volatility. Therefore, for a risk-averse investor with $\gamma = 2$, the high return is awarded more than the penalty for the high volatility, and consequently, the 100% market portfolio is the most successful static buy-and-hold strategy. Nevertheless, the dynamic models that capture asymmetric contemporaneous leverage effects continue to perform better. In this case, while the volatility of these portfolios soars, so do the returns. Because the penalty for volatility is less than the award for returns in absolute terms, the overall performance of these dynamic strategies enhances compared to the case $\gamma = 6$. Notice that when volatility is mildly penalized, it provides more room for better timing of the volatility. This better timing is indeed what we observed in our findings. When $\gamma = 2$ a mean-variance investor is willing to pay up to 780 basis points for switching from the HAR models, where returns and volatility are modeled independently, to the AJHAR models, where we consider asymmetric contemporaneous leverage effects. This sizable performance fee implies that for a mildly risk-averse investor, the AJHAR models are worth almost three times more than a typical risk-averse investor. When we do not allow for any short sales restricting the portfolio weight in the unit line, this outperformance is curbed partially. In this case, restricting portfolio weights lead to the inability to exploit the timing of volatility in portfolio performances fully.

For an excessively risk-averse investor with $\gamma = 10$, excess volatility is overly penalized. As a result, the static 50% and 0% market portfolios perform best due to their low volatility. For the models with volatility timing either independent of the return process or by just considering the direct contemporaneous and intertemporal leverage effects, the performance fees compared to these static portfolios are minuscule in the case when short-sales are allowed. In this case, the over-penalization of the volatility leads to under-performance, including the ones that also consider asymmetric contemporaneous leverage effects. The 100% market portfolio that bears high volatility is the only strategy beaten by all the dynamic portfolio strategies as expected. When we do not allow for any short-sales restricting the portfolio weight in the unit line, this restriction pays off, unlike the previous case when $\gamma = 2$ as it limits excess volatility in the portfolios. The performance fees for switching from the static 50% and 0% market portfolios to the dynamic strategies exploiting asymmetric contemporaneous leverage effects increase to around 150-200 basis points. Moreover, the performance fees for switching from the dynamic strategies that do not exploit asymmetric contemporaneous leverage effects to those capturing asymmetric contemporaneous leverage effects increase to around 75-100 basis points. These results indicate that our findings on the economic value of the leverage effects are robust to the risk aversion parameter of the representative investor.

4.1.2 Transaction costs

In Table 1 and 2 we display our findings in the absence of transaction costs. While the absence of these costs might have little impact on relative performances of dynamic strategies, it might have a considerable impact on the performance fees of dynamic strategies relative to buy-and-hold strategies. Therefore, in this section we evaluate model performances in the presence of performance fees. Specifically, at the start of day $t + 1$, the investor changes the allocation to stocks from w_t to w_{t+1} when new predictions are available. We assume that transaction costs are a fixed proportion c of the wealth invested, such that the portfolio return is reduced by

$$c_{t+1} = 2c|w_{t+1} - w_t|, \tag{15}$$

where the multiplication by 2 follows from the fact that the investor rebalances her positions in both stocks and the risk-free T-bills. We follow the practice in Fleming et al. (2003) and we assume that $c = 0.01$ in (15).[3] We display our findings in Table 3.

[Insert Table 3 about here]

Table 3 indicates that the performance fees for switching to dynamic strategies from static buy-and-hold strategies reduce on average 50 basis points when limited short sales are allowed. This reduction reflects the effects of the transactions costs incurred every day when rebalancing portfolio weights as in (15). When portfolio weights are restricted to lie in the unit line, rebalancing portfolio weights is also restricted. Consequently, the reduction in the performance fees drops to values around 20 basis points as the transactions are limited.

When we compare the performance fees for switching from the dynamic strategies that do not exploit asymmetric contemporaneous leverage effects to those capturing asymmetric contemporaneous leverage effects in Tables 3 and 1, we see that the fees are very close in both cases. These identical fees show that successful timing for asymmetric instantaneous leverage effects is not at the price of higher transaction costs.

5 Testing for alternative explanations

In the previous section, we provide strong evidence for the economic value of the timing of the joint distribution of the returns and volatility. This section extends our analysis to ensure that the documented economic value is due to the leverage effects rather than other potential underlying explanations that leverage effects might capture.

[3] Fleming et al. (2003) compute the one-way transaction cost in terms of the percentage of the transactions as 2.56% in annualized terms. These costs correspond roughly to 0.01% for the value of the parameter c.

5.1 Effect of jumps

We first consider potential jump effects. To elaborate further, we modify the diffusion process of the prices to incorporate jumps as follows,

$$p_t = \int_0^t \mu(s)ds + \sigma(s)dW(s) + \sum_{j=1}^{N(t)} \kappa(s_j), \tag{16}$$

where $\sum_{j=1}^{N(t)} \kappa(s_j)$ is the additional jump process with $\kappa(s_j)$ is the jump size and $N(t)$ is the counting process. In this case, realized volatility measure approximates

$$\lim_{M\to\infty} RV_{t,M} = \sigma^2(s)d(s) + \textstyle\sum_{j=1}^{N(t)} \kappa^2(s_j). \tag{17}$$

For an accurate measure of the integrated variance in the presence of jumps, we use Bipower Variation (BV) as a jump robust estimator of the volatility, see Barndorff-Nielsen and Shephard (2004, 2005); Barndorff-Nielsen et al. (2006), for details. BV measured for the day t can be computed as

$$BV_{t,M} = \tfrac{\pi}{2} \textstyle\sum_{i=2}^{M} |r_{t,i}||r_{t,i-1}|. \tag{18}$$

For taking volatility and jumps explicitly into account, we restructure the system by replacing the realized volatility measure with the measure of bipower variation in (18) and by including a further equation for capturing jump effects as follows,

$$\begin{aligned} \log BV_t &= \beta_0 + \beta_1 \log BV_{t-1} + \beta_2 \log BV_{t-1,t-5} + \beta_3 \log BV_{t-1,t-22} + v_t \\ \log\left(\tfrac{RV_t}{BV_t}\right) &= \gamma_0 + \textstyle\sum_{j=1}^{5} \log\left(\tfrac{RV_{t-j}}{BV_{t-j}}\right) + u_t. \end{aligned} \tag{19}$$

While long memory is captured in the volatility equation as before, the jump process is approximated using an AR(5) model as jumps bear much shorter memory, see Bollerslev et al. (2009); Maheu and McCurdy (2011) for a similar system of equations. In this case, the plain model where we use (4) and (19) i.e., returns, volatility, and jump processes are estimated independently, is called as HARBV model following the previous nomenclature. Accordingly, the equation system estimated jointly, taking the instantaneous leverage effect into account, is called the J-HARBV model. The models where we explicitly consider the asymmetry in the instantaneous leverage effect are called the AJ-HARBV models. We display the results in Table 4.

[Insert Table 4 about here]

The results in Table 4 are in accordance with those in Table 1. First, intertemporal leverage effects, either when asymmetry is captured or not, do not bear any significant information translated into profitable strategies. Second, the timing for asymmetric contemporaneous leverage effects pays off in the presence of jumps as well. In this case, the performance fees are around 130 basis points for switching to the portfolios constructed using the model capturing complete characteristics of the joint distribution of returns, volatility, and jumps from other dynamic strategies that do not time to these characteristics. When we allow for limited short-sales, it increases further to around 160 basis points. These results indicate that the economic value of the timing for the joint distribution of the returns and volatility is genuine, and it is not because of indirectly timing for the jump effects.

5.2 Timing for the downside risk

Next, we focus on whether the documented economic value of the leverage effects essentially stems from timing for the downside risks. That would imply that successful timing for the downside risk, i.e., prediction of the semivariance for the negative returns, is reflected as timing for the asymmetric instantaneous and intertemporal leverage effects, which to most extent emerge in case of negative returns. Accordingly, we replace the volatility equation with equations for semivariances to capture upside and downside risk separately with the following equations,

$$\begin{aligned} \log SV_t^- &= \beta_0 + \beta_1 \log SV_{t-1}^- + \beta_2 \log SV_{t-1,t-5}^- + \beta_3 \log SV_{t-1,t-22}^- + v_t \\ \log SV_t^+ &= \gamma_0 + \gamma_1 \log SV_{t-1}^+ + \gamma_2 \log SV_{t-1,t-5}^+ + \gamma_3 \log SV_{t-1,t-22}^+ + u_t. \end{aligned} \tag{20}$$

The semivariances are computed as $SV_t^- = \sum_{i=1}^M r_{t,i}^2 I_{[r_t<0]}$ and $SV_t^+ = \sum_{i=1}^M r_{t,i}^2 I_{[r_t>0]}$ where $I_{[A]}$ is an indicator function taking the value 1 if the argument A holds and 0 otherwise. We construct the final predictions of the volatility by adding up the predictions of the semivariances obtained using (20). The plain model where we use (4) and (20) i.e., the three equations of return and semivariances are estimated independently, is called as HARSV model. Accordingly, the equation system estimated jointly considering instantaneous leverage effect is called J-HARSV models. The models where the asymmetry in the instantaneous leverage effect is explicitly modeled are called as AJ-HARSV models. We display the results in Table 5.

[Insert Table 5 about here]

The results in Table 5 are qualitatively in line with those in Table 1. A minor difference is about the intertemporal leverage effects. It seems that modeling the semivariances pays off moderately in the sense that incorporating the asymmetric intertemporal leverage effects amounts to a performance fee of 40 basis points compared to the plain HARSV model when limited short sales are allowed, as can be seen in the last column of Panel A. Confirming our main findings in the previous sections, we observe a significant increase in the performance fees when we time for asymmetric contemporaneous leverage effects even if we model semivariances separately. In this case, in the presence of limited short-sales, the performance fees are around 220 basis points for switching to the portfolios constructed using the model capturing complete characteristics of the joint distribution of returns and volatility from other dynamic strategies that do not time to these characteristics. When short-sales are not allowed, this fee reduces to values around 170 basis points. These results indicate that the economic value of the timing for the joint distribution of the returns and volatility is genuine and still survives even when we model downside risk explicitly.

5.3 Observable factors of volatility

While (9) captures various stylized facts on volatility, Christiansen et al. (2012), for example, show that many variables might have additional information on predicting volatility. Therefore, we also extend our baseline model with those variable sets that proved useful in predicting volatility. The first set of variables includes the dummy variables capturing the effects of the negative returns in the slope and the intercept. By doing so, we allow for potential changes in the long-run level of volatility through the intercept. Second, we add several financial factors and measures of risk to the volatility equation (9). These include risk-free rate, Fama-French factors, short-term reversal factor, which is related to the VIX index, and various spreads as proxies of risk, including term spread (10-year minus 3-month US sovereign yields), default spread (Moody's BAA and AAA bonds yields) as a credit risk proxy and finally the TED spread (the spread between 3-month LIBOR and 3-month T-bill) to proxy liquidity risk. We gather these variables in the vector F_t for period t observations. Third, we include a dummy variable representing the days of the week, following the evidence in Martens et al. (2009) documenting daily seasonal patterns of volatility. Specifically, they argue that volatility has a U-shaped curve with a minimum on Wednesdays. Therefore, the day dummies are the centralized dummy variables as $D^*_{st} = D_{st} - D_{3t}$ where $D_{st} = 1$ if day t is the weekday s ($s = 1$ for Monday,..., $s = 5$ for Friday). We replace (9) with the following equation including these additional factors as,

$$\begin{aligned} \log(RV_t) &= \beta_0 + \beta_1 \log(RV_{t-1}) + \beta_2 \log(RV_{t-1,t-5}) + \beta_3 \log(RV_{t-1,t-22}) \\ &+ \beta_4 r_{t-1} + \beta_5 |r_{t-1}| \\ &+ \beta_6 I_{r_{t-1}<0} + F_{t-1}\beta_7 + \beta_8 D^*_{1t} + \beta_9 D^*_{2t} + \beta_{10} D^*_{4t} + \beta_{11} D^*_{5t} + v_t \end{aligned} \tag{21}$$

In this case, the plain model where we use (4) and (21) is called as HARX model following the previous nomenclature. Accordingly, the equation system estimated jointly, taking the instantaneous leverage effect into account, is called the J-HARX model. The models, where we explicitly consider the asymmetry in the instantaneous leverage effect, are called the AJ-HARX models. We display the results in Table 6.

[Insert Table 6 about here]

When we focus on the performance of the HARX relative to the static buy-and-hold strategies in case short-sales are allowed, as shown in the first row of Table 6, we observe that in general, performance fees are 20 basis points larger than the performance fees of the plain HAR model relative to the static strategies, as shown in the first row of Table 1. This difference indicates that the additional factors indeed improve the economic value of volatility prediction, albeit only marginally. This minor difference completely vanishes when we restrict the portfolio weight in the unit interval. In this case, there remains no room to exploit these additional gains. When we consider the models capturing leverage effects to the HARX model with independent return and volatility predictions, we see that the results are qualitatively similar with a substantial economic value of the AJ-HARX models, where asymmetric instantaneous leverage effects are explicitly modeled. We only observe some slight deterioration in the performance fees for switching from the HARX model to the AJHARX models reflecting this marginal increase in the economic value due to additional factors. These results reveal that the economic value of the timing for the joint distribution of the returns and volatility is genuine and still prevails even when we enhance the predictive model of volatility with further well-documented predictors.

6 Conclusion

This paper documents the considerable economic value of timing for the leverage effects between the returns and its volatility, measured using the realized variance metric. Departing from previous studies that provide evidence on the economic value of volatility and market timing independently, we show that there are sizeable economic gains in modeling the joint distribution of returns and volatility by capturing the dependence structure between returns and volatility properly.

Our results underline the importance of modeling contemporaneous leverage effects rather than the intertemporal leverage effects. Specifically, when we capture the contemporaneous leverage effects between returns and volatility distinctly for negative and positive returns, the economic value of this predictive system of regressions soars. In this case, a mean-variance investor would be willing to pay several hundred basis points to switch from the strategies based on conventional predictive regressions of mean and volatility in isolation of each other to the models of returns and its volatility together with asymmetric contemporaneous leverage in addition to the intertemporal leverage effects. Furthermore, our findings are robust to changes in the risk aversion parameter and the presence of transaction costs. We also provide various robustness checks for alternative explanations of these findings, such as the presence of jumps, upside and downside risk, and additional drivers of volatility.

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Tables and Figures

Table 1: Economic performance of active trading strategies with leverage timing

	μ	σ	SR	$\Delta_{100\%}$	$\Delta_{50\%}$	$\Delta_{0\%}$	Δ_{HAR}
100%	7.77	18.30	0.38				
50%	4.34	9.15	0.38				
0%	0.90	0.06					
Panel A: $\mathbf{w} \in [\mathbf{-1}, \mathbf{2}]$							
HAR	2.47	8.52	0.18	79.36	-149.56	3.09	
J-HAR I	2.46	8.51	0.18	79.36	-149.57	3.08	-0.01
J-HAR II	2.58	8.48	0.20	91.37	-137.55	15.09	12.01
J-HAR III	2.61	8.50	0.20	95.02	-133.90	18.74	15.66
AJ-HAR I	9.90	16.86	0.53	349.42	120.49	273.14	270.05
AJ-HAR II	9.34	16.67	0.51	313.35	84.42	237.07	233.98
AJ-HAR III	9.53	16.83	0.51	321.09	92.16	244.81	241.73
Panel B: $\mathbf{w} \in [\mathbf{0}, \mathbf{1}]$							
HAR	2.52	4.58	0.35	195.47	-33.45	119.20	
J-HAR I	2.52	4.58	0.35	195.77	-33.16	119.49	0.29
J-HAR II	2.44	4.55	0.34	187.98	-40.94	111.70	-7.85
J-HAR III	2.45	4.55	0.34	188.61	-40.31	112.33	-6.86
AJ-HAR I	6.71	10.51	0.55	403.72	174.79	327.44	208.24
AJ-HAR II	6.23	10.42	0.51	363.50	134.57	287.22	168.02
AJ-HAR III	6.28	10.53	0.51	363.27	134.34	286.99	167.79

Note: The table displays performance measures for the active mean-variance investment strategies based on daily excess return and (logarithm of) volatility prediction for January 2000 - December 2019. Columns headed μ and σ contain the annualized mean and standard deviation of the portfolio returns (in percent). SR denotes the Sharpe ratio. Columns headed Δ contain performance fees (in annualized basis points) for switching from the strategy indicated by the subscript to the strategy indicated by the corresponding row as in (14) where $\gamma = 6$. The subscript x with x =100, 50, and 0 refer to the static strategies with x% of wealth invested in stocks. The HAR model refers to the model where both the return and volatility processes are evaluated independently. JHAR-I model refers to the model where instantaneous leverage effects are considered by taking the contemporaneous correlation between the return and volatility innovations, i.e., the Joint distribution of returns and volatility innovations, into account. AJHAR-I model refers to the model where Asymmetric instantaneous leverage effects are considered by taking the contemporaneous correlation between the negative and positive returns together with volatility innovations separately into account. JHAR-II is the model where on top of the instantaneous leverage effects, intertemporal leverage effects are considered using (8). JHAR-III refers to the model, where on top of the instantaneous leverage effects, asymmetric intertemporal leverage effects are considered using (9). Finally, AJHAR-II and AJHAR-III are the counterparts of these models where Asymmetry in instantaneous leverage effects are explicitly modeled.

Table 2: Economic performance of active trading strategies with leverage timing and with alternative risk aversion parameter, γ, values

	μ	σ	SR	$\Delta_{100\%}$	$\Delta_{50\%}$	$\Delta_{0\%}$	Δ_{HAR}
Panel A: $\mathbf{w} \in [-\mathbf{1}, \mathbf{2}]$							
$\gamma = 2$							
HAR	2.41	14.39	0.10	-425.87	-273.22	6.61	
J-HAR I	2.41	14.30	0.11	-425.05	-272.40	7.44	0.82
J-HAR II	2.50	14.30	0.11	-414.83	-262.19	17.65	11.04
J-HAR III	2.52	14.30	0.11	-412.43	-259.78	20.05	13.44
AJ-HAR I	13.23	24.01	0.51	358.79	511.44	791.27	784.66
AJ-HAR II	12.08	23.78	0.47	254.15	406.80	686.64	680.02
AJ-HAR III	12.29	24.00	0.47	268.96	421.61	701.44	694.83
$\gamma = 10$							
HAR	2.16	5.57	0.23	603.00	-7.49	17.96	
J-HAR I	2.17	5.57	0.23	603.49	-7.01	18.44	0.48
J-HAR II	2.32	5.58	0.25	617.92	7.43	32.88	14.92
J-HAR III	2.32	5.59	0.25	618.86	8.37	33.82	15.86
AJ-HAR I	7.27	13.10	0.49	602.13	-8.37	17.09	-0.88
AJ-HAR II	6.80	12.92	0.46	574.83	-35.66	-10.21	-28.17
AJ-HAR III	6.89	13.04	0.46	573.33	-37.16	-11.71	-29.67
Panel B: $\mathbf{w} \in [\mathbf{0}, \mathbf{1}]$							
$\gamma = 2$							
HAR	4.03	6.88	0.45	-151.82	0.83	280.66	
J-HAR I	4.03	6.88	0.45	-151.86	0.79	280.63	-0.04
J-HAR II	3.98	6.84	0.45	-155.73	-3.09	276.75	-3.91
J-HAR III	3.97	6.84	0.45	-156.92	-4.27	275.56	-5.10
AJ-HAR I	7.54	13.03	0.51	99.09	251.74	531.58	250.91
AJ-HAR II	7.10	12.99	0.48	57.08	209.72	489.56	208.90
AJ-HAR III	7.15	13.08	0.48	60.00	212.65	492.49	211.82
$\gamma = 10$							
HAR	2.30	3.40	0.41	685.45	74.95	100.40	
J-HAR I	2.30	3.40	0.41	685.71	75.22	100.67	0.27
J-HAR II	2.24	3.38	0.39	679.37	68.87	94.33	-6.08
J-HAR III	2.23	3.38	0.39	679.45	68.96	94.41	-5.99
AJ-HAR I	5.89	8.94	0.56	785.39	174.90	200.35	99.95
AJ-HAR II	5.55	8.94	0.53	761.51	151.01	176.47	76.07
AJ-HAR III	5.60	8.94	0.53	760.51	150.02	175.47	75.07

Note: The table displays performance measures for the active mean-variance investment strategies based on daily excess return and (logarithm of) volatility prediction for January 2000 - December 2019. Columns headed μ and σ contain the annualized mean and standard deviation of the portfolio returns (in percent). SR denotes the Sharpe ratio. Columns headed Δ contain performance fees (in annualized basis points) for switching from the strategy indicated by the subscript to the strategy indicated by the corresponding row as in (14) where $\gamma = 2$ and 10. The subscript x with x =100, 50, and 0 refer to the static strategies with $x\%$ of wealth invested in stocks. The HAR model refers to the model where both the return and volatility processes are evaluated independently. JHAR-I model refers to the model where instantaneous leverage effects are considered by taking the contemporaneous correlation between the return and volatility innovations, i.e., the Joint distribution of returns and volatility innovations, into account. AJHAR-I model refers to the model where Asymmetric instantaneous leverage effects are considered by taking the contemporaneous correlation between the negative and positive returns together with volatility innovations separately into account. JHAR-II is the model where on top of the instantaneous leverage effects, intertemporal leverage effects are considered using (8). JHAR-III refers to the model, where on top of the instantaneous leverage effects, asymmetric intertemporal leverage effects are considered using (9). Finally, AJHAR-II and AJHAR-III are the counterparts of these models where Asymmetry in instantaneous leverage effects are explicitly modeled.

Table 3: Economic performance of active trading strategies with leverage timing and with transaction costs, $c = 0.01$

	μ	σ	SR	$\Delta_{100\%}$	$\Delta_{50\%}$	$\Delta_{0\%}$	Δ_{HAR}
100%	7.77	18.30	0.38				
50%	4.34	9.15	0.38				
0%	0.90	0.06					
Panel A: $\mathbf{w} \in [-\mathbf{1}, \mathbf{2}]$							
HAR	2.04	8.52	0.13	37.08	-191.85	-39.20	
J-HAR I	2.12	8.51	0.13	36.63	-192.30	-39.65	-0.45
J-HAR II	2.16	8.48	0.14	46.22	-182.70	-30.06	9.14
J-HAR III	2.61	8.50	0.15	50.37	-178.55	-25.90	13.29
AJ-HAR I	9.43	16.86	0.51	302.93	74.01	226.65	265.85
AJ-HAR II	8.82	16.67	0.48	261.64	32.71	185.36	224.56
AJ-HAR III	9.02	16.83	0.48	269.31	40.38	193.03	232.23
Panel B: $\mathbf{w} \in [\mathbf{0}, \mathbf{1}]$							
HAR	2.37	4.58	0.32	180.58	-48.34	104.30	
J-HAR I	2.37	4.58	0.32	180.80	-48.12	104.52	0.22
J-HAR II	2.28	4.56	0.30	171.90	-57.03	95.62	-8.68
J-HAR III	2.29	4.55	0.30	172.67	-56.26	96.39	-7.91
AJ-HAR I	6.57	10.51	0.54	390.25	161.32	313.97	209.67
AJ-HAR II	6.06	10.42	0.50	346.80	117.88	270.52	166.22
AJ-HAR III	6.11	10.53	0.49	346.54	117.61	270.26	165.96

Note: The table displays performance measures for the active mean-variance investment strategies based on daily excess return and (logarithm of) volatility prediction for January 2000 - December 2019. Columns headed μ and σ contain the annualized mean and standard deviation of the portfolio returns (in percent). SR denotes the Sharpe ratio. Columns headed Δ contain performance fees (in annualized basis points) for switching from the strategy indicated by the subscript to the strategy indicated by the corresponding row as in (14) where $\gamma = 6$ and $c = 0.01$. The subscript x with x =100, 50, and 0 refer to the static strategies with x% of wealth invested in stocks. The HAR model refers to the model where both the return and volatility processes are evaluated independently. JHAR-I model refers to the model where instantaneous leverage effects are considered by taking the contemporaneous correlation between the return and volatility innovations, i.e., the Joint distribution of returns and volatility innovations, into account. AJHAR-I model refers to the model where Asymmetric instantaneous leverage effects are considered by taking the contemporaneous correlation between the negative and positive returns together with volatility innovations separately into account. JHAR-II is the model where on top of the instantaneous leverage effects, intertemporal leverage effects are considered using (8). JHAR-III refers to the model, where on top of the instantaneous leverage effects, asymmetric intertemporal leverage effects are considered using (9). Finally, AJHAR-II and AJHAR-III are the counterparts of these models where Asymmetry in instantaneous leverage effects are explicitly modeled.

Table 4: Economic performance of active trading strategies with leverage timing in the presence of jumps

	μ	σ	SR	$\Delta_{100\%}$	$\Delta_{50\%}$	$\Delta_{0\%}$	Δ_{HARBV}
Panel A: $\mathbf{w} \in [-\mathbf{1}, \mathbf{2}]$							
HARBV	2.22	8.47	0.16	53.68	-175.74	-22.59	
J-HARBV I	2.31	8.45	0.16	53.61	-175.31	-22.67	-0.07
J-HARBV II	2.30	8.44	0.17	62.26	-166.66	-14.01	8.58
J-HARBV III	2.37	8.43	0.17	72.25	-156.68	-4.03	18.56
AJ-HARBV I	5.59	12.48	0.38	211.27	-17.66	134.99	157.58
AJ-HARBV II	5.40	12.29	0.37	204.19	-24.73	127.91	150.51
AJ-HARBV III	5.62	12.43	0.38	220.17	-8.76	143.89	166.48
Panel B: $\mathbf{w} \in [\mathbf{0}, \mathbf{1}]$							
HARBV	2.45	4.49	0.35	189.51	-39.42	113.23	
J-HARBV I	2.45	4.49	0.35	189.68	-39.25	113.40	0.17
J-HARBV II	2.40	4.49	0.33	184.48	-44.44	108.20	-5.03
J-HARBV III	2.42	4.47	0.34	186.96	-41.97	110.68	-2.55
AJ-HARBV I	4.66	7.64	0.49	326.96	98.04	250.69	137.46
AJ-HARBV II	4.52	7.55	0.48	317.00	88.07	240.72	127.49
AJ-HARBV III	4.60	7.69	0.48	320.24	91.31	243.96	130.73

Note: The table displays performance measures for the active mean-variance investment strategies based on daily excess return and (logarithm of) volatility prediction for January 2000 - December 2019. Columns headed μ and σ contain the annualized mean and standard deviation of the portfolio returns (in percent). SR denotes the Sharpe ratio. Columns headed Δ contain performance fees (in annualized basis points) for switching from the strategy indicated by the subscript to the strategy indicated by the corresponding row as in (14) where $\gamma = 6$. The subscript x with x =100, 50, and 0 refer to the static strategies with x% of wealth invested in stocks. The HARBV model refers to the model where the return, the volatility, and the jump processes are evaluated independently. JHARBV-I model refers to the model where instantaneous leverage effects are considered by taking the contemporaneous correlations, i.e., the Joint distribution of returns, volatility, and jump innovations, into account. AJHARBV-I model refers to the model where Asymmetric instantaneous leverage effects are considered by taking the contemporaneous correlation between the negative and positive returns together with volatility and jump innovations separately into account. JHARBV-II is the model where on top of the instantaneous leverage effects, intertemporal leverage effects are considered. JHARBV-III refers to the model, where asymmetric intertemporal leverage effects are considered on top of the instantaneous leverage effects. Finally, AJHARBV-II and AJHARBV-III are the counterparts of these models where Asymmetry in instantaneous leverage effects are explicitly modeled.

Table 5: Economic performance of active trading strategies with predictive equations of semivariances together with leverage timing

	μ	σ	SR	$\Delta_{100\%}$	$\Delta_{50\%}$	$\Delta_{0\%}$	Δ_{HARSV}
Panel A: $\mathbf{w} \in [-\mathbf{1}, \mathbf{2}]$							
HARSV	2.32	8.87	0.16	52.14	-176.78	-24.14	
J-HARSV I	2.24	8.96	0.15	41.46	-187.46	-34.18	-10.68
J-HARSV II	2.55	8.83	0.19	75.94	-152.98	-0.34	23.80
J-HARSV III	2.69	8.85	0.20	92.05	-136.87	15.77	39.91
AJ-HARSV I	6.72	13.40	0.43	278.06	49.13	201.78	225.92
AJ-HARSV II	6.51	13.21	0.42	268.08	39.15	191.80	215.93
AJ-HARSV III	6.57	13.27	0.43	273.06	44.13	196.78	220.92
Panel B: $\mathbf{w} \in [\mathbf{0}, \mathbf{1}]$							
HARSV	2.48	4.77	0.33	188.02	-40.90	111.75	
J-HARSV I	2.42	4.83	0.32	181.74	-47.18	105.46	-6.28
J-HARSV II	2.41	4.75	0.32	182.23	-46.69	105.95	-5.79
J-HARSV III	2.48	4.72	0.33	189.14	-39.79	112.86	1.11
AJ-HARSV I	5.22	8.23	0.52	359.32	130.40	283.05	171.30
AJ-HARSV II	5.00	8.16	0.50	341.23	112.31	264.96	153.21
AJ-HARSV III	4.98	8.23	0.50	336.83	107.91	260.56	148.81

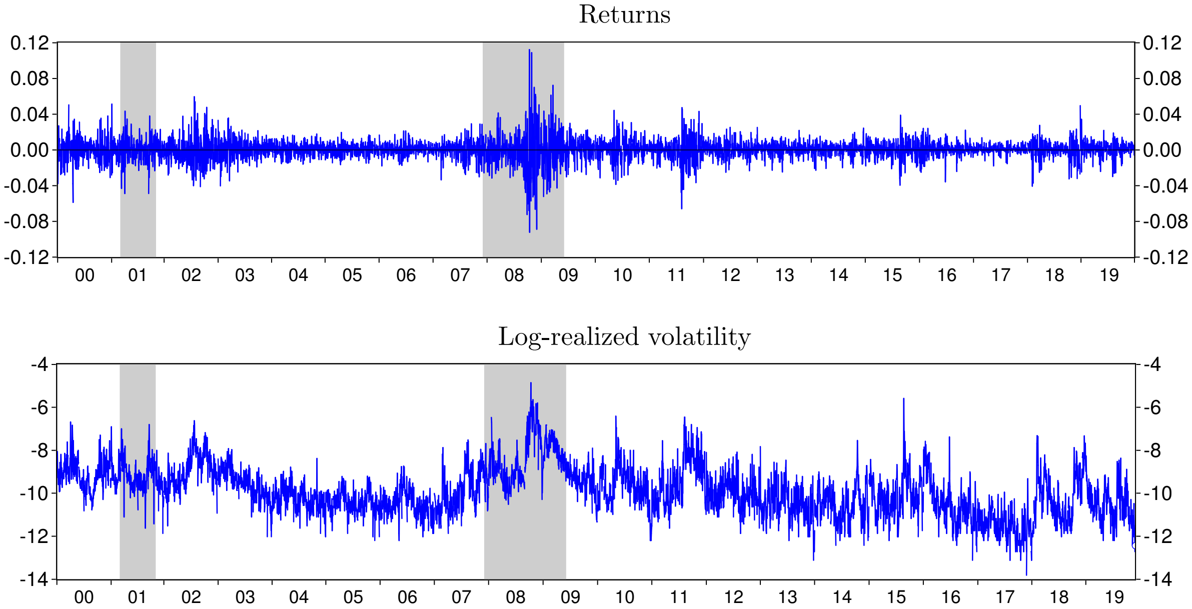
Note: The table displays performance measures for the active mean-variance investment strategies based on daily excess return, (logarithm of) semivariances for negative and positive returns for January 2000 - December 2019. Columns headed μ and σ contain the annualized mean and standard deviation of the portfolio returns (in percent). SR denotes the Sharpe ratio. Columns headed Δ contain performance fees (in annualized basis points) for switching from the strategy indicated by the subscript to the strategy indicated by the corresponding row as in (14) where $\gamma = 6$. The subscript x with x =100, 50, and 0 refer to the static strategies with $x\%$ of wealth invested in stocks. The HARSV model refers to the model where the return and semivariances are evaluated independently. JHARSV-I model refers to the model where instantaneous leverage effects are considered by taking the contemporaneous correlations, i.e., the Joint distribution of returns and semivariances, into account. AJHARSV-I model refers to the model where Asymmetric instantaneous leverage effects are considered by taking the contemporaneous correlation between the negative and positive returns together with the semivariances separately into account. JHARSV-II is the model where on top of the instantaneous leverage effects, intertemporal leverage effects are considered. JHARSV-III refers to the model, where asymmetric intertemporal leverage effects are considered on top of the instantaneous leverage effects. Finally, AJHARSV-II and AJHARSV-III are the counterparts of these models where Asymmetry in instantaneous leverage effects are explicitly modeled.

Table 6: Economic performance of active trading strategies with additional predictors of volatility

	μ	σ	SR	$\Delta_{100\%}$	$\Delta_{50\%}$	$\Delta_{0\%}$	Δ_{HARX}
Panel A: $\mathbf{w} \in [-\mathbf{1}, \mathbf{2}]$							
HARX	2.67	8.55	0.21	99.11	-129.81	22.83	
J-HARX I	2.65	8.53	0.20	97.09	-131.84	20.81	-2.02
J-HARX II	2.67	8.58	0.21	97.65	-131.27	21.37	-1.46
J-HARX III	2.69	8.85	0.21	99.54	-129.38	23.26	0.43
AJ-HARX I	9.28	16.74	0.50	300.47	71.55	224.20	201.36
AJ-HARX II	9.29	16.75	0.50	300.82	71.89	224.54	201.71
AJ-HARX III	9.45	16.91	0.51	305.95	77.02	229.67	206.83
Panel B: $\mathbf{w} \in [\mathbf{0}, \mathbf{1}]$							
HARX	2.55	4.68	0.35	197.18	-31.74	120.90	
J-HARX I	2.53	4.66	0.35	195.63	-33.29	119.36	-1.55
J-HARX II	2.54	4.66	0.35	196.08	-32.85	119.80	-1.10
J-HARX III	2.54	4.66	0.35	197.77	-32.16	120.49	0.41
AJ-HARX I	6.00	8.39	0.49	339.92	111.00	263.64	142.74
AJ-HARX II	6.00	8.40	0.49	340.47	111.55	264.19	143.29
AJ-HARX III	6.09	8.52	0.49	343.95	115.02	267.67	146.77

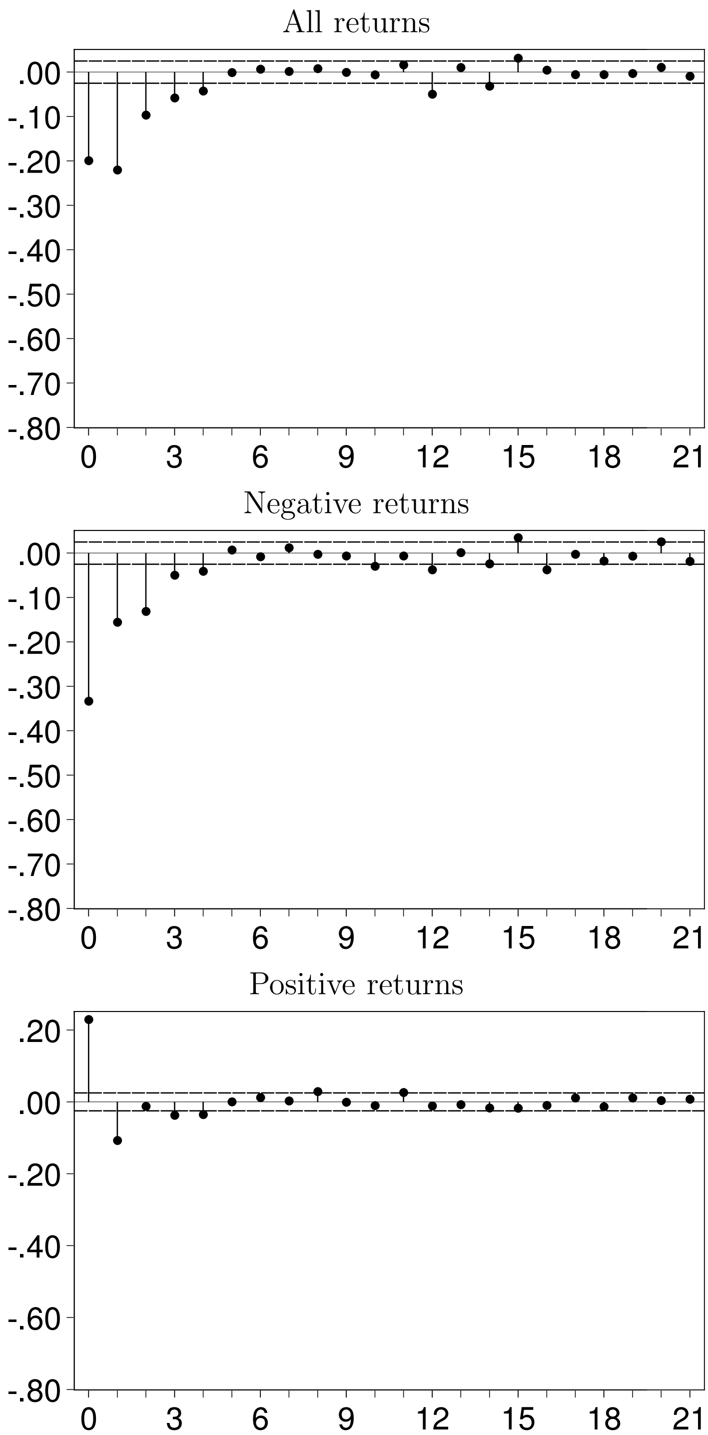
Note: The table displays performance measures for the active mean-variance investment strategies based on daily excess return and (logarithm of) volatility prediction using (4) and (21) for January 2000 - December 2019. Columns headed μ and σ contain the annualized mean and standard deviation of the portfolio returns (in percent). SR denotes the Sharpe ratio. Columns headed Δ contain performance fees (in annualized basis points) for switching from the strategy indicated by the subscript to the strategy indicated by the corresponding row as in (14) where $\gamma = 6$. The subscript x with x =100, 50, and 0 refer to the static strategies with x% of wealth invested in stocks. The HARX model refers to the model where both the return and volatility processes are evaluated independently. JHARX-I model refers to the model where instantaneous leverage effects are considered by taking the contemporaneous correlation between the return and volatility innovations, i.e., the Joint distribution of returns and volatility innovations, into account. AJHARX-I model refers to the model where Asymmetric instantaneous leverage effects are considered by taking the contemporaneous correlation between the negative and positive returns together with volatility innovations separately into account. JHARX-II is the model where on top of the instantaneous leverage effects, intertemporal leverage effects are considered using (8). JHARX-III refers to the model, where asymmetric intertemporal leverage effects are considered on top of the instantaneous leverage effects. Finally, AJHARX-II and AJHARX-III are the counterparts of these models where Asymmetry in instantaneous leverage effects are explicitly modeled.

Figure 1: Daily returns and realized volatility



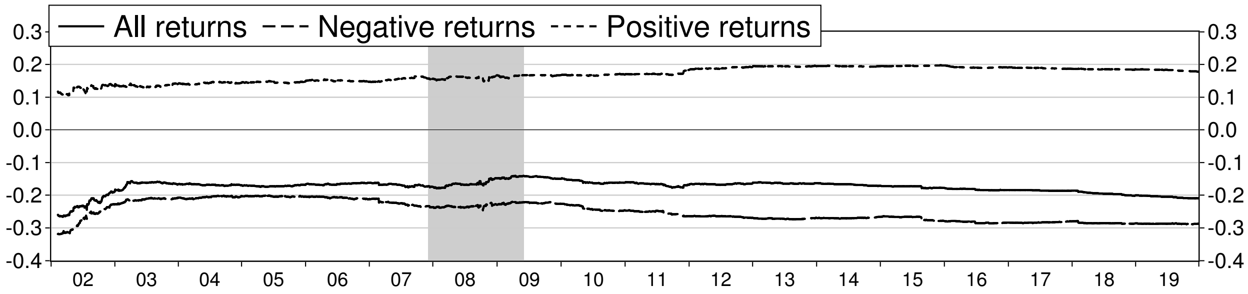
Note: The upper figure displays the daily S&P 500 market index return computed using the close prices of the days over the period from January 2000 to December 2019. The lower figure displays the daily realized volatility of these S&P 500 market index returns over the same period from January 2000 to December 2019. Realized volatilities are computed as in (3) using 5-minutes intervals subsampled by iterating the time of the first price, see Heber et al. (2009) for details. Gray shaded areas indicate US recession dates as documented by the NBER recession dates.

Figure 2: Conditional cross-autocorrelations between daily returns and realized volatility



Note: The figures display contemporaneous and intertemporal conditional correlations between daily returns, r_t and (log-)realized volatility, $\log(RV_t)$. Conditional means are computed using the regression equations in (4) and (6). The values on the $x-$axis denoted the order of correlation, i.e., $\text{Corr}(\log(RV_t), r_{t-x})$ for $x = 1, 2, \ldots, 21$. While the upper figure displays the correlations for all returns, the middle panel do this computation for only negative returns and the bottom panel for only positive returns.

Figure 3: The evolution of the contemporaneous leverage effects over time



Note: The figure shows the evolution of contemporaneous correlation of returns with the (log-)realized volatility over time. The correlations are computed starting with the sample using the first 500 observations for estimation and each time adding an observation recursively over time using (4) and (6). The solid line indicates the correlation between the (log-)realized volatility and all returns. The dashed line indicates the correlation between the (log-)realized volatility and negative returns, and the dotted line indicates the correlation between the (log-)realized volatility and positive returns. Gray shaded areas indicate US recession dates as documented by the NBER recession dates.