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Monetary Unions and National Welfare *

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Abstract

This paper studies monetary regime choice between monetary union and flexible exchange rate regime in a large open economy framework. The classical approach emphasizes that monetary unions are inherently costly because a single interest rate cannot respond effectively to different shocks of members of the union. Therefore, it is argued that countries with similar shocks should establish a monetary union so that the cost of one-size-fits-all monetary policy is minimized. This study reveals that when there are inefficient shocks (namely those which distort the economy asymmetrically and break the 'divine coincidence') and countries are large, the classical approach fails. In that case, monetary regime choice should depend on relative variation (mean preserving spread) of inefficient shocks rather than proximity of shocks. A union implicitly imposes cooperation in monetary policy between its members. This cooperation improves response to foreign inefficient shocks while it worsens responses to domestic inefficient shocks slightly less in terms of domestic welfare. Therefore, a country chooses monetary union over flexible exchange rate regime if variation of foreign shocks is close to or larger than variation of domestic shocks. That is on the condition that losing exchange rate flexibility is not costly or has a small cost. In this way, the domestic country 'ties the hand of the foreign country' and prevents foreign monetary policy actions which hurt domestic welfare. Both countries benefit from cooperation provided by the union, if variances (spreads) of domestic and foreign shocks are close enough. Then, a monetary union becomes Pareto Improvement. How close variances should be so that monetary union is welfare increasing or Pareto Improvement, and welfare loss or gain of losing exchange rate flexibility are contingent upon price rigidity and trade elasticity.

Keywords: monetary unions, flexible exchange rate, monetary policy, national welfare

JEL Codes: E52, E58, F33, F41, F42

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1 Introduction

This study revisits monetary regime choice between monetary union and flexible exchange rate regime in a large open economy (LOE) framework. The classical approach (Mundell (1961)) emphasizes that monetary unions are inherently costly because a single monetary policy tool cannot effectively respond to different shocks of the members of the union. On the other hand, monetary unions bring advantages in monetary policy and in trade.[1] In this sense, the conventional argument says that countries with similar shocks should establish a monetary union so that welfare cost of one-size-fits-all monetary policy is minimized while countries enjoy benefits of a monetary union.

The key contribution of this paper is to show that in the existence of inefficient shocks (i.e shocks which break divine coincidence (Blanchard and Galí (2007))) and with countries large enough to affect each other, the choice of monetary union should depend on relative variation (mean preserving spread) of inefficient shocks, not to proximity of shocks as the classical approach advocates. A monetary union is welfare increasing compared to flexible exchange rate regime if variation of foreign shocks is close to or larger than variation of domestic shocks. That is on the condition that adopting a common currency (fixed exchange rate) has low cost or is not costly at all.

The literature argues that a country 'ties its hand' (gives up its monetary autonomy) by joining a union to exploit benefits of the union (Giavazzi and Pagano (1988); Soffritti and Zanetti (2008); Clerc et al. (2011)). Countries without credible monetary policy suffer from welfare loss because of (i) inflationary bias à la Barro and Gordon (1983), and (ii) insufficiency in managing expectations (i.e discretionary monetary policy). They eliminate this welfare loss overnight by participating in a monetary union with a credible monetary authority. Giavazzi and Pagano (1988) states that this was the main motivation of many countries who joined European Monetary Union (EMU).

In this study, however, the main motivation of a country in joining a monetary union is to 'tie the hand of the other'. When countries are large enough, monetary policy responses of a central bank (CB) spillover to other countries and can damage their welfare, especially if the CB in question

[1] Dellas and Tavlas (2009) summarizes costs and benefits of monetary unions and discusses the monetary unions literature extensively.

disregards these spillovers. In that case, monetary union provides a tool to enforce a country to consider welfare spillovers. Monetary policy implicitly imposes cooperation. The CB of the union, maximizing overall welfare, works as a central planner except it is restricted with a single policy tool and fixed exchange rate. It considers welfare spillovers unlike national CBs in flexible exchange rate regime. Furthermore, strategic action to beggar-thy-neighbor (Corsetti and Pesenti (2001)) in flexible exchange rate regime cease to exist as monetary policy competition vanishes.

At national level, this cooperation particularly improves responses to inefficient foreign shocks. In return, it worsens responses to domestic ones slightly less.[2] Therefore, a country suffering from relatively close or larger variations of inefficient shocks in its neighbor prefers to establish a monetary union. It consents to transfer its monetary autonomy by establishing a monetary union because it expects larger welfare gain from other potential members losing their monetary autonomy. Since this relation is reciprocal, a monetary union is Pareto Improvement if inefficient shocks of countries have close mean preserving spreads. How close this spread should be and costs/benefits of fixed exchange rate depends strongly on price rigidity and trade elasticity.

To show my results, I use a two-country dynamic general equilibrium model with nominal price stickiness, producer's currency pricing (PCP), and complete asset markets based on Corsetti et al. (2010).[3] I compare two monetary regimes: monetary union and flexible exchange rate regime. In monetary union, the two countries adopt a common currency. Furthermore, they delegate their monetary policymaking capacity to a supranational authority which aims to maximize overall welfare. In flexible exchange rate regime, policymakers play a strategic game. The CB of each country maximizes welfare of their respective country and takes the other one's actions as given. Monetary authorities pursue monetary policy with commitment (they are credible) independent of monetary regime.

As many models in the field, monetary problems I defined cannot be solved in closed form.

[2]At international level, Clarida et al. (2002) shows that monetary policy cooperation weakly dominates monetary policy competition.

[3]Corsetti et al. (2010) constructs a representative agent new keynesian (RANK) model. I isolate this study from recent literature on so called heterogeneous agent new keynesian (HANK) models, see for example Debortoli and Galí (2017), Kaplan et al. (2018), Bilbiie (2018), because I specifically seek possible (in)efficiencies brought by monetary policy in monetary union and asymmetries across countries rather than (in)efficiencies originating from asymmetries within countries.

Therefore, I rely on linear quadratic (LQ) approach in welfare analysis based on Benigno and Benigno (2006) and numerical analysis.[4] I inspect national welfare loss differences between the monetary union and the flexible exchange rate regime.

This paper differs from the existing literature in two terms. First, it distinguishes between efficient and inefficient shocks. Efficient shocks do not create trade offs between output and inflation stabilizations. I use productivity shocks as efficient shocks in the model. With these shocks, the conventional Mundellian criterion (as Chari et al. (2019) calls it) remains intact. As Pappa (2004) also finds, both countries enjoy welfare gains in monetary union because they eliminate the beggar-thy-neighbor effect. In return, they suffer from one-size-fits-all monetary policy because different productivity shocks in each country require different monetary policy responses. As productivity shocks across counties approximate, the cost of one-size-fits-all policy is minimized.

Inefficient shocks create trade offs between output and inflation stabilizations, i.e they break the 'divine coincidence'. I use markups as inefficient shocks in the model. They distort the economy asymmetrically in the sense that a domestic shock hits only to the domestic aggregate supply equation and vice versa. That is why a country suffers significantly from its neighbor's responses to such shocks in flexible exchange rate regime. Consequently, how large these shocks are in the neighbor compared to domestic shocks becomes the main criterion for establishing a monetary union.

In this respect, this study emphasizes the distinction between efficient and inefficient shocks in monetary regime choice between monetary union and flexible exchange rate as Groll and Monacelli (2018) and Chari et al. (2019) even though it substantially differs from these papers in terms of its objective and its framework.

[4]LQ approach can broadly be summarized as welfare analysis using quadratic welfare criterion (social welfare function approximated up to second order) and linearized (approximated up to first order) structural equations. Even though dynamics of LQ approach is well determined, this analysis bears a particular difficulty. The standard method to derive welfare criteria provided by Rotemberg and Woodford (1998), Woodford (2011), and subsequent works, is insufficient to obtain an appropriate quadratic welfare criteria for each country and for overall economy together using a single steady state. Yet, to compare monetary regimes, a unique steady state is necessary. Therefore, I rely on an alternative method introduced by Benigno and Benigno (2006). Following Benigno and Woodford (2005), I use second approximation of structural equations to derive welfare criteria. Once welfare criteria are obtained, I proceed with linearized structural equations (approximated up to first order) to evaluate these criteria. Problems in LQ approach is beyond the scope of the study. Therefore, for further discussions of the problem, I refer to Corsetti and Pesenti (2001); Kim and Kim (2003); Benigno and Woodford (2006); Woodford (2011) Chapter 6.1.

Second, this study focuses on monetary unions of large economies. In SOE framework, the domestic country is a SOE and the rest of the world (RoW) is a LOE. From the domestic country's perspective, participation in a monetary union boils down to adopting completely RoW's policy. RoW (monetary union) remains indifferent to SOE's participation in the union. Therefore, monetary union turns out welfare decreasing without gains from credibility or trade. The cost of monetary union increases as monetary policy responses in the union diverge from the responses the domestic country requires. Consequently, even with inefficient shocks, the conventional argument (countries with similar shocks should establish monetary unions) prevails (Clerc et al. (2011)). However, in practice, countries establishing monetary unions may be sufficiently large to affect the monetary policy in the union, e.g early members of EMU.

The rest of the study has the following structure. Section 2 describes the model. Section 3 explains LQ approach and presents welfare criteria and structural equations. Monetary policy is solved in Section 4. I provide numerical comparisons in Section 5. Section 6 is the conclusion.

2 The Model

In this section, I describe the two-country model with nominal price rigidity, producer's currency pricing (PCP), and complete asset markets based on Corsetti et al. (2010).[5] There are two countries, Home (H) and Foreign (F), and infinitely many households aligned between 0 and 1. H's and F's shares in population are n and $1-n$ respectively. A household j in country i produces a differentiated good and consumes a bundle of domestic and foreign goods. It maximizes the following utility function.

$$\tilde{U}^j \equiv \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[U(C_t^j) - V(y_t(j); a_t^i) \right] \tag{2.1}$$

[5] The model is fairly similar the model of Benigno and Benigno (2006) and of Groll and Monacelli (2018). I isolate this model from home bias and consumption preference shocks existing in Groll and Monacelli (2018) and government spending in Benigno and Benigno (2006). Compared to Groll and Monacelli (2018) and using its notation, the model assumes $Z_{C,t} = Z^*_{C,t} = 1$, $\gamma = 1-n$, and $\gamma^* = n$. Compared to Benigno and Benigno (2006), it assumes $s_c = 1$ and $G_t = G^*_t = 0\ \forall t$. Throughout the study, I adopt the notation of the latter paper for convenience as I follow it in deriving welfare criteria.

$V(y_t(j), a_t^i) \equiv (a_t^i)^{-\eta} \frac{y_t(j)^{1+\eta}}{1+\eta}$ is disutility from labor. a_t^i denotes stochastic and country-dependent productivity shocks, $y_t(j)$ is the differentiated goods produced by household j at time t with inverse elasticity of goods production $\eta \geq 0$. $\beta \in (0; 1)$ is the intertemporal discount factor. $U(C_t^j) \equiv \frac{(C_t^j)^{1-\rho}}{1-\rho}$ is utility gained from consuming goods and $\rho > 0$ is the inverse elasticity of intertemporal substitution. C_t^j is a Dixit-Stiglitz aggregator for household j's consumption constituted of H's and F's goods bundles.

$$C_t^j \equiv \left[n^{1/\theta}(C_{H,t}^j)^{(\theta-1)/\theta} + (1-n)^{1/\theta}(C_{F,t}^j)^{(\theta-1)/\theta}\right]^{\theta/(\theta-1)} \tag{2.2}$$

$\theta > 0$ is the elasticity of substitution of domestic and foreign goods (also referred as the trade elasticity or the intratemporal elasticity of substitution). The model does not allow for home bias.[6] $C_{H,t}^j$ is the bundle of differentiated goods of country H.

$$C_{H,t}^j \equiv \left[\left(\frac{1}{n}\right)^{1/\sigma} \int_0^n c^j(h)^{(\sigma-1)/\sigma} dh\right]^{\sigma/(\sigma-1)} \tag{2.3}$$

$C_{F,t}^j$ is the bundle of differentiated goods of country F.

$$C_{F,t}^j \equiv \left[\left(\frac{1}{1-n}\right)^{1/\sigma} \int_n^1 c^j(f)^{(\sigma-1)/\sigma} df\right]^{\sigma/(\sigma-1)} \tag{2.4}$$

$\sigma > 0$ is the elasticity of substitution across goods produced within a country. Let $p_t^i(k)$ be the price of a differentiated good produced by a household k in terms of country i's currency. Then, the producer's price index (PPI) of country H in terms of country i's currency is

$$P_{H,t}^i \equiv \left[\left(\frac{1}{n}\right) \int_0^n p_t^i(h)^{1-\sigma} dh\right]^{1/(1-\sigma)} \tag{2.5}$$

Similarly, PPI of country F in terms of country i's currency is

$$P_{F,t}^i \equiv \left[\left(\frac{1}{1-n}\right) \int_n^1 p_t^i(f)^{1-\sigma} df\right]^{1/(1-\sigma)} \tag{2.6}$$

[6] For a similar model with home bias, I refer to Faia and Monacelli (2008).

The consumer price index (CPI) in terms of country i's currency is

$$P_t^i \equiv \left[n {P_{H,t}^i}^{1-\theta} + (1-n) {P_{F,t}^i}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{2.7}$$

S_t is the exchange rate at time t. I assume PCP meaning that sellers tag one price in their own currency for their goods in all markets. Then, the law of one price (LoOP) holds.[7]

$$p_t(k) = S_t p_t^*(k) \quad \forall t \tag{2.8}$$

(2.8) implies that $P_t = S_t P_t^* \ \forall t$. In the absence of home bias, this means that purchasing power parity (PPP) holds at any t. $T_t \equiv \frac{P_{F,t}}{P_{H,t}}$ is the terms of trade. Then, CPI can also be written as

$$\begin{aligned} P_t^i &= \left[(1-n) + n {T_t}^{1-\theta} \right]^{\frac{1}{1-\theta}} P_{H,t}^i \\ &= \left[(1-n) {T_t}^{-(1-\theta)} + n \right]^{\frac{1}{1-\theta}} P_{F,t}^i \end{aligned} \tag{2.9}$$

The law of motion of the exchange rate is

$$\frac{T_t}{T_{t-1}} = \frac{S_t}{S_{t-1}} \frac{\Pi_{F,t}^*}{\Pi_{H,t}} \quad \forall t \tag{2.10}$$

If the exchange rate is fixed, i.e $\Delta S_t = 0$, (2.10) becomes

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_{F,t}^*}{\Pi_{H,t}} \quad \forall t \tag{2.11}$$

Households trade one-period nominal state contingent assets denominated in H's currency. $v_{t,t+1} \equiv v(h^{t+1}|h^t)$ is the period-t risk adjusted price of an asset which gives 1 domestic currency if h^{t+1} is realized, 0 otherwise. I assume asset markets are complete both within and across countries. Households have identical initial (0) wealth. Then, consumption risk is perfectly insured between all agents implying

$$C_t^j = C_t \quad \forall j, t \tag{2.12}$$

[7] For simplicity, throughout the paper, I suppress superscripts of H's currency and denote F's currency with $*$. (2.8) means $p_t^H(k) = S_t p_t^F(k)$

H's one period nominal interest rate at time t is $(1+i_t) \equiv \Big[\mathbb{E}_t v_{t,t+1}\Big]^{-1}$ whereas F's one period nominal interest rate at time t is $(1+i_t^*) \equiv \Big[\mathbb{E}_t v_{t,t+1}\frac{S_{t+1}}{S_t}\Big]^{-1}$.

Household j is monopolistic producer of one of the differentiated goods. Total demand for the differentiated good h produced at H at time t is

$$y_t(h) = \Big(\frac{p_t(h)}{P_{H,t}}\Big)^{-\sigma}\Big(\frac{P_{H,t}}{P_t}\Big)^{-\theta} C_t \tag{2.13}$$

Similarly, total demand for the differentiated good f produced at F at time t is

$$y_t(f) = \Big(\frac{p_t(f)}{P_{F,t}}\Big)^{-\sigma}\Big(\frac{P_{F,t}}{P_t}\Big)^{-\theta} C_t \tag{2.14}$$

Producers have price setting mechanism à la Calvo (1983): Each producer is allowed to change the price of its good at any period with probability $1-\alpha$. A producer sets the price of its differentiated product, $\tilde{p}_t(j)$, to maximize the expected discounted value of its net profits taking $P_{H,t}$, $P_{F,t}$, P_t (or $P^*_{H,t}$, $P^*_{F,t}$, P^*_t) as given. Pricing problem of a seller which can change its price at time t is

$$\max_{\tilde{p}_t(j)} \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \Big[\frac{U(C_{t+k})}{P_{t+k}}(1-\tau_{i,t+k})\tilde{p}_t(j) y_{t+k}(j) - V(y_{t+k}(j); a^i_{t+k})\Big] \tag{2.15}$$

$\tau_{i,t}$ is an exogenous time-varying tax on sales of country i. Optimal choice of $\tilde{p}_t(j)$ is

$$\tilde{p}_t(j) = \frac{\mathbb{E}_t \sum_{k=0}^{\infty}(\alpha\beta)^k V_y(y_{t+k}(j); a_{t+k}) y_{t+k}(j)}{\mathbb{E}_t \sum_{k=0}^{\infty}(\alpha\beta)^k \frac{1}{\mu_{i,t+k}}\frac{U_C(C_{t+k})}{P_{t+k}} y_{t+k}(h)} \tag{2.16}$$

$\mu_{i,t+k} \equiv \frac{(1-\tau_{i,t})(\sigma-1)}{\sigma}$ is the time-varying markup shock of country i.[8] Given Calvo price setting mechanism, the evolution of the domestic price index of country i is

$$P_{i,t}{}^{1-\sigma} = \alpha P_{i,t-1}{}^{1-\sigma} + (1-\alpha)\tilde{p}_t(i)^{1-\sigma} \tag{2.17}$$

[8] Even though actual variation originates from sales tax, I will treat the model as if markups move stochastically not sales taxes.

2.1 Monetary Regimes and Strategies

In this section, I describe monetary regimes and monetary policy strategies in these regimes. I compare two monetary regimes: flexible exchange rate regime ($\mathcal{F}$) and monetary union ($\mathcal{M}$). To be able to compare the two, I also define a transitory regime called cooperative monetary policy with flexible exchange rate ($\mathcal{C}$).

In $\mathcal{F}$, two national CBs (monetary policymaker) exists, each is attached to a country. A CB aims to maximize national welfare of the country it is attached to: $W_H \equiv \int_0^n \tilde{U}^j dj$ for H and $W_F \equiv \int_n^1 \tilde{U}^j dj$ for F. National welfare (sum of the expected discounted utilities of households living in a particular country) of H is

$$W_H = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[U(C_{t+k}) - \frac{1}{n} \int_0^n V(y_{t+k}(h); \xi_{t+k}) dh \right] \tag{2.18}$$

$$= \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[U(C_{t+k}) - V(Y_{H,t+k}; \xi_{t+k}) \Delta_t \right] \tag{2.19}$$

where $Y_{H,t} \equiv \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t$ and $\Delta_t \equiv \frac{1}{n} \int_0^n \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\sigma(1+\eta)}$ is the price dispersion term of H. $\xi_t \equiv \{a_t^H, a_t^F, \mu_t^H, \mu_t^F, \}$ is the set of all exogenous shocks. Similarly, overall welfare of F is

$$W_F = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \left[U(C_{t+k}) - V(Y_{F,t+k}; \xi_{t+k}) \Delta_t \right] \tag{2.20}$$

with $Y^*_{F,t} \equiv \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t$ and $\Delta^*_t \equiv \frac{1}{1-n} \int_n^1 \left(\frac{p_t(f)}{P_{F,t}}\right)^{-\sigma(1+\eta)}$ is the price dispersion term of F. The price dispersion terms allows me to rewrite (2.16) and (2.17) as in Table 1. To maximize welfare of its country, a CB chooses a strategy to set $\{\Pi_{i,t}\}_{t=0}^{\infty}$ at $t = 0$ and implements it using one-period risk free nominal interest rate. It takes other CB's strategy as given. Policymakers commit to their strategies they declared at $t = 0$. The exchange rate floats.

In $\mathcal{M}$, H and F adopt a common currency and transfer their monetary policymaking powers to the CB of the union. The new monetary authority chooses a sequence $\{C_t, C^*_t, Y_{H,t}, Y^*_{F,t}, \Pi_{H,t}, \Pi_{F,t}\}_{t=0}^{\infty}$ to maximize welfare in the union ($W = \int_0^1 U^j dj$) using a single risk free nominal interest rate. Wel-

Table 1: Law of Motion fo Price Dispersion and Optimal Pricing

Price Dispersion	$\Delta_t = \alpha \Delta_{t-1} {\Pi_{H,t}}^{\sigma(1+\eta)} + (1-\alpha)\left(\frac{1-\alpha {\Pi_{H,t}}^{\sigma-1}}{1-\alpha}\right)^{\frac{-\sigma(1+\eta)}{1-\sigma}}$	(2.21)
	$\Delta^*_t = \alpha^* \Delta^*_{t-1} {\Pi^*_{F,t}}^{\sigma(1+\eta)} + (1-\alpha^*)\left(\frac{1-\alpha {\Pi^*_{F,t}}^{\sigma-1}}{1-\alpha^*}\right)^{\frac{-\sigma(1+\eta)}{1-\sigma}}$	(2.22)
Optimal Pricing	$\frac{1-\alpha {\Pi_{H,t}}^{\sigma-1}}{1-\alpha} = \left(\frac{F_t}{K_t}\right)^{\frac{\sigma-1}{1+\sigma\eta}}$	(2.23)
	$\frac{1-\alpha {\Pi^*_{F,t}}^{\sigma-1}}{1-\alpha} = \left(\frac{F^*_t}{K^*_t}\right)^{\frac{\sigma-1}{1+\sigma\eta}}$	(2.24)

$$K_t \equiv \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k {a_{t+k}}^{-\eta} {Y_{H,t+k}}^{1+\eta} \left(\frac{P_{H,t+k}}{P_{H,t}}\right)^{\sigma(1+\eta)}, \; F_t \equiv \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \frac{1}{\mu_{t+k}} {C_{t+k}}^{-\rho} Y_{H,t+k} \left(\frac{P_{H,t+k}}{P_{t+k}}\right)\left(\frac{P_{H,t+k}}{P_t}\right)^{\sigma-1}$$

$$K^*_t \equiv \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k {a^*_{t+k}}^{-\eta} {Y^*_{F,t+k}}^{1+\eta} \left(\frac{P^*_{F,t+k}}{P^*_{F,t}}\right)^{\sigma(1+\eta)}, \; F^*_t \equiv \mathbb{E}_t \sum_{k=0}^{\infty} (\alpha\beta)^k \frac{1}{\mu^*_{t+k}} {C_{t+k}}^{-\rho} Y^*_{F,t+k} \left(\frac{P^*_{F,t+k}}{P^*_{t+k}}\right)\left(\frac{P^*_{F,t+k}}{P^*_t}\right)^{\sigma-1}$$

fare in the union (corresponding to overall welfare, i.e $W = nW_H + (1-n)W_F$) can be written as

$$W = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \Big[U(C_{t+k}) - nV(Y_{H,t+k}, \xi_{t+k})\Delta_t - (1-n)V(Y^*_{F,t+k}, \xi_{t+k})\Delta^*_t \Big] \tag{2.25}$$

In this setting, fixed exchange rate rule (2.11) immediately implies a single policy rate and a common currency. In both regimes, $\mathcal{F}$ and $\mathcal{M}$, CBs pursue monetary policies under commitment meaning that they can efficiently alter expectations. This implies that credibility does not change across regimes. Similarly, the model does not anticipate any auxiliary changes (trade, factor mobility etc.) between the two regimes.

In this model, $\mathcal{M}$ differs from $\mathcal{F}$ in two terms. First, in $\mathcal{M}$ a common currency is adopted meaning that a single monetary policy tool (nominal interest rate) remains. Second, the nature of monetary policymaking changes. mIn $\mathcal{M}$, monetary policy competition disappears and monetary authority of the union aims to maximize overall welfare. In $\mathcal{F}$ CBs compete to maximize national welfare. In order to separate the two effects from one another, a third regime, cooperative flexible exchange rate regime ($\mathcal{C}$), is defined. In $\mathcal{C}$, exchange rate flexibility is preserved. However, monetary policy in guided by a central planner (CP) who chooses $\{C_t, C^*_t, Y_{H,t}, Y^*_{F,t}, \Pi_{H,t}, \Pi_{F,t}\}_{t=0}^{\infty}$ to maximize overall welfare W. This regime corresponds to the case when CBs cooperate to maximize overall welfare instead of competing with each other for national welfare as in $\mathcal{F}$. On the other hand, $\mathcal{C}$ differs from $\mathcal{M}$ only in terms of exchange rate regime. In both cases, monetary authorities aim to maximize overall welfare except in $\mathcal{M}$ the CB of the union is restricted with a single policy tool. $\mathcal{C}$ provides a transitory monetary policy problem between $\mathcal{F}$ and $\mathcal{M}$. By controlling welfare difference between $\mathcal{F}$ and $\mathcal{C}$, I distinguish the effect of monetary policy cooperation in $\mathcal{M}$ while welfare difference between $\mathcal{C}$ and $\mathcal{M}$ enables me to observe the effect of fixed exchange rate in the $\mathcal{M}$.

3 Linear Quadratic Approach

Monetary problems defined in the previous section cannot be solved in closed form. Therefore, I rely on LQ approach. [9] To avoid spurious welfare comparisons in LQ approach, I must ensure (i) welfare criteria appropriately rank policies in a monetary problem, and (ii) welfare loss functions in different monetary regimes are comparable.

The latter condition requires that welfare loss functions and structural equations are approximated around the same steady state. In Appendix A.1, I show that there exist zero inflation deterministic steady state for $\mathcal{F}$, $\mathcal{C}$, and $\mathcal{M}$ and they are identical. Given the steady state, Table 2 shows linearized structural equations when $\bar{\mu} = 1$. (3.9) is the law of motion of the exchange rate when the exchange rate is flexible. When the exchange rate is fixed, as supposed to be in $\mathcal{M}$, this equation is replaced with (3.10).

The former condition requires that welfare criteria are quadratic to evaluate welfare gains from different policies using linearized structural equations. Following Benigno and Benigno (2006), I

[9] LQ approach can broadly be defined as to evaluate quadratic functions, obtained by approximating objective functions up to second order, using linearized (approximated up to first order) structural equations.

Table 2: Structural Equations

Aggregate Demand	$\hat{C}_t = \mathbb{E}_t\hat{C}_{t+1} - \rho^{-1}(i_t - \mathbb{E}_t\pi_{H,t+1} - (1-n)\mathbb{E}_t\Delta\hat{T}_{t+1})$	(3.1)
	$\hat{C}^*_t = \mathbb{E}_t\hat{C}^*_{t+1} - \rho^{-1}(i^*_t - \mathbb{E}_t\pi^*_{F,t+1} + n\mathbb{E}_t\Delta\hat{T}_{t+1})$	(3.2)
Market Clearing Condition	$\hat{Y}_{H,t} = \hat{C}_t + \theta(1-n)\hat{T}_t$	(3.3)
	$\hat{Y}^*_{F,t} = \hat{C}^*_t - \theta n\hat{T}_t$	(3.4)
Risk Sharing	$\hat{C}_t = \hat{C}^*_t$	(3.5)
Aggregate Supply	$\pi_{H,t} = \beta\mathbb{E}_t\pi_{H,t+1} + \kappa\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big) + (1-n)\kappa\psi\big(\hat{T}_t - \tilde{T}^w_t\big) + u_t$	(3.6)
	$\pi^*_{F,t} = \beta\mathbb{E}_t\pi^*_{F,t+1} + \kappa^*\big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big) - n\kappa^*\psi\big(\hat{T}_t - \tilde{T}^w_t\big) + u^*_t$	(3.7)
Terms of Trade	$\hat{T}_t - \tilde{T}^w_t = \theta^{-1}\big[\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big) - \big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big)\big]$	(3.8)
Exchange Rate (flexible)	$\Delta\hat{S}_t = \Delta\hat{T}_t + \pi_{H,t} - \pi^*_{F,t}$	(3.9)
Exchange Rate (fixed)	$0 = \Delta\hat{T}_t + \pi_{H,t} - \pi^*_{F,t}$	(3.10)

$\kappa \equiv (\rho+\eta)\frac{(1-\alpha)(1-\alpha\beta)}{\alpha(1+\sigma\eta)}$, $\kappa^* \equiv (\rho+\eta)\frac{(1-\alpha^*)(1-\alpha^*\beta)}{\alpha^*(1+\sigma\eta)}$, $\psi \equiv \frac{1-\rho\theta}{\rho+\eta}$, $u_t \equiv \kappa\frac{\hat{\mu}_{H,t}}{\eta+\rho}$, $u^*_t \equiv \kappa^*\frac{\hat{\mu}_{F,t}}{\eta+\rho}$, $\tilde{Y}^w_{H,t} \equiv \tilde{C}_t + (1-n)\theta\tilde{T}^w_t$, $\tilde{Y}^w_{F,t} \equiv \tilde{C}_t + -n\theta\tilde{T}^w_t$, $\tilde{T}^w_t = \frac{\eta}{(1+\theta\eta)}[\hat{a}_{R,t}]$, $\tilde{C}_t \equiv \frac{\eta}{(\eta+\rho)}(\hat{a}_{W,t})$, $\hat{\mu}^W_t \equiv n\hat{\mu}_{H,t} + (1-n)\hat{\mu}_{F,t}$, $\hat{\mu}^R_t \equiv \hat{\mu}_{H,t} - \hat{\mu}_{F,t}$, $\hat{a}^W_t \equiv n\hat{a}_{H,t} + (1-n)\hat{a}_{F,t}$, $\hat{a}^R_t \equiv \hat{a}_{H,t} - \hat{a}_{F,t}$.

derive welfare loss functions.[10] For $\pi_{H,0} = \bar{\pi}_{H,0}$, H's loss function is

$$L = -\frac{1}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big[\lambda_{y_h}\big(\hat{Y}_{H,t} - \tilde{Y}^h_{H,t}\big)^2 + \lambda_{y_f}\big(\hat{Y}^*_{F,t} - \tilde{Y}^h_{F,t}\big)^2 + \lambda_q\big(\hat{T}_t - \tilde{T}^h_t\big)^2 + \lambda_{\pi_h}{\pi_{H,t}}^2 + \lambda_{\pi_F}{\pi^*_{F,t}}^2\Big] \tag{3.11}$$

Similarly, for a given $\pi_{F,0} = \bar{\pi}_{F,0}$, F's loss function is

$$L^* = -\frac{1}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big[\lambda^*_{y_h}\big(\hat{Y}_{H,t} - \tilde{Y}^f_{H,t}\big)^2 + \lambda^*_{y_f}\big(\hat{Y}^*_{F,t} - \tilde{Y}^f_{F,t}\big)^2 + \lambda^*_q\big(\hat{T}_t - \tilde{T}^f_t\big)^2 + \lambda^*_{\pi_h}{\pi_{H,t}}^2 + \lambda^*_{\pi_F}{\pi^*_{F,t}}^2\Big] \tag{3.12}$$

[10] A standard method for deriving a second order approximation to welfare criteria is presented by Rotemberg and Woodford (1998), Woodford (2011) and subsequent works. It requires second order approximation to social welfare function around efficient deterministic steady state. When approximated around this steady state, linear terms in the approximation vanish. However, efficient steady states in $\mathcal{F}$ and in $\mathcal{M}$ are different because of the contractionary bias discussed in Corsetti and Pesenti (2001) and Benigno (2002). Therefore, with this method a quadratic welfare criteria can be derived only using different steady states. Then, the former condition is violated. I circumvent this problem by adopting Benigno and Benigno (2006)'s approach which in turn uses Benigno and Woodford (2005)'s method. I replace linear terms in second order approximation to social welfare function with appropriate linear combination of second approximation of structural equations. Structural equations of second order are used only to replace linear terms in welfare criteria. Once I obtain quadratic objective functions, I return to linearized structural equations to evaluate these functions. For further discussion of the problem, I refer to Corsetti and Pesenti (2001), Kim and Kim (2003), Woodford (2011) Chapter 6.1.

Finally, for $\pi_{H,0} = \bar{\pi}_{H,0}$ and $\pi_{F,0} = \bar{\pi}_{F,0}$

$$
\begin{aligned}
L^W = -\frac{\lambda_y^w}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big[& n\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big)^2 + (1-n)\big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big)^2 \\
& + n(1-n)\theta\psi\big(\hat{T}_t - \tilde{T}^w_t\big)^2 + \frac{\sigma n}{\kappa}{\pi_{H,t}}^2 + \frac{\sigma(1-n)}{\kappa^*}{\pi^*_{F,t}}^2\Big]
\end{aligned}
\tag{3.13}
$$

with $\lambda_y^w \equiv (\eta + \rho)$. λ_{y_h}, λ_{y_f}, λ_{π_h}, λ_{π_f}, $\lambda^*_{y_h}$, $\lambda^*_{y_f}$, $\lambda^*_{\pi_h}$, $\lambda^*_{\pi_f}$, $\tilde{Y}^h_{H,t}$, $\tilde{Y}^h_{F,t}$, $\tilde{T}^h_t$, $\tilde{Y}^f_{H,t}$, $\tilde{Y}^f_{F,t}$, $\tilde{T}^f_t$ defined in Appendix A.3. $\bar{\pi}_{H,0}$ and $\bar{\pi}_{F,0}$ are arbitrary but for loss functions to be comparable between monetary regimes, under different regimes they must be identical. So, I impose that $\bar{\pi}_{H,0} = \bar{\pi}_{F,0} = 0$.[11]

4 Monetary Policy

In this section, I define monetary policy problems in LQ approach and present analytical solutions. In $\mathcal{F}$, the CB of H maximizes (3.11) subject to (3.6), (3.7), and (3.8). First order conditions of H's monetary policy problem are

$$-\lambda_{y_h}\big(\hat{Y}_{H,t} - \tilde{Y}^h_{H,t}\big) + \vartheta_{\pi,t}\kappa + \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{4.1}$$

$$-\lambda_{y_f}\big(\hat{Y}_{F,t} - \tilde{Y}^h_{F,t}\big) + \vartheta^*_{\pi,t}\kappa^* - \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{4.2}$$

$$-\lambda_{\pi_h}\pi_{H,t} - \vartheta_{\pi,t} + \vartheta_{\pi,t-1} = 0 \tag{4.3}$$

$$-\lambda_q\big(\hat{T} - \tilde{T}^h_t\big) + \vartheta_{\pi,t}(1-n)\kappa\psi - \vartheta^*_{\pi,t}n\kappa^*\psi - \vartheta_{\mathcal{T},t} = 0 \tag{4.4}$$

where $\vartheta_{\pi,t}$, $\vartheta^*_{\pi,t}$, $\vartheta_{\mathcal{T},t}$ are time dependent lagrange multipliers for (3.6), (3.7), (3.8) respectively. Similarly, the CB of F maximizes (3.12) subject to (3.6), (3.7), and (3.8).[12] (4.1) to (4.4) along with first order conditions of F's monetary policy problem and with (3.6), (3.7), (3.8) determine all endogenous variables for a given path of exogenous shocks $\{\xi_t\}_{t=0}^{\infty}$.

[11] In fact, Benigno and Woodford (2005) shows that this initial condition for inflation is compatible with monetary policy from timeless perspective described in Woodford (1999). Benigno and Benigno (2006) shows that initial conditions for domestic inflation in two-country model is also compatible with monetary policy from timeless perspective. However, to be able to compare the two monetary regimes initial inflation conditions must be identical in two monetary regimes. Timeless perspective does not necessarily supports this condition. For a comprehensive explanation, I refer to these two papers as well as Woodford (2011) Chapter 6.5.

[12] First order conditions of F's monetary problem are identical to first order conditions of H's monetary problem. Therefore, they are presented in Appendix B.1.

In $\mathcal{M}$, the monetary authority of the union maximizes (3.13) subject to (3.6), (3.7), (3.8), and (3.10). For $\alpha = \alpha^*$ (equivalent to $\kappa = \kappa^*$), the optimality condition of monetary policy problem of $\mathcal{M}$ is

$$-\sigma(n\pi_{H,t} + (1-n)\pi^*_{F,t}) = n\Delta\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big) + (1-n)\Delta\big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big) \tag{4.5}$$

(4.5) is identical to the optimality condition of monetary policy under commitment in monetary union defined in Groll and Monacelli (2018). As in the previous problem, given ξ_t, all endogenous variables are determined using (3.6), (3.7), (3.8), (3.10), and (4.5).

Finally, in $\mathcal{C}$ the CP maximizes (3.13) subject to (3.6), (3.7), and (3.8). Monetary policy problem in $\mathcal{C}$ differs from the problem in $\mathcal{M}$ only in terms of the exchange rate condition. In $\mathcal{C}$, monetary authority is not restricted with fixed exchange rate condition (3.10) which imposes a common currency. In that sense, monetary union can be considered as 'constrained cooperation' (Pappa (2004)). Therefore, in terms of overall welfare, W, monetary union is always worse off than (or can be at most as good as) $\mathcal{C}$, a result already found in Pappa (2004) and Groll and Monacelli (2018). However, this is not necessarily true for national welfare.

5 Welfare Analysis

In this section, I provide numerical welfare comparisons between $\mathcal{M}$ and $\mathcal{F}$. Throughout the study, I assume $\bar{\mu} = 1$ and symmetric price stickiness, i.e $\alpha = \alpha^*$. I take discount factor β as 0.99. H's relative population, n, is 0.5. Average price contract durations are 3 periods: $\alpha = \alpha^* = 0.66$. The elasticity of substitution between differentiated goods, σ, is 9 implying a steady-state markup of prices over marginal costs of 12.5 percent. Following Groll and Monacelli (2018), I set the intertemporal elasticity of substitution, ρ, to 1 and the trade elasticity to 2. The value of η has been a focus of debate in macroeconomics. Following Gali et al. (2007), Christiano et al. (2010) and other others, I take η as 0.89. Nevertheless, I check robustness of results for alternative values of η and results remain intact. I assume persistence of shocks are 0.7 and variations of shocks are 0.1. Table 3 summarizes the calibration. Unless stated otherwise, I use specifications at Table 3.

Table 3: Calibration

Parameter	Variable name	Value
β	Discount factor	0.99
ρ	Inverse elasticity of intertemporal substitution	1.00
η	Inverse elasticity of goods production	0.89
σ	Elasticity of substitution between goods produced in a country	9.00
θ	Elasticity of substitution between H and F goods	2.00
α	Calvo parameter for $i = \{H, F\}$	0.66
n	Relative size of domestic country	0.50
ρ_i	Persistence of shocks $\rho_{\mu_h} = \rho_{\mu_f} = \rho_{a_h} = \rho_{a_f}$	0.70
σ_k^2	Variance of shocks $\sigma_{\mu_h}^2 = \sigma_{\mu_f}^2 = \sigma_{a_h}^2 = \sigma_{a_f}^2$	0.10

First, I close productivity shocks. Only inefficient cost-push shocks $\{\hat{\mu}_{H,t}, \hat{\mu}_{F,t}\}_{t=0}^{\infty}$ which create a trade off between output gap and inflation remain (Clarida et al. (1999)). Figure 1 shows H's welfare loss differences between $\mathcal{M}$ and $\mathcal{F}$ as a function of nominal price rigidity α for different values of θ. In the region above zero line, $\mathcal{M}$ dominates $\mathcal{F}$ meaning that welfare gains in $\mathcal{M}$ outweighs gains in $\mathcal{F}$. Below the zero line, $\mathcal{F}$ dominates $\mathcal{M}$. I find that neither $\mathcal{M}$ nor $\mathcal{F}$ globally

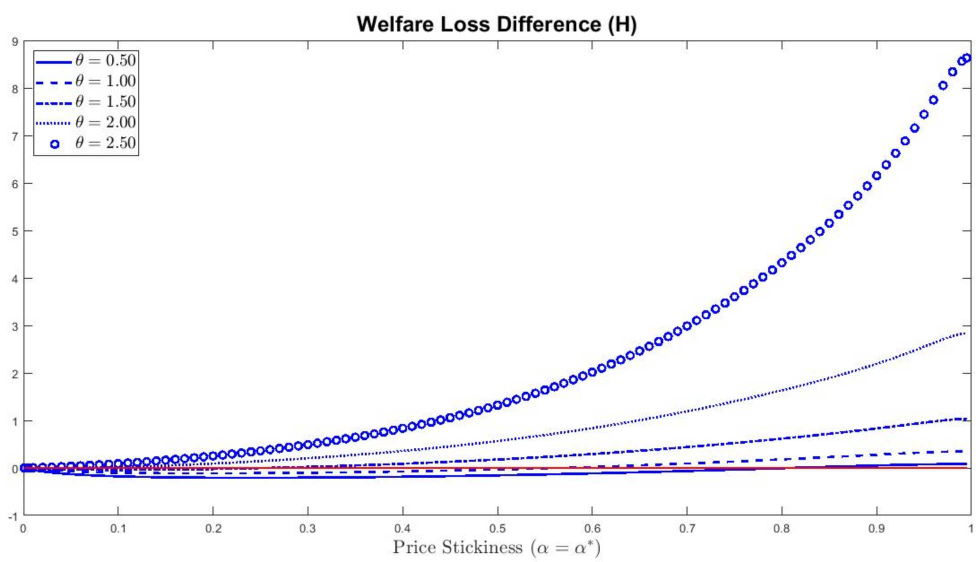

Figure 1: Welfare loss difference of H as a function of nominal price rigidity for the parametrization given at Table 3 and for alternative values of θ.

dominates one another. It depends on parameters. For example, at $\theta = 1$, $\mathcal{F}$ dominates $\mathcal{M}$ for

$\alpha \leq 0.56$ in terms of H's welfare. For $0.56 \leq \alpha$, $\mathcal{M}$ dominates $\mathcal{F}$ in terms of H's welfare.[13]

Dynamics of welfare change can easily be grasped by analyzing different parts which contribute to welfare. Figure 2 presents contributions of different parts of the loss function of H as a function of price rigidity for the specification at Table 3. At the limiting case of fully flexible prices ($\alpha =$

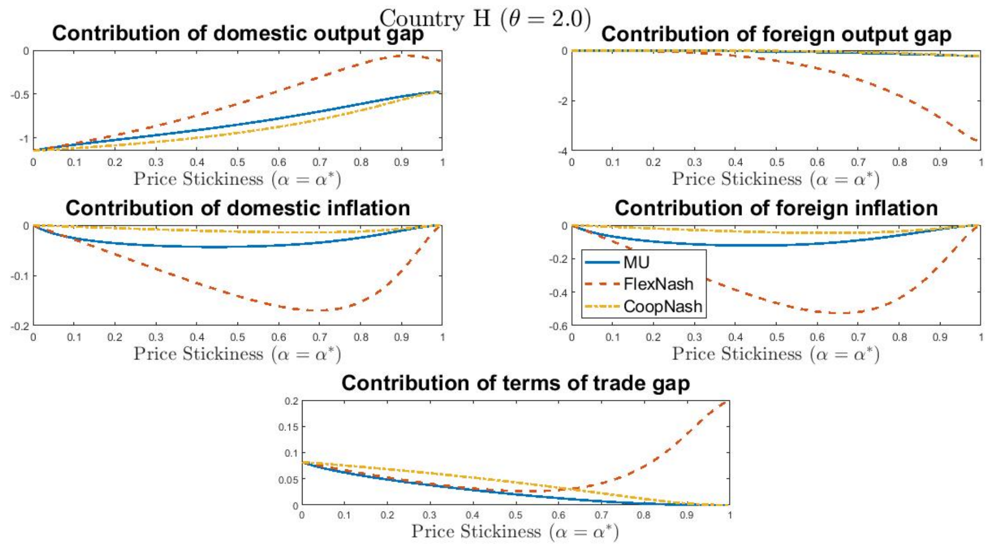

Figure 2: Contributions of the different parts to welfare loss function of H as a function of nominal price rigidity for the parametrization given at Table 3.

$\alpha^* = 0$), prices compensate any exchange rate inertia and monetary policy shock independent of underlying parameters. Therefore, monetary regime becomes irrelevant. As $\alpha \to 0$, output and trade gaps in $\mathcal{M}$ and $\mathcal{F}$ (also in $\mathcal{C}$) converge to each other. For the same reason, both domestic and foreign inflation contribute 0 as $\alpha \to 0$. Figure 1 demonstrates that welfare loss difference between $\mathcal{F}$ and $\mathcal{M}$ disappears as α converges to 0 for any level of trade elasticity. Domestic and foreign PPIs have U-shape non-monotonic contribution. They do not have any contribution to welfare when prices are fully fixed just like when prices are fully flexible. That is because under full price rigidity ($\alpha = \alpha^* = 1$) inflation is always 0 irrespective of monetary regime. In $\mathcal{M}$, the terms of trade converges to 0 as $\alpha \to 1$ for it depends only on domestic and foreign inflation under fixed exchange rate as can be seen from (3.10). Under flexible exchange rate, however, the terms of trade

[13] In the Appendix C, I provide the same figure for F. By symmetry of parameters and shock processes, F's welfare loss difference is similar to H's. Nevertheless, it is also possible that $\mathcal{M}$ dominates $\mathcal{F}$ for a country while $\mathcal{F}$ dominates $\mathcal{M}$ for the other. I provide such examples in Section 5.3.

depends also on exchange rate which is affected by monetary policy which in turn changes output. Therefore, welfare loss difference do not necessarily converge to 0 when $\alpha \to 1$.

A key point derived from Figure 1 is that a monetary union can be welfare increasing or Pareto Improvement (together with Figure 12 in the Appendix C) even in the absence of auxiliary gains from establishing a monetary union such as credibility or trade. It is expected that transferring monetary policymaking power to a new CB whose objective is not solely maximizing welfare of domestic country has a welfare reducing effect. However, losing monetary autonomy is not necessarily welfare reducing in the context of LOEs. In the next chapter, I show that this is because a monetary union inherently imposes a cooperation between its members. A country may choose to abandon its monetary autonomy because it expects welfare gains from the foreign country transferring its monetary autonomy to the CB of the union.

5.1 Monetary Union as a Cooperation

In this chapter, I show that main national welfare difference between the monetary union and the flexible exchange rate regime originates from inherent cooperation imposed by the union.

5.1.1 A special case

To give an intuition, I begin by analyzing the specific case of $\rho = \theta = 1$, i.e the intertemporal elasticity of substitution is equal to the intratemporal elasticity of substitution and both are equal to 1. In this case, the terms of trade is insulated from welfare criteria, i.e. $\lambda_q = \lambda_q^* = 0$. Welfare loss function of H is[14]

$$\tilde{\tilde{W}}_H = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \left[\lambda_{y_h} \left(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^h_{H,t} \right)^2 + \lambda_{y_f} \left(\hat{Y}^*_{F,t} - \tilde{\tilde{Y}}^h_{F,t} \right)^2 + \lambda_{\pi_h} {\pi_{H,t}}^2 + \lambda_{\pi_f} {\pi^*_{F,t}}^2 \right] \tag{5.1}$$

[14] Double tilde refers to the respective values evaluated at $\rho = \theta = 1$. In Appendix A.3, I present $\tilde{\tilde{Y}}^W_{H,t}$, $\tilde{\tilde{Y}}^W_{F,t}$, $\tilde{\tilde{Y}}^h_{H,t}$, $\tilde{\tilde{Y}}^h_{F,t}$, $\tilde{\tilde{Y}}^f_{H,t}$, $\tilde{\tilde{Y}}^f_{F,t}$ explicitly.

Similarly, welfare loss function of F becomes

$$\tilde{\tilde{W}}_F = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Big[\lambda^*_{y_h}\big(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^f_{H,t}\big)^2 + \lambda^*_{y_f}\big(\hat{Y}^*_{F,t} - \tilde{\tilde{Y}}^f_{F,t}\big)^2 + \lambda^*_{\pi_h}{\pi_{H,t}}^2 + \lambda^*_{\pi_f}{\pi^*_{F,t}}^2\Big] \tag{5.2}$$

Furthermore, the terms of trade is eliminated from aggregate supply equations because $\psi = 0$. (3.6) boils down to

$$\pi_{H,t} = \beta\mathbb{E}_t\pi_{H,t+1} + \kappa\big(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^w_{H,t}\big) + u_t \tag{5.3}$$

(3.7) becomes

$$\pi^*_{F,t} = \beta\mathbb{E}_t\pi^*_{F,t+1} + \kappa^*\big(\hat{Y}_{F,t} - \tilde{\tilde{Y}}^w_{F,t}\big) + u^*_t \tag{5.4}$$

This means that in $\mathcal{F}$, monetary authorities cannot change the other country's variables through the terms of trade. A national monetary authority is able to set only its own output and PPI. Therefore, objective function of H's CB is

$$\tilde{\tilde{L}} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Big[\lambda_{y_h}\big(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^h_{H,t}\big)^2 + \lambda_{\pi_h}{\pi_{H,t}}^2\Big] \tag{5.5}$$

Objective function of F's CB is

$$\tilde{\tilde{L}}^* = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Big[\lambda^*_{y_f}\big(\hat{Y}^*_{F,t} - \tilde{\tilde{Y}}^f_{F,t}\big)^2 + \lambda^*_{\pi_f}{\pi^*_{F,t}}^2\Big] \tag{5.6}$$

In this special case, I am able to distinguish the objective function of a national CB from welfare loss function of its country. (5.5) ((5.6)) is the part of (5.1) ((5.2)) that domestic monetary policy can affect, i.e domestic output and PPI. The rest, foreign output and PPI, is under the control of the foreign CB, therefore, it is outside of the domestic CB's objective function. Consequently, monetary policy problem of H is to determine a path for $\{\hat{Y}_{H,t}, \pi_{H,t}\}_{t=0}^{\infty}$ to maximize (5.5) subject to (5.3). The optimal condition for this problem is

$$-\sigma\pi_{H,t} = \Delta\big(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^h_{H,t}\big) \tag{5.7}$$

The CB of F determines $\{\hat{Y}^*_{F,t}, \pi^*_{F,t}\}_{t=0}^{\infty}$ to maximize (5.6) subject to (5.4). Optimality conditions for the CB of F's problem is

$$-\sigma\pi_{F,t} = \Delta\big(\hat{Y}_{F,t} - \tilde{\tilde{Y}}^f_{F,t}\big) \tag{5.8}$$

(5.7) and (5.8) pin down all endogenous variables along with (5.3) and (5.4). Obvious from (5.1) and (5.2), national welfare still depends on monetary policy decisions of the other country.

The first best monetary policy for H, on the other hand, can easily be found by determining the path of $\{\hat{Y}_{H,t}, \hat{Y}_{F,t}, \pi_{H,t}, \pi^*_{F,t}\}_{t=0}^{\infty}$ which maximizes $\tilde{\tilde{W}}_H$. Optimality conditions indicate

$$-\sigma\pi_{H,t} = \Delta\big(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^h_{H,t}\big) \tag{5.9}$$

$$-\sigma\pi_{F,t} = \Delta\big(\hat{Y}_{F,t} - \tilde{\tilde{Y}}^h_{F,t}\big) \tag{5.10}$$

(5.9) and (5.10) combined with (5.3) and (5.4) pin down all endogenous variables. Comparing (5.7) and (5.8) to (5.9) and (5.10), the relationship between domestic output and domestic PPI matches with the first best choice of H as it is decided by the CB of H itself. The relationship between foreign output and foreign PPI, however, is not the first best for H because it is chosen by the CB of F to maximize $\tilde{\tilde{L}}^*$.

In $\mathcal{C}$, the CP maximizes overall welfare $\tilde{\tilde{W}}$.

$$\tilde{\tilde{L}}^W = \tilde{\tilde{W}} = -\frac{\lambda^w_y}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big[n\big(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^w_{H,t}\big)^2 + (1-n)\big(\hat{Y}_{F,t} - \tilde{\tilde{Y}}^w_{F,t}\big)^2 + \frac{\sigma n}{\kappa}{\pi_{H,t}}^2 + \frac{\sigma(1-n)}{\kappa^*}{\pi^*_{F,t}}^2\Big] \tag{5.11}$$

$\tilde{\tilde{Y}}^w_{i,t}$ corresponds to the sum of output targets of H and F averaged by population. The CP chooses a path for $\{\hat{Y}_{H,t}, \hat{Y}_{F,t}, \pi_{H,t}, \pi^*_{F,t}\}_{t=0}^{\infty}$ which maximizes $\tilde{\tilde{W}}$ subject to (5.3) and (5.4). Optimality conditions are

$$-\sigma\pi_{H,t} = \Delta\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big) \tag{5.12}$$

$$-\sigma\pi_{F,t} = \Delta\big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big) \tag{5.13}$$

In $\mathcal{C}$, H diverges from the first best relationship between domestic PPI and domestic output.

Consequently, H loses welfare with this new cooperative equilibrium in terms of domestic parts, $\tilde{\tilde{L}}$. On the other hand, (5.13) improves welfare compared to (5.8) because the CP tries to stabilize F's output at a level closer to H's target compared to CB of F's target. If welfare gains from (5.13) with respect to (5.8) outweigh welfare losses from (5.12) with respect to (5.7) this loss in autonomy benefits H.

In $\mathcal{M}$, the monetary authority of the union chooses $\{\hat{Y}_{H,t}, \hat{Y}_{F,t}, \pi_{H,t}, \pi^*_{F,t}\}$ to maximize $\tilde{\tilde{L}}^W$ as in $\mathcal{C}$ but is additionally restricted by fixed exchange rate (3.10). The optimality condition is

$$-\sigma\big(n\pi_{H,t} + (1-n)\pi_{F,t}\big) = n\Delta\big(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^w_{H,t}\big) + (1-n)\Delta\big(\hat{Y}_{F,t} - \tilde{\tilde{Y}}^w_{F,t}\big) \tag{5.14}$$

Single policy tool restraints monetary policy in $\mathcal{M}$ compared to $\mathcal{C}$ in the following way. For example, in the special case ($\rho = \theta = 1$), a foreign markup shock does not require any change in domestic interest rate as it is independent to domestic monetary policy. However, in $\mathcal{M}$, stabilization in domestic (foreign) output and PPI cannot be insulated from foreign (domestic) output and PPI: H's variables are automatically affected from a monetary response to F's markup shock even though it would have been unnecessary if the exchange rate was flexible. Nevertheless, the targets are identical to the targets in $\mathcal{C}$. Consequently, $\mathcal{M}$ inherently imposes cooperation albeit less efficiently due to single monetary tool and fixed exchange rate.

Figure 3 depicts impulse responses of H's domestic and foreign output gaps, domestic and foreign inflation, and the terms of trade gap to a domestic markup shock under $\mathcal{F}$, $\mathcal{C}$ and $\mathcal{M}$ given the specification at Table 3 except I set $\theta = 1$.[15] In $\mathcal{F}$, a positive shock to H's markup pushes inflation and the terms of trade upwards. Foreign output and foreign inflation does not move because the terms of trade is insulated from aggregate supply relation of H (meaning that the terms of trade does not affect F's output and PPI) and from welfare losses. Therefore, F's monetary authority remains unresponsive to a markup shock in H. The CB of H responds to cost-push shock optimally considering its output gap, $\hat{Y}_{H,t} - \tilde{\tilde{Y}}^h_{H,t}$, and domestic PPI, $\pi_{H,t}$. In $\mathcal{C}$, a positive shock to H's markup pushes the terms of trade and the domestic PPI upwards. F's

[15] ρ is 1. Therefore, I obtain $\rho = \theta = 1$ when I set $\theta = 1$. Furthermore, I note that in the absence of prductivity shocks $\tilde{T}^i_t = 0$ for $i \in \{H, F\}$ and $\tilde{Y}^i_{F,t} = 0$ for $i \in \{H, F\}$ if $\hat{\mu}_{F,t} = 0$.

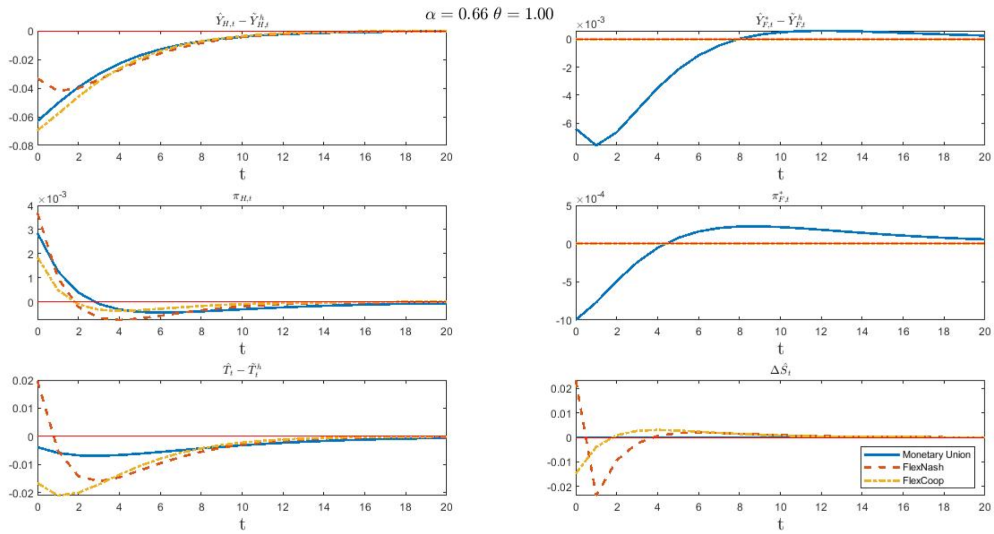

Figure 3: Impulse response functions of a positive domestic markup shock ($\hat{\mu}_h$) for the parametrization at Table 3 with $\theta = 1$ (% deviations from steady state).

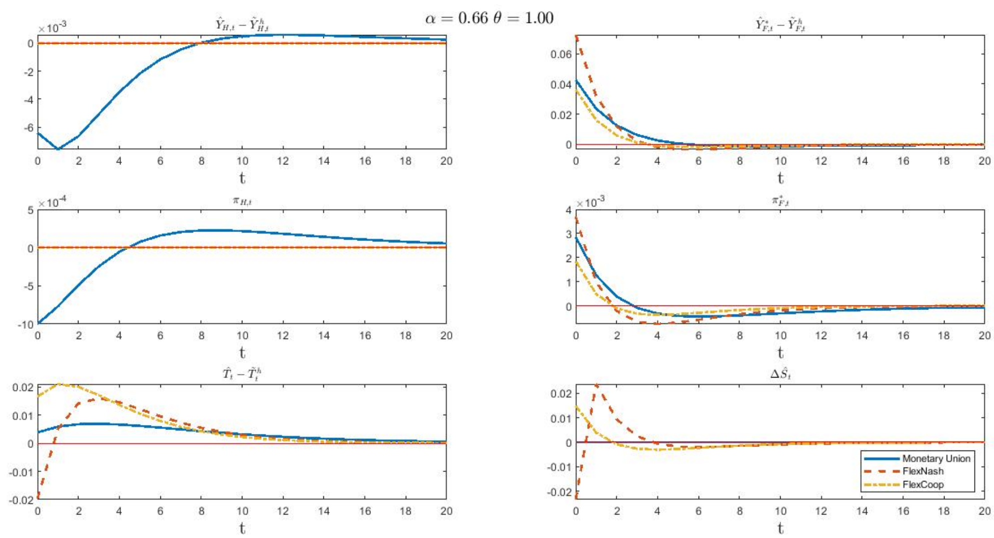

Figure 4: Impulse response functions of a positive foreign markup shock ($\hat{\mu}_f$) for the parametrization at Table 3 with $\theta = 1$ (% deviations from steady state).

output and PPI remain unchanged again because mark-up shock in H does not affect F's output and PPI in any way. However, H's CB, cooperating to maximize overall welfare rather than its own country, stabilizes output to minimize deviation from domestic output target of the overall economy, $\tilde{\tilde{Y}}^w_{H,t}$, rather than domestic output target of H, $\tilde{\tilde{Y}}^h_{H,t}$. This causes a higher deviation for H from its own domestic output target. Consequently, cooperation causes loss in welfare for H

when a domestic markup shock hits the economy. In $\mathcal{M}$, a response to a shock (an interest rate increase or decrease) to stabilize H's output and inflation automatically incites an unwanted shift in foreign output and inflation. Therefore, CB reacts less aggressively then it does compared to $\mathcal{C}$. This, in turn, tempers H's loss from inefficient response of domestic output gap.

Figure 4 presents same graphs for a positive foreign markup shock. This time, with the same reasoning, $\mathcal{C}$ improves H's foreign output gap. In $\mathcal{M}$, H gives away some its improvement in $\mathcal{C}$. Nevertheless, in $\mathcal{M}$, H experiences a smaller deviation from its foreign output target compared to $\mathcal{F}$.

Figure 5 shows contributions of different parts for the same specification. I observe that $\mathcal{C}$

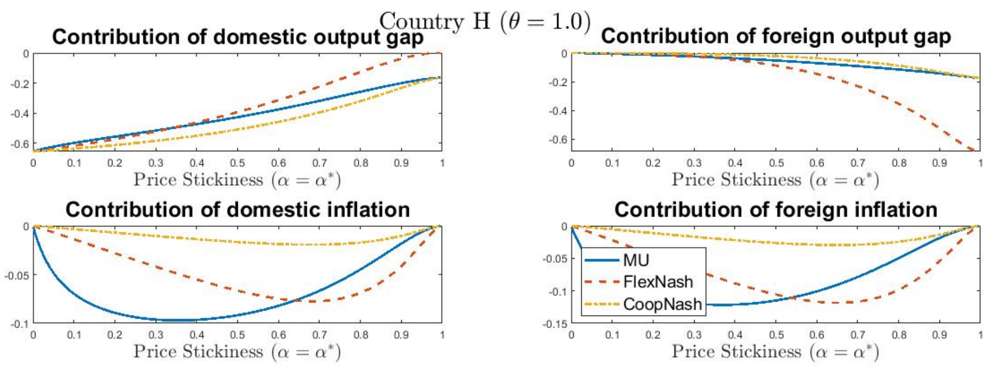

Figure 5: Contributions of the different parts to welfare loss function of H as a function of nominal price rigidity for the parametrization given at Table 3 and $\theta = 1$

strongly improves contributions from foreign output gap while damaging welfare from domestic output gap. The improvement on the former comes from the fact that cooperative policy considers effects of F's output and inflation on H and deterioration on the latter originates from cooperative policy in turn considering effects of H's output and inflation on F.[16]

5.1.2 General Case

I switch back to the specification at Table 3 (I set $\theta = 2$). The terms of trade affects both countries through aggregate supply equations. It is also in welfare criteria. Figure 6 presents

[16]In fact, improvement in welfare is in contribution of sum of foreign inflation and output gap, i.e the part $\tilde{\tilde{L}}$ does not cover in $\tilde{\tilde{W}}_H$, whereas the deterioration is in sum of domestic inflation and output gap.

impulse responses of H's domestic and foreign output gaps, domestic and foreign inflation, and the terms of trade gap to a positive domestic markup shock in $\mathcal{F}$, $\mathcal{C}$, and $\mathcal{M}$. In $\mathcal{F}$, a domestic

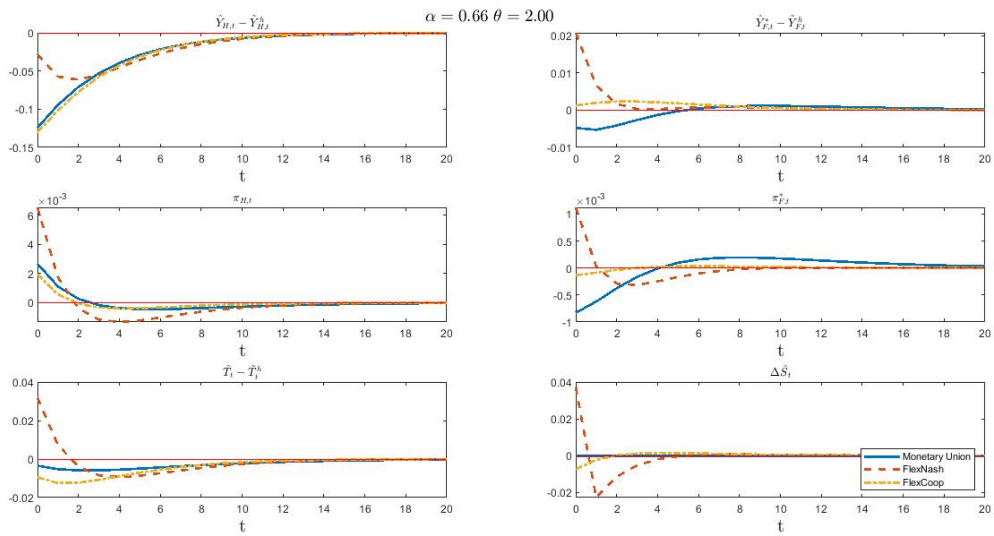

Figure 6: Impulse response functions of a positive domestic markup shock ($\hat{\mu}_h$) for the parametrization at Table 3 with $\theta = 1$ (% deviations from steady state).

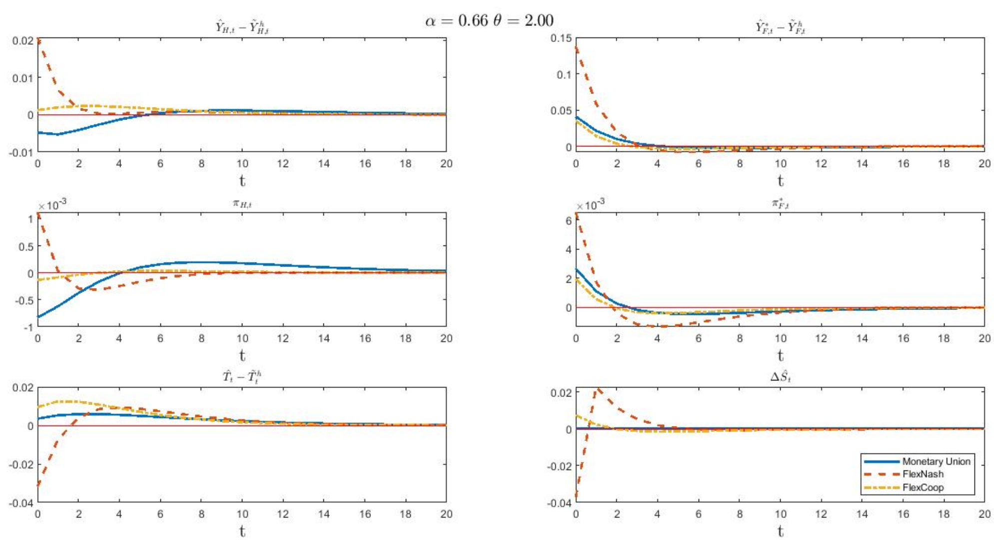

Figure 7: Impulse response functions of a positive domestic markup shock ($\hat{\mu}_f$) for the parametrization at Table 3 with $\theta = 1$ (% deviations from steady state).

positive shock generates domestic inflation and an increase in the terms of trade due to increase in domestic prices. This time, different than $\rho = \theta = 1$ case, a change in the terms of trade affects

F's output and PPI. Therefore, the CB of F responds by increasing inflation in order to adjust to the change in the terms of trade. This reaction moves F's output and expands H's foreign output gap. In $\mathcal{C}$, through the same mechanism I described in the previous section, deviation from domestic output target increases, however, deviation from foreign output target decreases. When a positive foreign markup shock hits the economy as shown at Figure 7, $\mathcal{C}$ improves responses in terms of domestic and foreign output gap compared to $\mathcal{F}$. Furthermore, in both shocks, $\mathcal{C}$ generates less domestic and foreign inflation than $\mathcal{F}$. In $\mathcal{M}$, output gaps (except the domestic output gap when domestic markup shock hits) and PPIs are larger compared to $\mathcal{C}$. Nevertheless, they are still improved relative to $\mathcal{F}$ in terms of H's welfare.

Looking at Figure 2, it is clear that gains in $\mathcal{M}$ originates from foreign output gap whereas losses are from domestic output gap. Figure 8 shows welfare loss difference between $\mathcal{C}$ and $\mathcal{F}$ as a function of price rigidity for different values of θ. Above the zero line, $\mathcal{C}$ dominates $\mathcal{F}$ and vice versa below the zero line. Comparing Figure 1 to Figure 8, I observe that cooperation constitutes

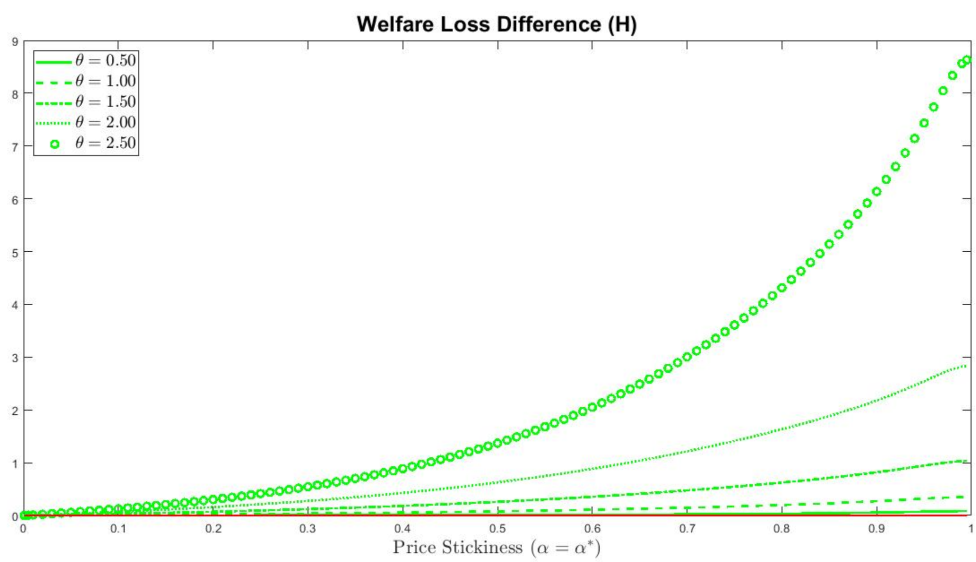

Figure 8: Welfare loss differences of H between $\mathcal{C}$ and $\mathcal{F}$ as a function of nominal price rigidity for the parametrization given at Table 3 and for alternative values of θ. Above the zeroline $\mathcal{C}$, below $\mathcal{F}$ dominates.

main gains from monetary union.

5.2 Trade Elasticity

Welfare loss differences between monetary regimes are strongly related with the elasticity of substitution of domestic and foreign goods θ. The trade elasticity should be considered in two levels: First, as substitutability between international goods decrease, outputs become less sensitive to fluctuations in the terms of trade which can be observed from (3.8). Therefore, the effect of cooperation on welfare diminishes. In the limiting case $\theta \to 0$, each country consumes a constant share of domestic and foreign goods. Second, as θ decreases, inefficiency in the adjustment of the international relative prices increase which in turn, coupled with price rigidity, puts more importance to exchange rate regime. The effect of trade elasticity on monetary regime choice depends on the interplay of the two forces on welfare.

With symmetric parameters presented at Table 3 and symmetric shocks, as θ increases national welfare loss difference shifts on behalf of $\mathcal{M}$ as can be observed from Figure 1. However, this cannot be generalized. For example, Figure 9 presents welfare loss differences of H between $\mathcal{M}$ and $\mathcal{F}$, $\mathcal{M}$ and $\mathcal{C}$, and $\mathcal{C}$ and $\mathcal{F}$ as a function of trade elasticity when variation in H's markup shocks is relatively high. Fixed exchange rate becomes costly as trade elasticity increases. On the other hand,

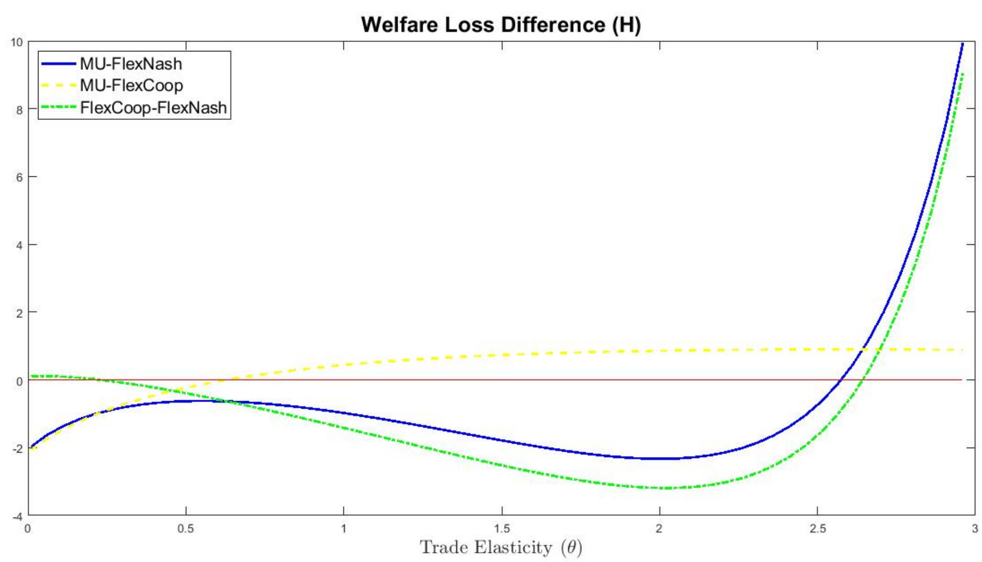

Figure 9: Welfare loss differences of H as a function of trade elasticity for parametrization given at Table 3 when variation in F's markup shocks is relatively high ($\sigma_{\mu_h} = 0.4$, $\sigma_{\mu_f} = 0.1$).

the cost of cooperation surges as the trade elasticity increases up to $\theta = 2.05$. After this point, the cost starts to decrease and eventually cooperation becomes welfare increasing. As a composition of the two forces, the effect of trade elasticity on welfare loss difference is not monotonic. It is also possible that the cost of fixed exchange is non-monotonic.[17] Therefore, depending on relative variation of mark-up shocks, it is possible that higher trade elasticity causes flexible exchange rate regime to dominate monetary union.

5.3 Shocks

I have shown that main welfare gain in $\mathcal{M}$ comes from responses to foreign markup shocks whereas the main welfare loss derives from responses to domestic markup shocks. Therefore, welfare loss difference shifts on behalf of $\mathcal{M}$ as foreign markup shocks vary more than or close to domestic markup shocks. Figure 10 and Figure 11 show welfare loss difference of H and F respectively for alternative standard deviations of H's markup shocks. Under our specification, as variation in H's

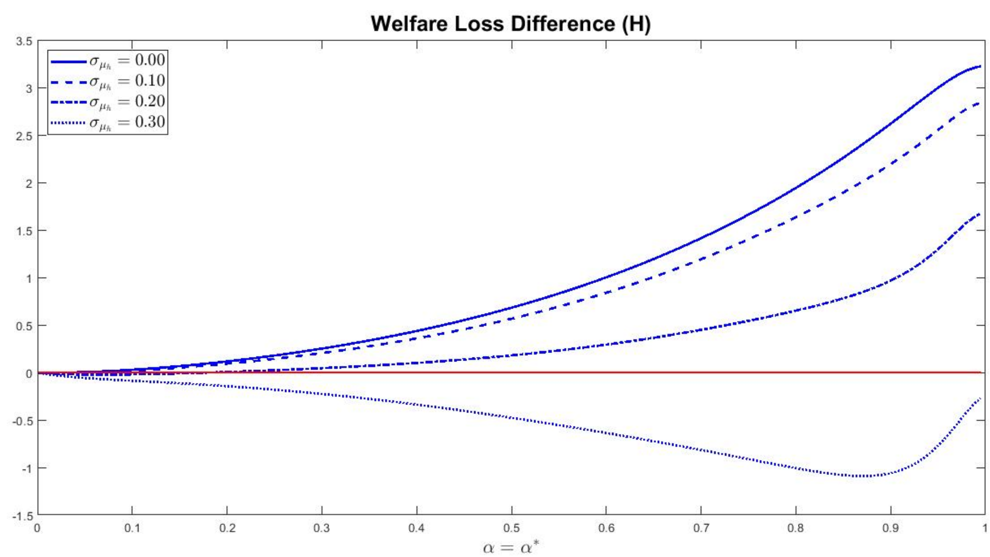

Figure 10: Welfare loss differences of H as a function of nominal price rigidity for the parametrization given at Table 3 and for alternative standard deviations for H's shocks (σ_{μ_h}).

markup shocks outweighs variation in F's markup shocks, $\mathcal{F}$ becomes preferable for H. That is

[17] In Appendix C, I present welfare loss difference of F between $\mathcal{M}$ and $\mathcal{C}$ as a function of the trade elasticity for the same specification in Figure 9. The cost of fixed exchange rate decreases with the trade elasticity up to $\theta = 1.1$, than it starts increasing with θ.

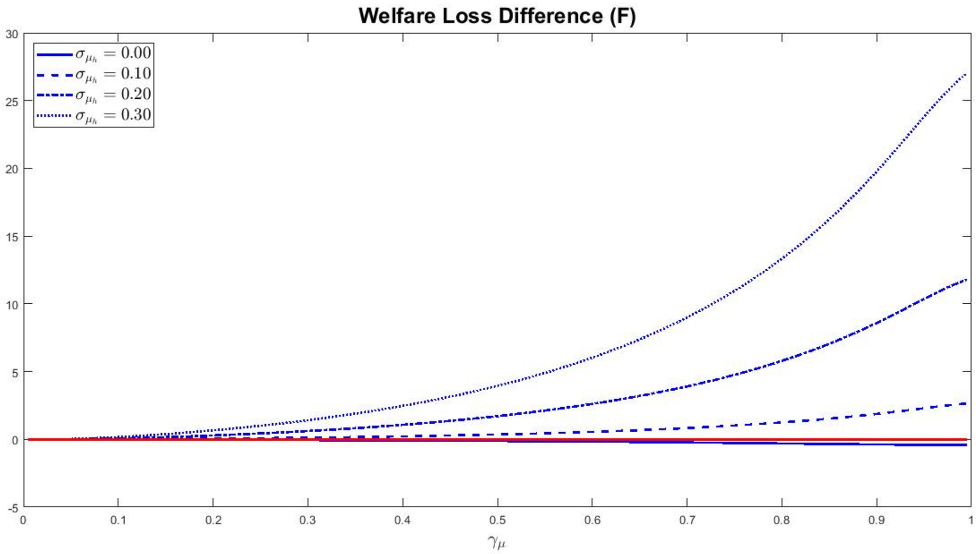

Figure 11: Welfare loss differences of F as a function of nominal price rigidity for the parametrization given at Table 3 and for alternative standard deviations for H's shocks (σ_{μ_h}).

because H prefers to dismiss F when it responds to large domestic markup shocks. It is willing to bear the relatively small (because F's markup shocks are close to or smaller than H's markup shocks) cost of F's monetary responses which ignores H's welfare. If variation in F's markups is high but H's markup is low, then H particularly suffers from markup shocks in F because F disregards welfare spillovers of its policy actions. Then, H prefers $\mathcal{M}$ over $\mathcal{F}$ because in $\mathcal{M}$ the CB considers welfare spillovers of F's shocks on H and responds accordingly. In this way, by the nature of monetary union, H compels F to cooperate in monetary policy. A direct reference to 'tying one's hand', I call this 'tying the other's hand'. Since this mechanism applies to F as well, $\mathcal{M}$ becomes preferable for F as variation in H's markups increases and vice versa. Consequently, if variation of markup shocks are close in H and F, then monetary union is Pareto Improvement. For example, if $\sigma_{\mu_h} = 0.3$ and $\alpha = \alpha^* = 0.5$, then H prefers $\mathcal{F}$ but F prefers $\mathcal{M}$. In fact, when $\sigma_{\mu_h} = 0.3$, monetary union is welfare decreasing for H at any level of price rigidity.[18] However, for $\sigma_{\mu_h} = 0.2$, monetary union is Pareto Improvement if $0.19 \leq \alpha$.

[18]In fact, Figure 10 and Figure 11 exemplify many cases in which $\mathcal{M}$ is preferable for a country but not for the other.

5.3.1 Productivity Shocks

Unlike markup shocks which distort the economy asymmetrically (through aggregate supply equations), productivity shocks create symmetric deviations from targets: for example a positive productivity shock in H or F causes the same deviations from inflation and output targets for H and F but in the opposite directions.[19] Therefore, differences in the mean preserving spread of productivity shocks are irrelevant to monetary regime choices. Cooperation becomes beneficial only to terminate beggar-thy-neighbor effect. In return, a single interest rate fails to respond optimally to two different productivity shocks. However, optimal monetary policy responses converge as shocks become similar.

Consequently, I reaffirm Pappa (2004)'s findings. A country exploits cooperation in monetary union but suffers from losing exchange rate flexibility or from one-size-fits-all monetary policy. As variation in productivity shocks increases both the need for cooperation and for flexibility in exchange rate surge. The choice of monetary regime depends on whichever dominates the other which in turn is determined by model parameters, e.g price rigidity and trade elasticity.[20] However, for a given variation of productivity shocks, the cost of fixed exchange rate decreases as shocks become similar.

If $\rho = \theta = 1$, then monetary union is welfare decreasing in national level (implying that it is welfare decreasing economy-wide). That is because domestic output targets of both countries, therefore, cooperative targets coincide, (ii) gains from shifting the terms of trade is 0. Then, $\mathcal{F}$ mimics $\mathcal{C}$'s allocations. $\mathcal{M}$, being a constrained $\mathcal{C}$ can produce welfare only as high as $\mathcal{C}$. If productivity shocks are perfectly correlated, then welfare loss difference between $\mathcal{F}$ and $\mathcal{M}$ is 0 because in addition to (i) and (ii) inefficiency due to having a single policy tool disappears.

However, with markup shocks along with productivity shocks, results remain unchanged.[21] If

[19]Impulse response functions are presented in Appendix C.

[20]In Appendix C, I present welfare loss differences between $\mathcal{M}$ and $\mathcal{F}$ as a function of price rigidity for various calibrations when only productivity shocks exists.

[21]In Appendix C, I present welfare loss differences of H and F as a function of nominal price rigidity for the parametrization given at Table 3 and for alternative values of θ. Variances of markup and productivity shocks are equal as presented in Table 3.

variance of productivity (efficient) shocks outweigh variance of markup (inefficient) shocks, the classical criterion stands: Countries with similar shocks should establish monetary unions. Otherwise, if inefficient shocks dominate the economy, countries should form a monetary union depending on relative variation (mean preserving spread) of these shocks.

6 Conclusion

This paper explores monetary regime choice between monetary union and flexible exchange rate regime in a LOE framework. I construct a two-country dynamic general equilibrium model based on Corsetti et al. (2010). I introduce two shocks: productivity shocks (efficient) which do not create trade offs in macroeconomic stabilization, and markup (inefficient) shocks which cause trade offs between output and inflation stabilizations. I assume that in flexible exchange rate regime, national CBs compete to maximize welfare of their own country. In monetary union, the CB aims to maximize overall welfare. In both regimes, monetary authorities are credible, i.e they pursue monetary policy with commitment. The model cannot be solved in closed form, therefore, I rely on LQ approach following Benigno and Benigno (2006) and numerical analysis.

The classical approach argues that monetary unions should be established between countries with similar shocks. This study reveals that proximity of shocks is not a good criterion when there are inefficient shocks creating asymmetric trade offs in macroeconomic stabilization and countries are large enough to have an impact on each other. In that case, monetary regime choice should depend on relative variation (mean preserving spread) of inefficient shocks.

In flexible exchange rate regime, a country suffers significantly from foreign monetary authority's policy responses to foreign inefficient shocks. Monetary union, however, inherently imposes cooperation in the sense that the CB of the union takes into account welfare spillovers across countries. Cooperation improves responses to foreign inefficient shocks whereas it worsens responses to domestic ones slightly less. Therefore, a country chooses to establish a monetary union with another if variation of the foreign country's inefficient shocks are close to or larger than variation of domestic inefficient shocks on the condition that adopting a common currency (fixed exchange rate) has low cost or is not costly at all. In this way, a country uses the union as a tool 'to tie the hand of the foreign country', i.e to compel the foreign country to cooperate and to consider monetary policy spillovers. Since this is a reciprocal relation, a monetary union is Pareto Improvement if inefficient shocks of its members have close mean preserving spreads (variance). How close these spreads should be depends strongly on price rigidity and trade elasticity.

References

Alesina, A. and Barro, R. J. (2002). Currency unions*. *The Quarterly Journal of Economics*, 117(2):409.

Barro, R. J. and Gordon, D. B. (1983). Rules, discretion and reputation in a model of monetary policy. *Journal of monetary economics*, 12(1):101–121.

Benigno, G. and Benigno, P. (2006). Designing targeting rules for international monetary policy cooperation. *Journal of Monetary Economics*, 53(3):473–506.

Benigno, P. (2002). A simple approach to international monetary policy coordination. *Journal of International Economics*, 57(1):177–196.

Benigno, P. (2004). Optimal monetary policy in a currency area. *Journal of international economics*, 63(2):293–320.

Benigno, P. and Woodford, M. (2005). Inflation stabilization and welfare: The case of a distorted steady state. *Journal of the European Economic Association*, 3(6):1185–1236.

Benigno, P. and Woodford, M. (2006). Optimal taxation in an rbc model: A linear-quadratic approach. *Journal of Economic Dynamics and Control*, 30(9-10):1445–1489.

Bilbiie, F. O. (2018). Monetary policy and heterogeneity: An analytical framework.

Blanchard, O. and Galí, J. (2007). Real wage rigidities and the new keynesian model. *Journal of money, credit and banking*, 39:35–65.

Calvo, G. A. (1983). Staggered prices in a utility-maximizing framework. *Journal of monetary Economics*, 12(3):383–398.

Chari, V. V., Dovis, A., and Kehoe, P. J. (2019). Rethinking optimal currency areas. *Journal of Monetary Economics.*

Christiano, L. J., Trabandt, M., and Walentin, K. (2010). Dsge models for monetary policy analysis. In *Handbook of monetary economics*, volume 3, pages 285–367. Elsevier.

Clarida, R., Gali, J., and Gertler, M. (1999). The science of monetary policy: a new keynesian perspective. *Journal of economic literature*, 37(4):1661–1707.

Clarida, R., Galı, J., and Gertler, M. (2002). A simple framework for international monetary policy analysis. *Journal of monetary economics*, 49(5):879–904.

Clerc, L., Dellas, H., and Loisel, O. (2011). To be or not to be in monetary union: A synthesis. *Journal of International Economics*, 83(2):154–167.

Corsetti, G., Dedola, L., and Leduc, S. (2010). Optimal monetary policy in open economies. In *Handbook of monetary economics*, volume 3, pages 861–933. Elsevier.

Corsetti, G. and Pesenti, P. (2001). Welfare and macroeconomic interdependence. *The Quarterly Journal of Economics*, 116(2):421–445.

Debortoli, D. and Galí, J. (2017). Monetary policy with heterogeneous agents: Insights from tank models. *Manuscript, September.*

Dellas, H. and Tavlas, G. S. (2009). An optimum-currency-area odyssey. *Journal of International Money and Finance*, 28(7):1117–1137.

Faia, E. and Monacelli, T. (2008). Optimal monetary policy in a small open economy with home bias. *Journal of Money, credit and Banking*, 40(4):721–750.

Gali, J., Gertler, M., and Lopez-Salido, J. D. (2007). Markups, gaps, and the welfare costs of business fluctuations. *The review of economics and statistics*, 89(1):44–59.

Giavazzi, F. and Pagano, M. (1988). The advantage of tying one's hands: Ems discipline and central bank credibility. *European Economic Review*, 32(5):1055–1075.

Groll, D. and Monacelli, T. (2018). The inherent benefit of monetary unions.

Kaplan, G., Moll, B., and Violante, G. L. (2018). Monetary policy according to hank. *American Economic Review*, 108(3):697–743.

Kennen, P. B. (1969). The optimum currency area: An eclectic view. *Mundell, R., Swoboda, A.(eds.): Monetary Problems in the International Economy.*

Kim, J. and Kim, S. H. (2003). Spurious welfare reversals in international business cycle models. *journal of International Economics*, 60(2):471–500.

McKinnon, R. I. (1963). Optimum currency areas. *The American Economic Review*, 53(4):717–725.

Mundell, R. A. (1961). A theory of optimum currency areas. *The American Economic Review*, 51(4):657–665.

Pappa, E. (2004). Do the ecb and the fed really need to cooperate? optimal monetary policy in a two-country world. *Journal of Monetary Economics*, 51(4):753–779.

Rotemberg, J. J. and Woodford, M. (1998). An optimization-based econometric framework for the evaluation of monetary policy: expanded version.

Soffritti, M. and Zanetti, F. (2008). The advantage of tying one's hands: Revisited. *International Journal of Finance & Economics*, 13(2):135–149.

Woodford, M. (1999). Commentary: How should monetary policy be conducted in an era of price stability? *New challenges for monetary policy*, 277316.

Woodford, M. (2011). *Interest and prices: Foundations of a theory of monetary policy*. princeton university press.

A Appendix for the Model

A.1 Deterministic Steady State

In this part, I show that there exist a zero-inflation steady state for deterministic problem of $\mathcal{F}$, $\mathcal{C}$ and $\mathcal{M}$. If government spending is closed i.e, $G_t = \forall t$ and $\bar{G} = 0$ (equivalent to $s_c = 1$), then Benigno and Benigno (2006)'s model boils down to my model. Flexible exchange rate regime ($\mathcal{F}$) and coopertive regime ($\mathcal{C}$) in my model correspond to non-cooperative regime and cooperative regime in Benigno and Benigno (2006) respectively. Therefore, following Benigno and Benigno (2006), it is trivial that for $F_0 = \bar{F}_0$, $F_0^* = \bar{F}_0^*$, $K_0 = \bar{K}_0$, $K_0^* = \bar{K}_0^*$ such that $\frac{\bar{F}_0}{\bar{K}_0} = 1$ and $\frac{\bar{F}_0^*}{\bar{K}_0^*} = 1$, there exist a zero inflation steady state for $\mathcal{F}$ and $\mathcal{C}$. This steady state can be identified by

$$\bar{a}^{-\eta}(\bar{C} + \bar{G})^{\eta} = \bar{\mu}^{-1}\bar{C}^{-\rho} \tag{A.1}$$

$$\bar{Y}_H = \bar{Y}_F = \bar{Y} = \bar{C} \tag{A.2}$$

$$\bar{F} = \bar{K} = \bar{\mu}^{-1}\bar{C}^{-\rho}\bar{Y} \tag{A.3}$$

and $\frac{P_{H,t}}{P_t} = \frac{P_{F,t}}{P_t} = \Pi_{H,t} = \Pi_{F,t} = 1$, $C_t = \bar{C}$, $Y_{H,t} = Y_{F,t} = \bar{Y}_H = \bar{Y}_F = \bar{Y}$, $F_t = K_t = F_t^* = K_t^* = \bar{F} = \bar{K} = \bar{F}^* = \bar{K}^*$.

Then, I must show that $\mathcal{M}$ has the same deterministic steady state. $\mathcal{M}$ differs from $\mathcal{C}$ only in terms of exchange rate regime: it additionally has fixed exchange rate condition (2.11). Therefore, in order to show that $\mathcal{M}$ has the same steady state, it is sufficient to prove that this steady state satisfies (2.11).[22] To show that, I take the fixed exchange rate condition.

$$\begin{aligned}
&\frac{T_t}{T_{t-1}} = \frac{S_t}{S_{t-1}}\frac{\Pi_{F,t}^*}{\Pi_{H,t}} \\
\iff &\frac{P_{F,t}}{P_{H,t}}\frac{P_{H,t-1}}{P_{F,t-1}} = \frac{S_t}{S_{t-1}}\frac{\Pi_{F,t}^*}{\Pi_{H,t}} \\
\iff &\frac{P_{F,t}}{P_t}\frac{P_t}{P_{H,t}}\frac{P_{t-1}}{P_{F,t-1}}\frac{P_{H,t-1}}{P_{t-1}} = \frac{S_t}{S_{t-1}}\frac{\Pi_{F,t}^*}{\Pi_{H,t}}
\end{aligned}$$

[22]This comes from the following result: Let $x^* = \arg\max f(x)$ subject to $g(x) = 0$ and also let $h(x^*) = 0$. Then, $x^* = \arg\max f(x)$ subject to $g(x) = 0$ and $h(x^*) = 0$.

Given the steady state variables, the above equation becomes

$$\frac{S_t}{S_{t-1}} = 1 \quad \forall t$$
$$\iff \Delta S_t = 0 \quad \forall t$$

Then, the steady state defined above satisfies (2.11) meaning that $\mathcal{M}$ has the same zero-inflation steady state as $\mathcal{F}$ and $\mathcal{C}$ and it is the one defined above.

A.2 Structural Equations

I derive structural equations provided at Table 2. Aggregate Supply relations (3.6) and (3.7), and the terms of trade - output relation (3.8) are derived in Benigno and Benigno (2006). I additionally derive risk sharing condition, aggregate demand equation, market clearing conditions and law of motion of exchange rate.

By market completeness assumption and consumer's problem, for any household pair $\{j,k\}$

$$\frac{C^j_{t+1}}{C^k_{t+1}} = \frac{C^j_t}{C^k_t} \quad \forall t$$
$$\iff \frac{C^j_t}{C^k_t} = \frac{C^j_0}{C^k_0} \quad \forall t$$

Ex-ante identical household assumption leads to $C^j_t = C^k_t \implies \hat{C}_t = \hat{C}^*_t$.

For aggregate demand equation, I refer again to consumer's problem. By (Woodford, 2011, Chapter 2), I find that

$$\mathbb{E}_t \frac{{C_t}^{-\rho}}{{C_{t+1}}^{-\rho}} \frac{P_{t+1}}{P_t} = (1+i_t)^{-1}$$

Similarly, I have $(1+i^*_t)^{-1} = \mathbb{E}_t \frac{{C^*_t}^{-\rho}}{{C^*_{t+1}}^{-\rho}} \frac{P^*_{t+1}}{P^*_t}$. Using (2.9), the above equation can be written as

$$\mathbb{E}_t \frac{{C_t}^{-\rho}}{{C_{t+1}}^{-\rho}} \frac{\left[n + (1-n){T_{t+1}}^{1-\theta}\right]^{\frac{1}{1-\theta}}}{\left[n + (1-n){T_t}^{1-\theta}\right]^{\frac{1}{1-\theta}}} \Pi_{H,t+1} = (1+i_t)^{-1} \tag{A.4}$$

I log-linearize market clearing condition $Y_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_{H,t}$. I rewrite this condition as follows:

$$Y_{H,t} = \left(\frac{1}{\left[n + (1-n){T_{t+1}}^{1-\theta}\right]^{1/(1-\theta)}}\right)^{-\theta} C_{H,t} \tag{A.5}$$

$$\begin{aligned}
&\bar{Y}\hat{Y}_{H,t} = \bar{C}\hat{C}_t + (1-n)\theta\bar{C}\hat{T}_t \\
\Longleftrightarrow\ &\bar{Y}\hat{Y}_{H,t} = \bar{C}\hat{C}_t + (1-n)\theta\bar{C}\hat{T}_t && \text{(A.6)} \\
\Longleftrightarrow\ &\hat{Y}_{H,t} = \hat{C}_t + \theta(1-n)\hat{T}_t && \text{(A.7)}
\end{aligned}$$

The foreign counterpart is

$$\hat{Y}_{F,t} = \hat{C}_t - \theta n\hat{T}_t$$

Law of motion for exchange rate is the following:

$$\begin{aligned}
&\frac{P_{F,t}}{P_{H,t}}\frac{P_{H,t-1}}{P_{F,t-1}} = \frac{S_t}{S_{t-1}}\frac{P^*_{F,t}}{P_{H,t}}\frac{P_{H,t-1}}{P^*_{F,t-1}} \\
\Longleftrightarrow\ &\frac{T_t}{T_{t-1}} = \frac{S_t}{S_{t-1}}\frac{\Pi^*_{F,t}}{\Pi_{H,t}} && \text{(A.8)}
\end{aligned}$$

The above equation is law of motion for flexible exchange rate. First order log-linearization of (A.8) is (3.9). When exchange rate is fixed, I have $S_t = S_{t-1}\ \forall t$. Then, I have the following condition.

$$\frac{T_t}{T_{t-1}} = \frac{\Pi^*_{F,t}}{\Pi_{H,t}} \tag{A.9}$$

Similarly, (3.10) is loglinearized (A.9).

A.3 Welfare Criteria

Welfare criteria are identical to the ones in Benigno and Benigno (2006). Nevertheless, for convenience I provide a sketch of derivations. For details, I refer to Technical Appendix of the relevant paper.[23] Second order approximation to period t utility is

$$
\begin{aligned}
w_t = \bar{U}_C\bar{C}\Big[&\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 - \bar{\mu}^{-1}\hat{Y}_{H,t} \\
&- \frac{1}{2}\bar{\mu}^{-1}(1+\eta)\hat{Y}_{H,t}^2 + \bar{\mu}^{-1}\eta\hat{a}\hat{Y}_{H,t} \\
&- \frac{1}{2}\bar{\mu}^{-1}(\sigma^{-1}+\eta)\mathrm{var}_h\hat{y}_t(h)\Big] \\
&+ \mathrm{t.i.p} + \mathcal{O}(||\xi||^3)
\end{aligned} \tag{A.10}
$$

Using (Woodford, 2011, Chapter 6), I find that

$$
\sum_{t=0}^{\infty}\beta^t\mathrm{var}_h\hat{y}_t(h) = \sum_{t=0}^{\infty}\beta^t\mathrm{var}_h\hat{y}_t(h) = \frac{1}{k(1+\sigma\eta)}\sigma^2\sum_{t=0}^{\infty}\beta^t{\pi_{H,t}}^2 = \frac{\sigma^2}{k(1+\sigma\eta)}\sum_{t=0}^{\infty}\beta^t{\pi_{H,t}}^2 \tag{A.11}
$$

Merging the two equations above and summing over t with discount factor β, I obtain

$$
\begin{aligned}
W_H = \bar{U}_C\bar{C}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[&\hat{C}_t + \frac{1}{2}(1-\rho)\hat{C}_t^2 - \bar{\mu}^{-1}\hat{Y}_{H,t} \\
&- \frac{1}{2}\bar{\mu}^{-1}(1+\eta)\hat{Y}_{H,t}^2 + \bar{\mu}^{-1}\eta\hat{a}\hat{Y}_{H,t} \\
&- \frac{1}{2}\bar{\mu}^{-1}\sigma k^{-1}{\pi_{H,t}}^2\Big] \\
&+ \mathrm{t.i.p} + \mathcal{O}(||\xi||^3)
\end{aligned} \tag{A.12}
$$

I keep following Benigno and Benigno (2006) for the rest of the derivation. It basically requires second order approximation to structural equations and incorporating them into second order approximation to utility to cancel linear terms

$$
\begin{aligned}
W_H = -\frac{1}{2}\bar{U}_C\bar{C}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t&\left[x_t'Q_x x_t + 2x_t'Q_\xi\xi_t + q_{\pi_h}{\pi_{H,t}}^2 + q_{\pi_f}{\pi_{F,t}^*}^2\right] \\
&- \bar{U}_C\bar{C}\zeta_1\left[a_x'x_0 + a_\xi'\xi_0 + \frac{1}{2}x_0'A_\xi\xi_0 + \frac{1}{2}a_{\pi_h}{\pi_{H,0}}^2\right] \\
&- \bar{U}_C\bar{C}\zeta_2\left[b_x'x_0 + b_\xi'\xi_0 + \frac{1}{2}b_0'B_\xi\xi_0 + \frac{1}{2}b_{\pi_f}{\pi_{F,0}}^2\right] \\
&K_0 + \mathrm{t.i.p} + \mathcal{O}(||\xi||^3)
\end{aligned} \tag{A.13}
$$

[23]The technical appendix can be found at `http://personal.lse.ac.uk/BENIGNO/`

where $K_0 \equiv \bar{U}_C \bar{C}\left[\zeta_1 V_0 + \zeta_2 V_0^*\right]$ and $x_t' \equiv [\hat{Y}_{H,t}\ \hat{C}_t\ \hat{p}_{H,t}\ \hat{Y}_{H,t}^*\ \hat{C}_t^*\ \hat{p}_{F,t}^*\ \hat{T}_t]$. Following the same process for foreign country I have

$$
\begin{aligned}
W_F = -\frac{1}{2}\bar{U}_C\bar{C}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[x_t'Q_x^*x_t + 2x_t'Q_\xi^*\xi_t + q_{\pi_h}^*{\pi_{H,t}}^2 + q_{\pi_f}^*{\pi_{F,t}^*}^2\right]& \\
- \bar{U}_C\bar{C}\zeta_1^*\left[a_x'x_0 + a_\xi'\xi_0 + \frac{1}{2}x_0'A_\xi\xi_0 + \frac{1}{2}a_{\pi_h}{\pi_{H,0}}^2\right]& \\
- \bar{U}_C\bar{C}\zeta_2^*\left[b_x'x_0 + b_\xi'\xi_0 + \frac{1}{2}b_0'B_\xi\xi_0 + \frac{1}{2}b_{\pi_f}{\pi_{F,0}}^2\right]& \\
K_0^* + \text{t.i.p} + \mathcal{O}(||\xi||^3)&
\end{aligned} \tag{A.14}
$$

where $K_0^* \equiv \bar{U}_C\bar{C}\left[\zeta_1^* V_0 + \zeta_2^* V_0^*\right]$. Then, again following Benigno and Benigno (2006), I obtain

$$
\begin{aligned}
W_H = -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[\lambda_{y_h}\big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^h\big)^2 + \lambda_{y_f}\big(\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^h\big)^2& \\
+ \lambda_q\big(\hat{T}_t - \tilde{T}_t^h\big)^2 + \lambda_{\pi_h}{\pi_{H,t}}^2 + \lambda_{\pi_F}{\pi_{F,t}^*}^2\Big]& \\
+ K_0 + \text{t.i.p} + \mathcal{O}(||\xi||^3)&
\end{aligned} \tag{A.15}
$$

$$
\begin{aligned}
W_F = -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[\lambda_{y_h}^*\big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^f\big)^2 + \lambda_{y_f}^*\big(\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^f\big)^2& \\
+ \lambda_q^*\big(\hat{T}_t - \tilde{T}_t^f\big)^2 + \lambda_{\pi_h}^*{\pi_{H,t}}^2 + \lambda_{\pi_F}^*{\pi_{F,t}^*}^2\Big]& \\
+ K_0^* + \text{t.i.p} + \mathcal{O}(||\xi||^3)&
\end{aligned} \tag{A.16}
$$

Welfare criterion of monetary authority of the union is

$$
\begin{aligned}
W = -\frac{\lambda_y^w}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[n\big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w\big)^2 + (1-n)\big(\hat{Y}_{F,t} - \tilde{Y}_{F,t}^w\big)^2& \\
+ n(1-n)\theta\psi\big(\hat{T}_t - \tilde{T}_t^w\big)^2 + \frac{\sigma n}{\kappa}{\pi_{H,t}}^2 + \frac{\sigma(1-n)}{\kappa^*}{\pi_{F,t}^*}^2\Big]& \\
K_0^W + \text{t.i.p} + \mathcal{O}(||\xi||^3)&
\end{aligned} \tag{A.17}
$$

where $K_0^W \equiv nK_0 + (1-n)K_0^*$. V_0 and V_0^* are defined in Benigno and Woodford (2005) and Benigno and Benigno (2006). They depend on $t = 0$ inflation and on monetary policy in succeeding periods. They are constituted of $t = 0$ inflation, and output at $t = 0$ and in succeeding terms. This means that they depend on monetary policy. Let V_0 and V_0^* be given, then K_0, K_0^*, and K_0^W are also determined. It follows that first order approximations to V_0 and V_0^* are $\pi_{H,0}$ and $\pi_{H,0}^*$ respectively. Then, conditional on $\pi_{H,0}$ and $\pi_{F,0}^*$, I have

$$\begin{aligned} W_H = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[& \lambda_{y_h} \big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^h\big)^2 + \lambda_{y_f} \big(\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^h\big)^2 \\ & + \lambda_q \big(\hat{T}_t - \tilde{T}_t^h\big)^2 + \lambda_{\pi_h} {\pi_{H,t}}^2 + \lambda_{\pi_F} {\pi_{F,t}^*}^2 \Big] \\ & + \text{t.i.p} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{A.18}$$

$$\begin{aligned} W_F = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[& \lambda_{y_h}^* \big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^f\big)^2 + \lambda_{y_f}^* \big(\hat{Y}_{F,t}^* - \tilde{Y}_{F,t}^f\big)^2 \\ & + \lambda_q^* \big(\hat{T}_t - \tilde{T}_t^f\big)^2 + \lambda_{\pi_h}^* {\pi_{H,t}}^2 + \lambda_{\pi_F}^* {\pi_{F,t}^*}^2 \Big] \\ & + \text{t.i.p} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{A.19}$$

$$\begin{aligned} W = -\frac{\lambda_y^w}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Big[& n\big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w\big)^2 + (1-n)\big(\hat{Y}_{F,t} - \tilde{Y}_{F,t}^w\big)^2 \\ & + n(1-n)\theta\psi\big(\hat{T}_t - \tilde{T}_t^w\big)^2 + \frac{\sigma n}{\kappa} {\pi_{H,t}}^2 + \frac{\sigma(1-n)}{\kappa^*} {\pi_{F,t}^*}^2 \Big] \\ & + \text{t.i.p} + \mathcal{O}(||\xi||^3) \end{aligned} \tag{A.20}$$

Weights of output, inflation, and the terms of trade targets are

$\lambda_{y_h} \equiv \lambda_y^w \Big[1 - (1-n)\frac{\theta(1+\eta)}{1+\eta\theta}\Big]$	$\lambda_{y_h}^* \equiv n\Big[\lambda_y^w \frac{\theta(1+\eta)}{1+\eta\theta}\Big]$
$\lambda_{y_f} \equiv (1-n)\Big[\lambda_y^w \frac{\theta(1+\eta)}{1+\eta\theta}\Big]$	$\lambda_{y_f}^* \equiv \lambda_y^w \Big[1 - n\frac{\theta(1+\eta)}{1+\eta\theta}\Big]$
$\lambda_q \equiv (1-\theta\rho)\theta(1-n)\Big[n\frac{\theta(1+\eta)}{1+\eta\theta} + (1-n)\frac{1-\theta}{1+\eta\theta}\Big]$	$\lambda_q^* \equiv (1-\theta\rho)\theta n\Big[(1-n)\frac{\theta(1+\eta)}{1+\eta\theta} + n\frac{1-\theta}{1+\eta\theta}\Big]$
$\lambda_{\pi_h} \equiv \frac{\sigma}{k}\Big[n\frac{\theta(1+\eta)}{1+\eta\theta} + \frac{1-\theta}{1+\eta\theta}\Big]$	$\lambda_{\pi_h}^* \equiv \frac{\sigma}{k} n \frac{\theta(1+\eta)}{1+\eta\theta}$
$\lambda_{\pi_f} \equiv \frac{\sigma}{k}(1-n)\frac{\theta(1+\eta)}{1+\eta\theta}$	$\lambda_{\pi_f}^* \equiv \frac{\sigma}{k}\Big[(1-n)\frac{\theta(1+\eta)}{1+\eta\theta} + \frac{1-\theta}{1+\eta\theta}\Big]$

Output and the terms of trade targets are

$$\tilde{Y}^h_{H,t} \equiv \frac{1}{\lambda_{y_h}}\Big[\eta\frac{1-\theta+n\theta(1+\eta)-n(1-n)(1-\theta\rho)}{1+\eta\theta}\hat{a}_H + \frac{(1-n)n\eta(1-\rho\theta)}{1+\eta\theta}\hat{a}_{F,t} + (1-n)\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{H,t}\Big] \tag{A.21}$$

$$\tilde{Y}^h_{F,t} \equiv \frac{1}{\lambda_{y_f}}\Big[\frac{(1-n)n\eta(1-\rho\theta)}{1+\eta\theta}\hat{a}_{H,t} + (1-n)\eta\frac{\theta(1+\eta)-n(1-\theta\rho)}{1+\eta\theta}\hat{a}_{F,t} - (1-n)\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{F,t}\Big] \tag{A.22}$$

$$\tilde{T}^h_t \equiv \frac{1}{\lambda_q}\eta\theta\frac{1-\theta\rho}{1+\eta\theta}(\hat{a}_{H,t}-\hat{a}_{F,t}) \tag{A.23}$$

$$\tilde{Y}^f_{H,t} \equiv \frac{1}{\lambda^*_{y_h}}\Big[n\eta\frac{\theta(1+\eta)-(1-n)(1-\theta\rho)}{1+\eta\theta}\hat{a}_{H,t} + \frac{(1-n)n\eta(1-\rho\theta)}{1+\eta\theta}\hat{a}_F - n\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{H,t}\Big] \tag{A.24}$$

$$\tilde{Y}^f_{F,t} \equiv \frac{1}{\lambda^*_{y_f}}\Big[\frac{(1-n)n\eta(1-\rho\theta)}{1+\eta\theta}\hat{a}_{H,t} + \eta\frac{1-\theta+(1-n)\theta(1+\eta)-n(1-n)(1-\theta\rho)}{1+\eta\theta}\hat{a}_{F,t} + n\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{F,t}\Big] \tag{A.25}$$

$$\tilde{T}^f_t \equiv \frac{1}{\lambda^*_q}\eta\theta\frac{1-\theta\rho}{1+\eta\theta}(\hat{a}_{H,t}-\hat{a}_{F,t}) \tag{A.26}$$

Below, I also present national targets when $\rho\theta = 1$. Beware that this is different than $\rho = \theta = 1$ case.

$$\tilde{\tilde{Y}}^h_{H,t} \equiv \frac{1}{\lambda_{y_h}}\Big[\eta\frac{1-\theta+n\theta(1+\eta)}{1+\eta\theta}\hat{a}_{H,t} + (1-n)\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{H,t}\Big] \tag{A.27}$$

$$\tilde{\tilde{Y}}^h_{F,t} \equiv \frac{1}{\lambda_{y_f}}\Big[(1-n)\eta\frac{\theta(1+\eta)}{1+\eta\theta}\hat{a}_{F,t} - (1-n)\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{F,t}\Big] \tag{A.28}$$

$$\tilde{\tilde{T}}^h_t \equiv \frac{\eta}{(1-n)(n\theta(1+\eta)+(1-n)(1-\theta))}(\hat{a}_{H,t}-\hat{a}_{F,t}) \tag{A.29}$$

$$\tilde{\tilde{Y}}^f_{H,t} \equiv \frac{1}{\lambda^*_{y_h}}\Big[n\eta\frac{\theta(1+\eta)}{1+\eta\theta}\hat{a}_{H,t} - n\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{H,t}\Big] \tag{A.30}$$

$$\tilde{\tilde{Y}}^f_{F,t} \equiv \frac{1}{\lambda^*_{y_f}}\Big[\eta\frac{1-\theta+(1-n)\theta(1+\eta)}{1+\eta\theta}\hat{a}_{F,t} + n\frac{\theta(1+\eta)}{1+\eta\theta}\hat{\mu}_{F,t}\Big] \tag{A.31}$$

$$\tilde{\tilde{T}}^f_t \equiv \frac{\eta}{n((1-n)\theta(1+\eta)+n(1-\theta))}(\hat{a}_{H,t}-\hat{a}_{F,t}) \tag{A.32}$$

B Appendix for Monetary Policy Problems

The conventional approach is to assume that countries are at steady state for $t < 0$. I additionally assume that there are no shocks at $t = 0$. Therefore, monetary policy from timeless perspective satisfies $\pi_{H,0} = \pi_{F,0} = 0$ for all monetary regimes.

I do not derive the monetary problem in monetary union in the special case of $\rho = \theta = 1$ because it is almost identical to its general case except in the special case I have $\psi = 0$.

B.1 Monetary Policy Problem in Flexible Exchange Rate Regime

In flexible exchange rate regime, optimal monetary problem for H's monetary authority is to maximize (5.1) subject to (3.6), (3.7), (3.8). Lagrangian of the problem can be written as

$$\begin{aligned}\mathcal{L} = -\frac{1}{2}\mathbb{E}_0 \sum_{t=1}^{\infty} \beta^t \Big[& \lambda_{y_h} (\hat{Y}_{H,t} - \tilde{Y}^h_{H,t})^2 + \lambda_{y_f} (\hat{Y}^*_{F,t} - \tilde{Y}^h_{F,t})^2 + \lambda_q (\hat{T}_t - \tilde{T}^h_t)^2 + \lambda_{\pi_h} {\pi_{H,t}}^2 + \lambda_{\pi_F} {\pi^*_{F,t}}^2 \Big] \\ & + \vartheta_{\pi,t} \Big[\beta \mathbb{E}_t \pi_{H,t+1} + \kappa (\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}) + (1-n)\kappa\psi(\hat{T}_t - \tilde{T}^w_t) + u_t - \pi_{H,t} \Big] \\ & + \vartheta^*_{\pi,t} \Big[\beta \mathbb{E}_t \pi^*_{F,t+1} + \kappa^* (\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}) - n\kappa^*\psi(\hat{T}_t - \tilde{T}^w_t) + u^*_t - \pi^*_{F,t} \Big] \\ & + \vartheta_{\mathcal{T},t} \Big[\theta^{-1} \big[(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}) - (\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}) \big] - (\hat{T}_t - \tilde{T}^w_t) \Big]\end{aligned} \tag{B.1}$$

First order conditions with respect to $\hat{Y}_{H,t}$, $\hat{Y}_{F,t}$, $\pi_{H,t}$, and $\hat{T}_t$ are respectively

$$-\lambda_{y_h}\big(\hat{Y}_{H,t} - \tilde{Y}^h_{H,t}\big) + \vartheta_{\pi,t}\kappa + \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.2}$$

$$-\lambda_{y_f}\big(\hat{Y}_{F,t} - \tilde{Y}^h_{F,t}\big) + \vartheta^*_{\pi,t}\kappa^* - \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.3}$$

$$-\lambda_{\pi_h}\pi_{H,t} - \vartheta_{\pi,t} + \vartheta_{\pi,t-1} = 0 \tag{B.4}$$

$$-\lambda_q\big(\hat{T} - \tilde{T}^h_t\big) + \vartheta_{\pi,t}(1-n)\kappa\psi - \vartheta^*_{\pi,t}n\kappa^*\psi - \vartheta_{\mathcal{T},t} = 0 \tag{B.5}$$

F's monetary problem is symmetric to H's problem. First order conditions to F's problem are

$$-\lambda_{y_h}^* \left(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^f\right) + \varpi_{\pi,t}\kappa + \varpi_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.6}$$

$$-\lambda_{y_f}^* \left(\hat{Y}_{F,t} - \tilde{Y}_{F,t}^f\right) + \varpi_{\pi,t}^*\kappa^* - \varpi_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.7}$$

$$-\lambda_{\pi_f}^* \pi_{F,t}^* - \varpi_{\pi,t}^* + \beta\varpi_{\pi,t-1}^* = 0 \tag{B.8}$$

$$-\lambda_q^* \big(\hat{T} - \tilde{T}_t^f\big) + \varpi_{\pi,t}(1-n)\kappa\psi - \varpi_{\pi,t}^* n\kappa^*\psi - \varpi_{\mathcal{T},t} = 0 \tag{B.9}$$

where $\varpi_{\pi,t}$, $\varpi_{\pi,t}^*$, $\varpi_{\mathcal{T},t}$ are lagrange multipliers for (3.6), (3.7), (3.8) respectively.

B.2 Monetary Policy Problem in Monetary Union

Monetary problem in the union is to maximize W subject to (3.6), (3.7), (3.8), (3.10). Lagrangian of the problem is

$$\begin{aligned}
\mathcal{L} = -\frac{\lambda_y^w}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big[& n\big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w\big)^2 + (1-n)\big(\hat{Y}_{F,t} - \tilde{Y}_{F,t}^w\big)^2 + n(1-n)\theta\psi\big(\hat{T}_t - \tilde{T}_t^w\big)^2 + \frac{\sigma n}{\kappa}\pi_{H,t}{}^2 + \frac{\sigma(1-n)}{\kappa^*}\pi_{F,t}^*{}^2\Big] \\
&+ \vartheta_{\pi,t}\Big[\beta\mathbb{E}_t\pi_{H,t+1} + \kappa\big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w\big) + (1-n)\kappa\psi\big(\hat{T}_t - \tilde{T}_t^w\big) + u_t - \pi_{H,t}\Big] \\
&+ \vartheta_{\pi,t}^*\Big[\beta\mathbb{E}_t\pi_{F,t+1}^* + \kappa^*\big(\hat{Y}_{F,t} - \tilde{Y}_{F,t}^w\big) - n\kappa^*\psi\big(\hat{T}_t - \tilde{T}_t^w\big) + u_t^* - \pi_{F,t}^*\Big] \\
&+ \vartheta_{\mathcal{T},t}\Big[\theta^{-1}\big[\big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w\big) - \big(\hat{Y}_{F,t} - \tilde{Y}_{F,t}^w\big)\big] - \big(\hat{T}_t - \tilde{T}_t^w\big)\Big] \\
&+ \vartheta_{\mathrm{Ex},t}\Big[\Delta\hat{T}_t + \pi_{H,t} - \pi_{F,t}^* = 0\Big]
\end{aligned} \tag{B.10}$$

First order conditions are

$$-n\big(\hat{Y}_{H,t} - \tilde{Y}_{H,t}^w\big) + \vartheta_{\pi,t}\kappa + \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.11}$$

$$-(1-n)\big(\hat{Y}_{F,t} - \tilde{Y}_{F,t}^w\big) + \vartheta_{\pi,t}\kappa^* - \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.12}$$

$$-\frac{\sigma n}{\kappa}\pi_{H,t} - \vartheta_{\pi,t} + \beta\vartheta_{\pi,t-1} + \vartheta_{\mathrm{Ex},t} = 0 \tag{B.13}$$

$$-\frac{\sigma(1-n)}{\kappa^*}\pi_{F,t}^* - \vartheta_{\pi,t} + \beta\vartheta_{\pi,t-1} - \vartheta_{\mathrm{Ex},t} = 0 \tag{B.14}$$

$$-n(1-n)\theta\psi\big(\hat{T}_t - \tilde{T}_t^w\big) + (1-n)\kappa\psi\vartheta_{\pi,t} - n\kappa^*\psi\vartheta_{\pi,t}^* - \vartheta_{\mathcal{T},t} + \vartheta_{\mathrm{Ex},t} - \vartheta_{\mathrm{Ex},t+1} = 0 \tag{B.15}$$

Under the assumption that $\kappa = \kappa^*$, I obtain that

$$-\sigma(n\pi_{H,t} + (1-n)\pi^*_{F,t}) = n\Delta\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big) + (1-n)\Delta\big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big) \tag{B.16}$$

B.3 Monetary Policy Problem in Cooperative Regime

Monetary problem in the cooperative regime is to maximize W subject to (3.6), (3.7), and (3.8). Lagrangian of the problem is

$$\begin{aligned}\mathcal{L} = -\frac{\lambda^w_y}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big[& n\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big)^2 + (1-n)\big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big)^2 + n(1-n)\theta\psi\big(\hat{T}_t - \tilde{T}^w_t\big)^2 + \frac{\sigma n}{\kappa}{\pi_{H,t}}^2 + \frac{\sigma(1-n)}{\kappa^*}{\pi^*_{F,t}}^2\Big] \\ &+ \vartheta_{\pi,t}\Big[\beta\mathbb{E}_t\pi_{H,t+1} + \kappa\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big) + (1-n)\kappa\psi\big(\hat{T}_t - \tilde{T}^w_t\big) + u_t - \pi_{H,t}\Big] \\ &+ \vartheta^*_{\pi,t}\Big[\beta\mathbb{E}_t\pi^*_{F,t+1} + \kappa^*\big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big) - n\kappa^*\psi\big(\hat{T}_t - \tilde{T}^w_t\big) + u^*_t - \pi^*_{F,t}\Big] \\ &+ \vartheta_{\mathcal{T},t}\Big[\theta^{-1}\big[\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big) - \big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big)\big] - \big(\hat{T}_t - \tilde{T}^w_t\big)\Big]\end{aligned} \tag{B.17}$$

First order conditions are

$$-n\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big) + \vartheta_{\pi,t}\kappa + \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.18}$$

$$-(1-n)\big(\hat{Y}_{F,t} - \tilde{Y}^w_{F,t}\big) + \vartheta_{\pi,t}\kappa^* - \vartheta_{\mathcal{T},t}\theta^{-1} = 0 \tag{B.19}$$

$$-\frac{\sigma n}{\kappa}\pi_{H,t} - \vartheta_{\pi,t} + \beta\vartheta_{\pi,t-1} = 0 \tag{B.20}$$

$$-\frac{\sigma(1-n)}{\kappa^*}\pi^*_{F,t} - \vartheta_{\pi,t} + \beta\vartheta_{\pi,t-1} = 0 \tag{B.21}$$

$$-n(1-n)\theta\psi\big(\hat{T}_t - \tilde{T}^w_t\big) + (1-n)\kappa\psi\vartheta_{\pi,t} - n\kappa^*\psi\vartheta^*_{\pi,t} - \vartheta_{\mathcal{T},t} = 0 \tag{B.22}$$

B.4 Monetary Policy Problem in Flexible Exchange Rate ($\rho = \theta = 1$ Case)

Under the assumption of $\rho = \theta = 1$, H's monetary policy is to maximize (5.1) subject to (5.4). Lagrangian of the problem is

$$\begin{aligned}\mathcal{L} = -\frac{1}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big[& \lambda_{y_h}\big(\hat{Y}_{H,t} - \tilde{Y}^f_{H,t}\big)^2 + \lambda y_f\big(\hat{Y}_{F,t} - \tilde{Y}^f_{F,t}\big)^2 + \lambda_{\pi_h}{\pi_{H,t}}^2 + \lambda_{\pi_F}{\pi^*_{F,t}}^2\Big] \\ &+ \vartheta_{\pi,t}\Big[\beta\mathbb{E}_t\pi_{H,t+1} + \kappa\big(\hat{Y}_{H,t} - \tilde{Y}^w_{H,t}\big) + u_t - \pi_{H,t}\Big]\end{aligned} \tag{B.23}$$

First order conditions are

$$-\lambda_{y_h}\left(\hat{Y}_{H,t}-\tilde{Y}^{f}_{H,t}\right)+\vartheta_{\pi,t}\kappa=0 \tag{B.24}$$

$$-\lambda_{\pi_h}\pi_{H,t}-\vartheta_{\pi,t}+\vartheta_{\pi,t-1}=0 \tag{B.25}$$

Combining the first order conditions gives

$$\begin{aligned}
-\lambda_{\pi_h}\pi_{H,t} &= \frac{\lambda_{y_h}}{\kappa}\Delta\left(\hat{Y}_{H,t}-\tilde{Y}^{h}_{H,t}\right) \\
\Leftrightarrow -\sigma\pi_{H,t} &= \Delta\left(\hat{Y}_{H,t}-\tilde{Y}^{h}_{H,t}\right)
\end{aligned}$$

Following the same process for F, I obtain

$$-\sigma\pi_{F,t}=\Delta\left(\hat{Y}_{F,t}-\tilde{Y}^{f}_{F,t}\right) \tag{B.26}$$

The two optimality condition along with (5.3) and (5.4) pin down all endogenous variables.

B.5 Monetary Policy Problem in Cooperative Regime ($\rho=\theta=1$ Case)

Monetary problem in the cooperative flexible exchange rate regime is to maximize W subject to (3.6), (3.7). Lagrangian of the problem is

$$\begin{aligned}
\mathcal{L}=-\frac{\lambda^w_y}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^t\Big[&n\big(\hat{Y}_{H,t}-\tilde{\tilde{Y}}^{w}_{H,t}\big)^2+(1-n)\big(\hat{Y}_{F,t}-\tilde{\tilde{Y}}^{w}_{F,t}\big)^2+\frac{\sigma n}{\kappa}\pi_{H,t}{}^2+\frac{\sigma(1-n)}{\kappa^*}\pi^*_{F,t}{}^2\Big] \\
&+\vartheta_{\pi,t}\Big[\beta\mathbb{E}_t\pi_{H,t+1}+\kappa\big(\hat{Y}_{H,t}-\tilde{\tilde{Y}}^{w}_{H,t}\big)+u_t-\pi_{H,t}\Big] \\
&+\vartheta^*_{\pi,t}\Big[\beta\mathbb{E}_t\pi^*_{F,t+1}+\kappa^*\big(\hat{Y}_{F,t}-\tilde{\tilde{Y}}^{w}_{F,t}\big)+u^*_t-\pi^*_{F,t}\Big]
\end{aligned} \tag{B.27}$$

First order conditions are

$$
\begin{aligned}
& -n\big(\hat{Y}_{H,t}-\tilde{\tilde{Y}}^{w}_{H,t}\big)+\vartheta_{\pi,t}\kappa=0 && \text{(B.28)}\\
& -(1-n)\big(\hat{Y}_{F,t}-\tilde{\tilde{Y}}^{w}_{F,t}\big)+\vartheta_{\pi,t}\kappa^{*}=0 && \text{(B.29)}\\
& -\frac{\sigma n}{\kappa}\pi_{H,t}-\vartheta_{\pi,t}+\beta\vartheta_{\pi,t-1}=0 && \text{(B.30)}\\
& -\frac{\sigma(1-n)}{\kappa^{*}}\pi^{*}_{F,t}-\vartheta_{\pi,t}+\beta\vartheta_{\pi,t-1}=0 && \text{(B.31)}
\end{aligned}
$$

Using first order condition, I obtain

$$
\begin{aligned}
-\sigma\pi_{H,t} &= \Delta\big(\hat{Y}_{H,t}-\tilde{\tilde{Y}}^{w}_{H,t}\big) && \text{(B.32)}\\
-\sigma\pi_{F,t} &= \Delta\big(\hat{Y}_{F,t}-\tilde{\tilde{Y}}^{w}_{F,t}\big) && \text{(B.33)}
\end{aligned}
$$

B.6 First Best Solution for H in Flexible Exchange Rate ($\rho=\theta=1$ Case)

The first best monetary policy for H is the policy which maximizes W_H. Then, monetary policy problem is to maximize $\tilde{\tilde{W}}_H$ subject to (5.3) and (5.4). Lagrangian of the problem is

$$
\begin{aligned}
\mathcal{L}=\tilde{\tilde{W}}_H=-\frac{1}{2}\mathbb{E}_0\sum_{t=1}^{\infty}\beta^{t}\Big[&\lambda_{y_h}\big(\hat{Y}_{H,t}-\tilde{\tilde{Y}}^{h}_{H,t}\big)^2+\lambda_{y_f}\big(\hat{Y}^{*}_{F,t}-\tilde{\tilde{Y}}^{h}_{F,t}\big)^2+\lambda_{\pi_h}{\pi_{H,t}}^2+\lambda_{\pi_f}{\pi^{*}_{F,t}}^2\Big]\\
&+\vartheta_{\pi,t}\Big[\beta\mathbb{E}_t\pi_{H,t+1}+\kappa\big(\hat{Y}_{H,t}-\tilde{\tilde{Y}}^{w}_{H,t}\big)+u_t-\pi_{H,t}\Big]\\
&+\vartheta^{*}_{\pi,t}\Big[\beta\mathbb{E}_t\pi^{*}_{F,t+1}+\kappa^{*}\big(\hat{Y}_{F,t}-\tilde{\tilde{Y}}^{w}_{F,t}\big)+u^{*}_t-\pi^{*}_{F,t}\Big]
\end{aligned}
\qquad \text{(B.34)}
$$

First order conditions of this problem are

$$
\begin{aligned}
& -\lambda_{y_h}\big(\hat{Y}_{H,t}-\tilde{\tilde{Y}}^{h}_{H,t}\big)+\vartheta_{\pi,t}\kappa=0 && \text{(B.35)}\\
& -\lambda_{\pi_h}\pi_{H,t}-\vartheta_{\pi,t}+\vartheta_{\pi,t-1}=0 && \text{(B.36)}\\
& -\lambda_{y_f}\big(\hat{Y}_{F,t}-\tilde{\tilde{Y}}^{h}_{F,t}\big)+\vartheta_{\pi,t}\kappa=0 && \text{(B.37)}\\
& -\lambda_{\pi_f}\pi_{F,t}-\vartheta_{\pi,t}+\vartheta_{\pi,t-1}=0 && \text{(B.38)}
\end{aligned}
$$

First order conditions I obtain

$$-\sigma\pi_{H,t} = \Delta\big(\hat{Y}_{H,t} - \tilde{\tilde{Y}}^{h}_{H,t}\big) \tag{B.39}$$

$$-\sigma\pi_{F,t} = \Delta\big(\hat{Y}_{F,t} - \tilde{\tilde{Y}}^{h}_{F,t}\big) \tag{B.40}$$

C Additional Figures

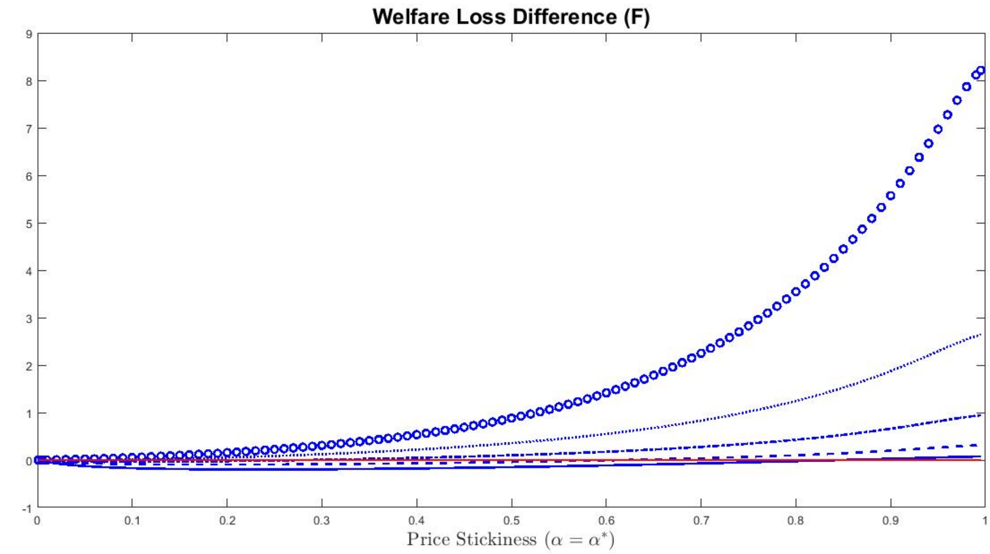

Figure 12: Welfare loss difference of F as a function of nominal price rigidity for the parametrization given at Table 3 and for alternative values of θ with markup shocks only.

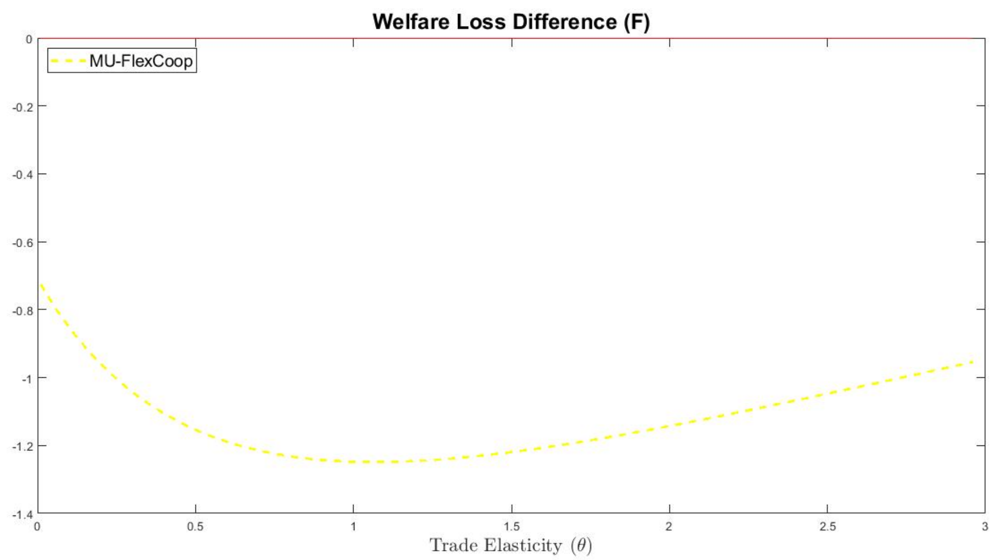

Figure 13: Welfare loss differences of F between $\mathcal{M}$ and $\mathcal{C}$ as a function of trade elasticity for the parametrization given at Table 3 when variation in F's markup shocks is relatively high ($\sigma_{\mu_h} = 0.4$, $\sigma_{\mu_f} = 0.1$).

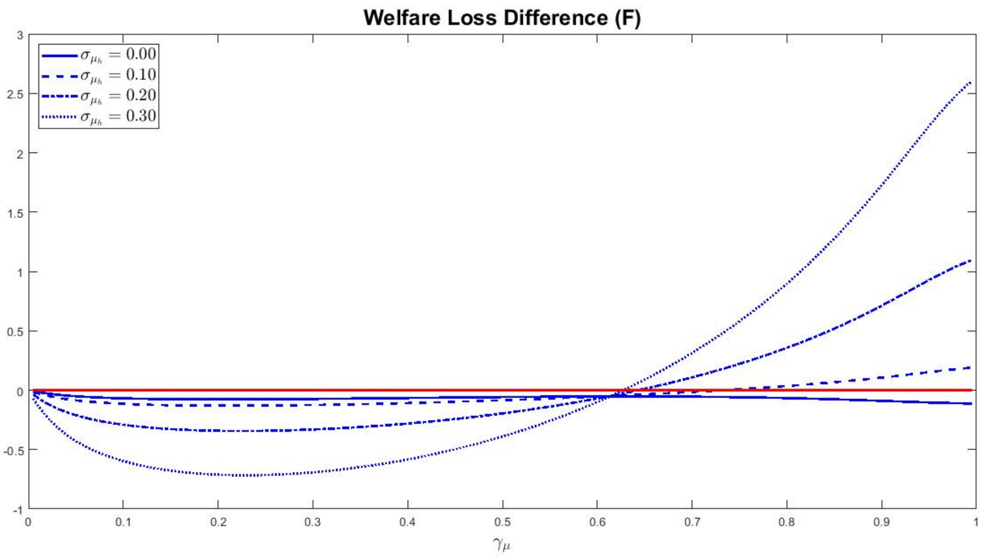

Figure 14: Welfare loss differences of F as a function of price stickiness for parametrization given at Table 3 and $\theta = 0.8$.

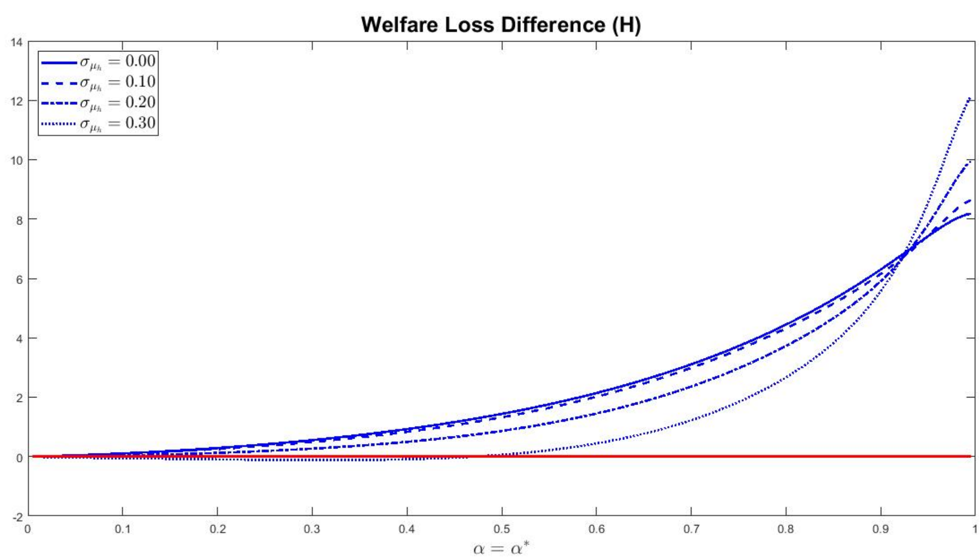

Figure 15: Welfare loss differences of H as a function of price stickiness for parametrization given at Table 3 and $\theta = 2.5$.

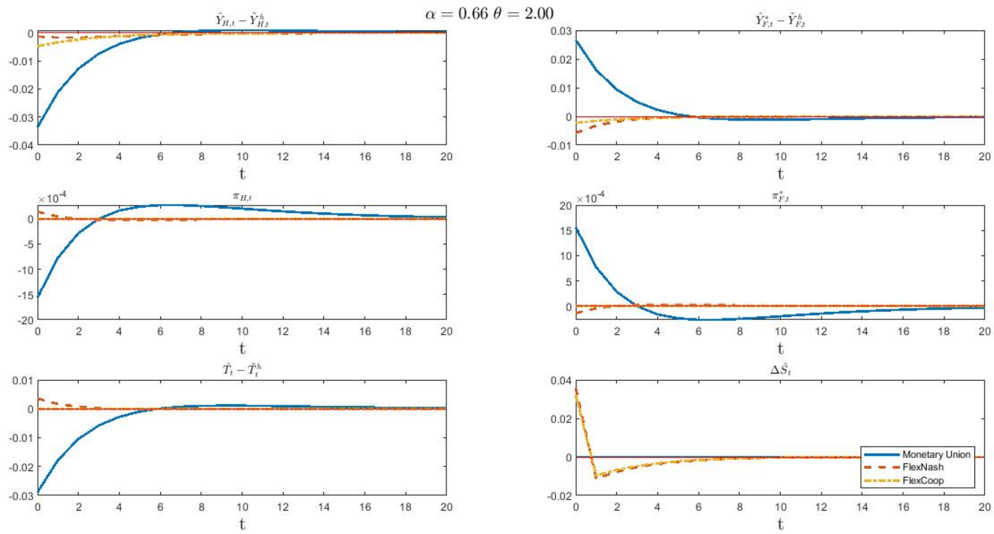

Figure 16: Impulse response functions to deviation from steady state domestic productivity ($\hat{a}_h$) for parametrization at Table 3

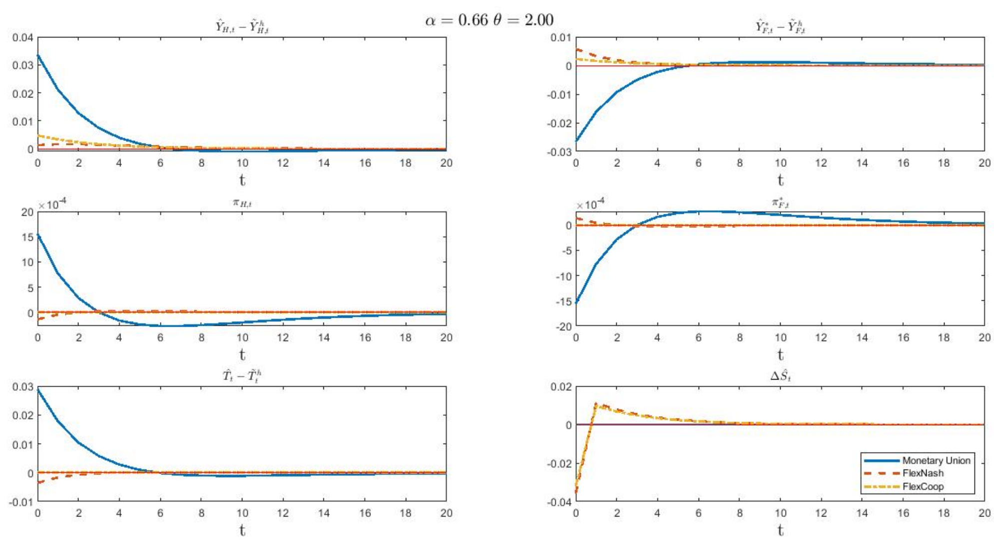

Figure 17: Impulse response functions to deviation from steady state foreign productivity ($\hat{a}_f$) for parametrization at Table 3

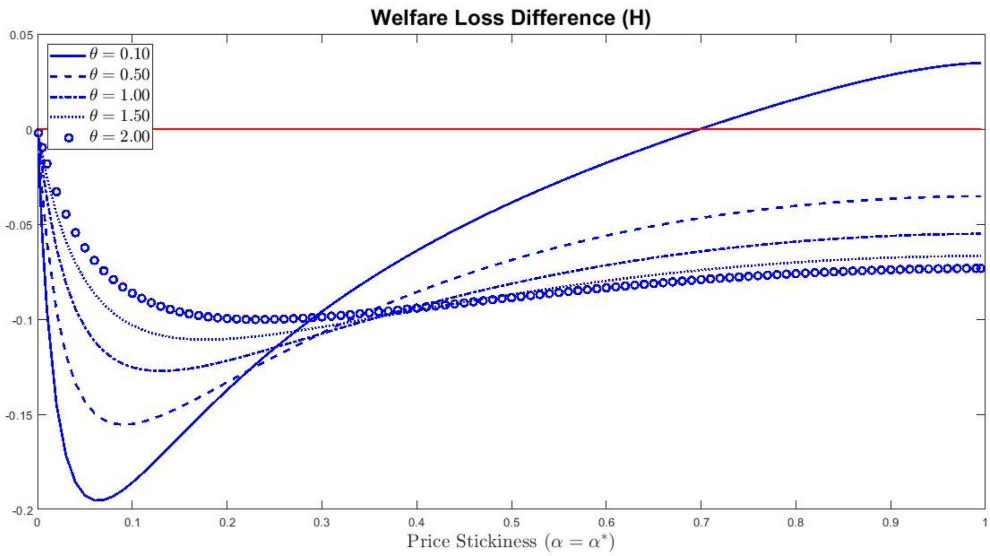

Figure 18: Welfare loss differences of H as a function of nominal price rigidity with only productivity shocks. The parametrization is different than the one in Table 3.

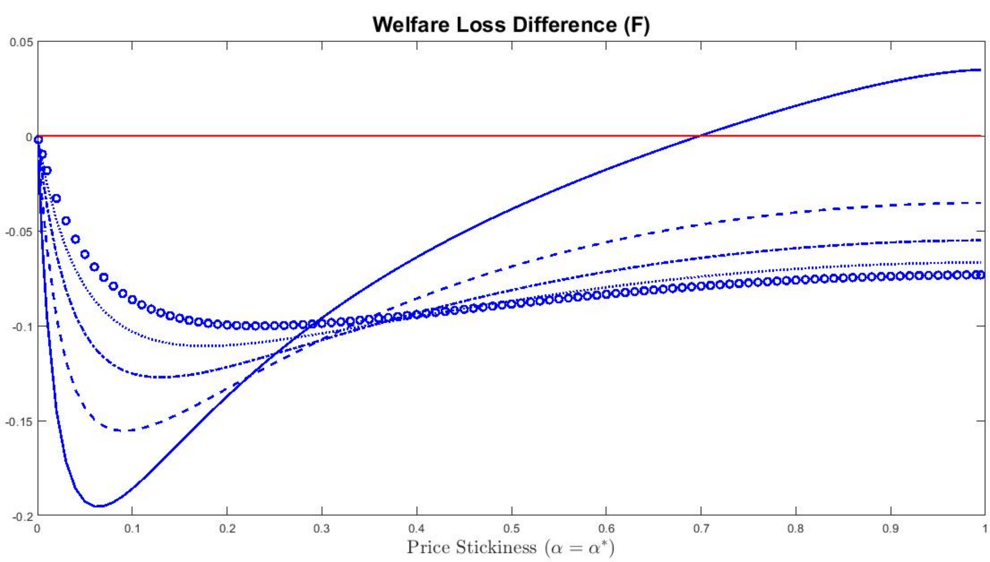

Figure 19: Welfare loss differences of F as a function of nominal price rigidity with only productivity shocks. The parametrization is identical to the one in Figure 18.

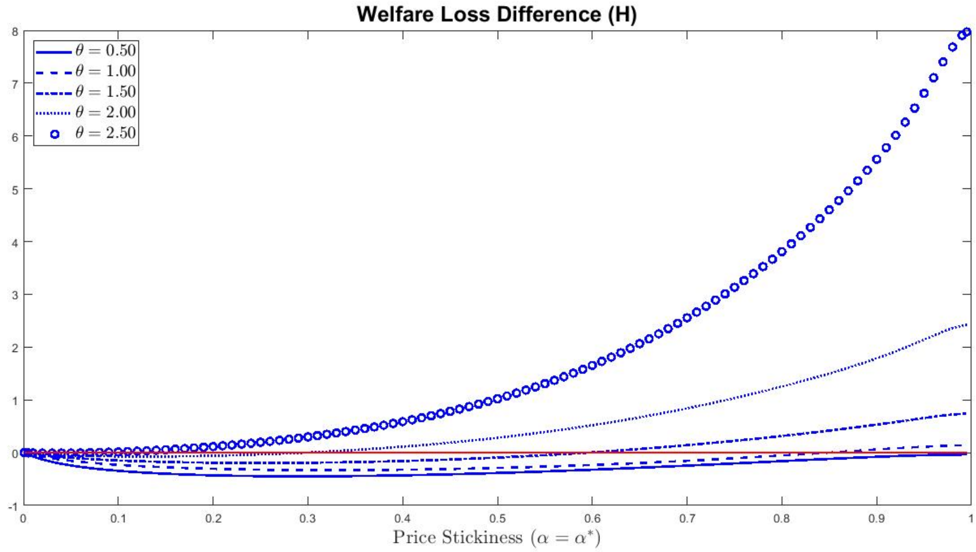

Figure 20: Welfare loss differences of H as a function of nominal price rigidity for the parametrization given at Table 3 and for alternative values of θ when both productivity and markup shocks exist.

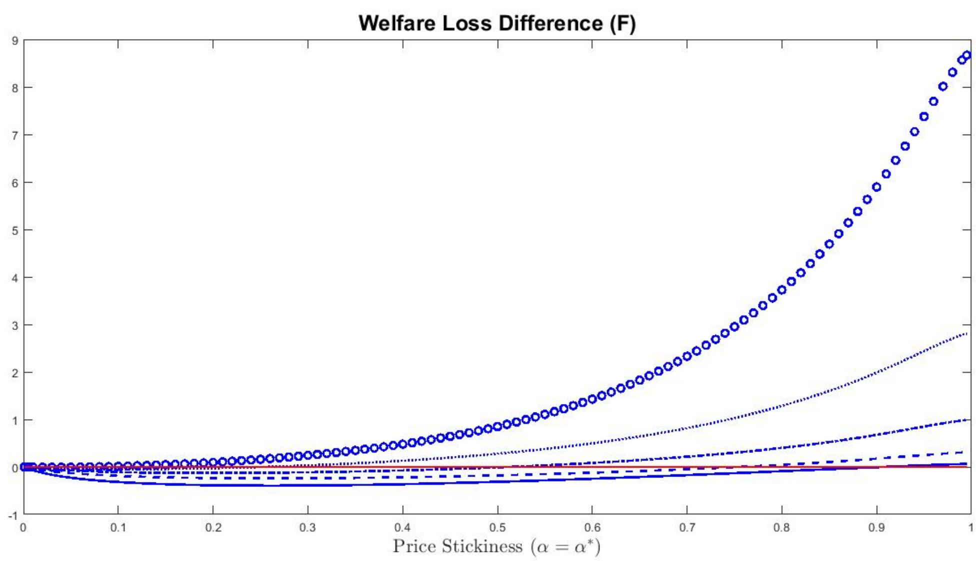

Figure 21: Welfare loss differences of F as a function of nominal price rigidity for the parametrization given at Table 3 and for alternative values of θ when both productivity and markup shocks exist.