Financial Development, International Capital Flows, and Aggregate Output*

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This Version: September 2012
First Version: November 2008

Abstract

We develop a tractable two-country overlapping-generations model and show that cross-country differences in financial development can explain three recent empirical patterns of international capital flows: Financial capital flows from relatively poor to relatively rich countries, while foreign direct investment flows in the opposite direction; net capital flows go from poor to rich countries; despite its negative net international investment positions, the United States receives a positive net investment income.

International capital mobility affects output in each country directly through the size of domestic investment and indirectly through the aggregate saving rate. Under certain conditions, the indirect effect may dominate the direct effect so that international capital mobility raises output in the poor country and globally, although net capital flows are in the direction to the rich country. We also explore the welfare and distributional effects of international capital flows and show that the patterns of capital flows may reverse along the convergence process of a developing country. Our model adds to the understanding of the benefits of international capital mobility in the presence of domestic financial frictions.

Keywords: Capital account liberalization, financial development, foreign direct investment, symmetry breaking
JEL Classification: E44, F41

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*We appreciate the comments and suggestions from the editor and two anonymous referees as well as the participants at the 40th Konstanz Seminar, the Econometric Society NASM 2009 in Boston, the SED 2009 Annual Meeting in Istanbul, the 13th ZEI Summer School, the AEA 2010 meeting in Atlanta, and seminar participants at the University of Bonn, the University of Illinois, Indiana University, National University of Singapore, Queen's University. Financial support from Singapore Management University and the German Research Foundation (DFG) are sincerely acknowledged.

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1 Introduction

Standard international macroeconomics predicts that capital flows from capital-rich countries, where the marginal product of capital (MPK, henceforth) is low, to capital-poor countries, where the MPK is high. Furthermore, there should be no difference between gross and net capital flows, as capital movements are unidirectional.

The patterns of international capital flows observed in the past 20 years, however, stand in stark contrast to these predictions (Lane and Milesi-Ferretti, 2001, 2007b,c). First, since 1998, the average per-capita income of countries running current account surpluses has been below that of the deficit countries, i.e., net capital flows have been "uphill" from poor to rich countries (Prasad, Rajan, and Subramanian, 2006, 2007). Second, many developing economies, including China, Malaysia, and South Africa, are net importers of foreign direct investment (hereafter, FDI) and net exporters of financial capital at the same time, while developed countries such as France, the United Kingdom, and the United States exhibit the opposite pattern (Ju and Wei, 2010). Third, despite its negative net international investment position since 1986, the U.S. has been receiving a positive net investment income until 2005 (Gourinchas and Rey, 2007; Hausmann and Sturzenegger, 2007; Higgins, Klitgaard, and Tille, 2007).

Recent research offers two main explanations to these empirical facts. Devereux and Sutherland (2009) and Tille and van Wincoop (2010) focus on the cross-country risk-sharing investors can achieve by diversifying their portfolios globally. International portfolio investment is determined by the cross-correlation patterns of aggregate shocks at the country level. These models do not distinguish between FDI and portfolio equity investment and, therefore, offer no explanation for the second pattern.

The other strand of literature focuses on domestic financial market imperfections (Aoki, Benigno, and Kiyotaki, 2009; Caballero, Farhi, and Gourinchas, 2008; Smith and Valderrama, 2008). Matsuyama (2004) shows that, in the presence of credit market imperfections, financial market globalization may lead to a steady-state equilibrium in which fundamentally identical countries end up with different levels of per capita output, a result he calls "symmetry breaking". Furthermore, financial capital flows from poor to rich countries in the steady state. However, Matsuyama (2004) does not address FDI flows. Mendoza, Quadrini, and Rios-Rull (2009) analyze the joint determination of financial capital flows and FDI in a heterogeneous-agent model with uninsurable idiosyncratic endowment and investment risks. The precautionary savings motive plays the crucial role. Ju and Wei (2010) show in a static model that, when both FDI and financial capital flows are allowed, all financial capital leaves the country where credit market imperfections are more severe, while FDI flows into this country. Thus, capital mobility allows investors to fully *bypass* the underdeveloped financial system. The models mentioned above explain only one or two of the three facts.

While the literature does not explicitly address the implications of international capital mobility for aggregate output, it seems intuitively plausible that, due to the declining MPK, "uphill" capital flows make the poor countries and the world economy poorer.[1] The policy implications seem to be clear: The world would be better off without international capital movements between rich and poor countries.

We extend the second strand of literature and explain simultaneously all three empirical facts. Following Matsuyama (2004), we take the tightness of the borrowing constraints as a measure of a country's level of financial development. The two countries in our model differ fundamentally only in the level of financial development. Under international financial autarky (hereafter, IFA), interest rates are affected by two factors. First, for a given level of financial development, a lower capital-labor ratio implies a higher MPK and higher interest rates. We call this the *neoclassical* effect, as it arises from the concavity of the neoclassical production function with respect to the capital-labor ratio. Second, for a given capital-labor ratio, a lower level of financial development implies the less efficient enforcement of credit contract and monitoring of borrowers. In this case, agents face tighter borrowing constraints and the lower aggregate credit demand leads to a lower loan rate and a higher equity rate. We call this the *financial-underdevelopment* effect. If aggregate saving is interest-elastic, domestic financial frictions distort aggregate saving through the interest rates, leading to the inefficiently low investment and high MPK. Thus, domestic financial frictions affect interest rates directly through the financial-underdevelopment effect and indirectly through the neoclassical effect. In the less financially developed country, the steady-state loan rate is lower, as the financial-underdevelopment effect dominates the neoclassical effect; as the two effects work in the same direction, the steady-state equity rate is strictly higher.

Suppose that the two countries are initially in the steady state under IFA. Upon full capital mobility, the more financially developed country receives net capital inflows, thanks to its larger credit market. In other words, net capital flows are "uphill" from the poor to the rich country. The initial cross-country interest rate differentials drive financial capital flows from the poor to the rich country and FDI flows in the opposite direction. Since the rich country receives a higher return on its FDI assets than it pays on its foreign debts, it gets a positive net investment income despite its negative net international investment position. Intuitively, by "exporting" its superior financial services through two-way capital flows, the rich country receives a positive net reward, accordingly. Thus, our model predictions are consistent with the three empirical facts mentioned above. Building upon this model, we make four contributions to the literature.

First, we show that full capital mobility can raise output in the poor country as well as globally, despite "uphill" net capital flows. Intuitively, financial frictions depress the return on and, hence, the level of aggregate saving. Allowing for international capital mo-

[1]Matsuyama (2004) and von Hagen and Zhang (2010) show that this may indeed be the case.

bility provides domestic households with better returns on savings. Thus, by ameliorating the interest rate distortions, capital mobility indirectly raises aggregate savings in the less financially developed country. If saving is sufficiently interest-elastic, the rise in aggregate saving may exceeds net capital outflows so that aggregate investment and output in the less financially developed country as well as globally can be higher than under IFA.

The interest-elastic saving is key to output gains in our model and deserves special attention. Given the Cobb-Douglas preference, the income effect and the substitution effect of interest rates on saving exactly offset each other. The interest-elastic saving in our model results from the positive future labor income, which is defined as the human wealth effect by Summers (1981). Our model predicts that, in the country with a higher growth rate of the labor income, aggregate saving is more interest-elastic so that full capital mobility is more likely to raise output. The interest elasticity of saving has been the focus of the debates on the effectiveness of tax reform (Bernheim, 2002; Evans, 1983; Summers, 1981), financial liberalization (Bandiera, Caprio, Honohan, and Schiantarelli, 2000), and other public policies (Corbo and Schmidt-Hebbel, 1991) on capital accumulation. The empirical evidences on the magnitude of the interest elasticity of savings are rather mixed (Giovannini, 1983; Loayza, Schmidt-Hebbel, and Serven, 2000). In particular, Ogaki, Ostry, and Reinhart (1996) provide the empirical evidences that savings are more responsive to rates of return at higher income levels. Instead of arguing for the empirical significance of the interest elasticity of saving, our analysis complements the existing literature by emphasizing the theoretical relevance of the interest-elastic saving to the output implications of capital account liberalization policy.

As our second contribution, we show that financial capital flows affect the owners of credit capital and equity capital in opposite ways and so do FDI flows. Capital flows also affect the intergenerational income distribution. Our model points out such distributional effects of capital flows and offers an explanation for why capital account liberalization often encounters both support and opposition in a given country.

Third, we also analyze a scenario where one country is more financially developed and in its steady state, while the other country is less financially developed and below its steady state before capital account liberalization. We study the interactions of international capital flows and the economic convergence of the second country and show that the pattern of international capital flows may reverse along the convergence process of the less financially developed country.

We assume that the mass of individuals who can produce is fixed in each country, while the investment size of each producer is endogenously determined. Thus, aggregate investment occurs on the intensive margin instead of on the extensive margin as in Matsuyama (2004). Countries with identical fundamentals have the same, unique, and stable steady state under capital mobility in our model. As our fourth contribution, we show that Matsuyama's *symmetry-breaking* depends critically on the assumption of the fixed project size and thus, investment occurs along the extensive margin.

Our model differs from the existing literature in the following aspects. The static model of Ju and Wei (2010) is useful for analyzing the immediate impacts of capital account liberalization, while our OLG model facilitates the short-run and the long-run analysis. Devereux and Sutherland (2009); Mendoza, Quadrini, and Rios-Rull (2009); Tille and van Wincoop (2010) capture international capital flows in the settings with aggregate or idiosyncratic uncertainty, while our model features international capital flows in the deterministic setting. Angeletos and Panousi (2011); Buera and Shin (2010); Carroll and Jeanne (2011); Sandri (2010); Song, Storesletten, and Zilibotti (2011) address "uphill" financial capital flows, while we focus on the joint determination of financial capital and FDI flows. Caballero, Farhi, and Gourinchas (2008); Mendoza, Quadrini, and Rios-Rull (2009) analyze the joint determination of financial capital and FDI flows in an endowment-economy model, while endogenous capital accumulation is crucial in our model. Caballero, Farhi, and Gourinchas (2008) assume that foreign direct investors from the more financially developed country have an advantage in capitalizing the return on investment in the host country and Mendoza, Quadrini, and Rios-Rull (2009) assume that investors from the more financially developed country can insure their foreign direct investment using the better risk-sharing opportunities in their home country. We do not need these extra assumptions. Carroll and Jeanne (2011); Sandri (2010) feature the precautionary savings channel in a model with idiosyncratic risk and incomplete markets, while interest-elastic savings in our model result from limited commitment.

Caselli and Feyrer (2007) present alternative estimates of cross-country MPK differences to assess the importance of international credit market frictions. They implicitly assume away domestic financial frictions so that the MPK is the rate of return to investors and the driving force behind international capital flows. They find that, if one focuses on reproducible capital and adjusts for the higher relative prices of capital goods in poor countries, the MPK does not differ much between developed and developing countries. Thus, they conclude that international credit market frictions cannot go far in explaining observed capital flows between these countries. Our analysis abstracts from international credit market frictions and focuses on domestic financial frictions which creates a wedge between the private rates of return (i.e., the rates of return to credit capital and equity capital) and the social rate of return (i.e., MPK). The private rates of return are the driving forces behind international capital flows in our model, which allows us to distinguish between financial capital and FDI flows.

The rest of the paper is structured as follows. Section 2 sets up the model and shows the distortions of financial frictions on interest rates and output under IFA. Section 3 analyzes the output and welfare implications of capital mobility. Section 4 concludes with some remarks. Appendix collects the technical proofs and relevant discussions.

2 The Model under International Financial Autarky

The world economy consists of two countries, N (North) and S (South), which are fundamentally identical except in the level of financial development as specified later. In the following, variables in country $i \in \{N, S\}$ are denoted with the superscript i. A final good can be consumed or transformed into capital goods. The final good is internationally tradable and chosen as the numeraire, while capital goods are non-tradable.

Individuals live for two periods, young and old. There is no population growth and the size of each generation is normalized to one in each country. Each individual is endowed with one unit of labor when young and $\epsilon \geq 0$ units of labor when old, which are supplied to aggregate production. Aggregate labor supply is $L = 1 + \epsilon$ in each period.

At the beginning of each period, final goods Y_t^i are produced with capital goods K_t^i and labor L in a Cobb-Douglas fashion. Capital goods fully depreciate after production. Capital goods and labor are priced at their respective marginal products. To summarize,

$$Y_t^i = \left(\frac{K_t^i}{\alpha}\right)^{\alpha} \left(\frac{L}{1-\alpha}\right)^{1-\alpha}, \qquad \text{where} \qquad \alpha \in (0,1), \tag{1}$$

$$R_t^i K_t^i = \alpha Y_t^i \qquad \text{and} \qquad \omega_t^i L = (1-\alpha) Y_t^i, \tag{2}$$

where ω_t^i denotes the wage rate and R_t^i denotes the MPK. There is no uncertainty in the economy. In this section, we assume that international capital flows are not allowed.

Each generation consists of two types of individuals, *entrepreneurs* and *households*, of mass η and $1-\eta$, respectively. They have the Cobb-Douglas preference over consumption,

$$u_t^{i,j} = \left(\frac{c_{y,t}^{i,j}}{1-\beta}\right)^{1-\beta} \left(\frac{c_{o,t+1}^{i,j}}{\beta}\right)^{\beta}, \tag{3}$$

where superscript $j \in \{e, h\}$ denotes the identity of entrepreneur or household; $c_{y,t}^{i,j}$ and $c_{o,t+1}^{i,j}$ denote individual j's consumption when young and when old; $\beta \in (0,1)$ is the patience factor, i.e., a larger β means that individuals are more patient and care more about consumption when old. If $\beta = 1$, they only consume when old, $u_t^{i,j} = c_{o,t+1}^{i,j}$.

An individual j born in period t and country i receives a labor income ω_t^i, consumes $c_{y,t}^{i,j}$, and saves $s_t^{i,j} = \omega_t^i - c_{y,t}^{i,h}$ at a gross interest rate of $R_t^{i,j}$ in period t. In period $t+1$, after receiving the financial income $R_t^{i,j} s_t^{i,j}$ and a labor income $\epsilon \omega_{t+1}^i$, the individual consumes its total wealth $c_{o,t+1}^{i,j} = R_t^{i,j} s_t^{i,j} + \epsilon \omega_{t+1}^i$ and exits from the economy. Its lifetime budget constraint is $c_{y,t}^{i,j} + \frac{c_{o,t+1}^{i,j}}{R_t^{i,j}} = \mathbb{W}_t^{i,j}$, where $\mathbb{W}_t^{i,j} \equiv \omega_t^i + \frac{\epsilon \omega_{t+1}^i}{R_t^{i,j}}$ denotes its discounted lifetime wealth when young. The component $\frac{\epsilon \omega_{t+1}^i}{R_t^{i,j}}$ captures the *human wealth* defined by Summers (1981). Given the Cobb-Douglas preference, its optimal consumption-saving choices are

$$c_{y,t}^{i,j} = (1-\beta) \mathbb{W}_t^{i,j} \quad \text{and} \quad c_{o,t+1}^{i,j} = R_t^{i,j} \beta \mathbb{W}_t^{i,j}, \tag{4}$$

$$s_t^{i,j} = \omega_t^i - c_{y,t}^{i,j} = \beta \omega_t^i - (1-\beta) \frac{\epsilon \omega_{t+1}^i}{R_t^{i,j}}. \tag{5}$$

Plug the solutions to consumption back into the utility function (3), the individual's indirect lifetime utility function is $u_t^{i,j} = \mathbb{W}_t^{i,j}(R_t^{i,j})^\beta$.

Households and entrepreneurs may get different interest rates on their savings and the determination of interest rates is key to our results. We assume that only entrepreneurs can use final goods to produce capital one-to-one and the production takes one period. Thus, the gross rate of return to the entrepreneurial investment made in period t is equal to the MPK in period $t+1$, R_{t+1}^i. With no other investment opportunity available, households lend their entire savings to the credit market at the gross interest rate $R_t^{i,h}$ in period t. As long as $R_{t+1}^i \geq R_t^{i,h}$, an entrepreneur prefers to finance its investment i_t^i using loans $d_t^{i,h}$. However, due to limited commitment, the entrepreneur can borrow only up to a fraction of its future project revenues,

$$R_t^{i,h} d_t^{i,h} = R_t^{i,h}(i_t^i - d_t^{i,e}) \leq \theta^i R_{t+1}^i i_t^i. \tag{6}$$

where $d_t^{i,e}$ denotes the entrepreneur's own funds in the project. In other words, an entrepreneurial project with the investment size i_t^i demands for equity capital $d_t^{i,e}$ and credit capital $d_t^{i,h}$. Following Matsuyama (2004, 2007), we use $\theta^i \in [0,1]$ as a measure of financial development or the severity of credit market imperfections in country i. It captures a wide range of institutional factors and is higher in countries with more sophisticated financial and legal systems, better creditor protection, and more liquid asset market, etc.

Define the equity rate as the rate of return to the entrepreneurial equity capital,

$$R_t^{i,e} \equiv \frac{R_{t+1}^i i_t^i - R_t^{i,h} d_t^{i,h}}{d_t^{i,e}} = R_{t+1}^i + (R_{t+1}^i - R_t^{i,h})(\lambda_t^i - 1) \geq R_t^{i,h}, \tag{7}$$

where $\lambda_t^i \equiv \frac{i_t^i}{d_t^{i,e}}$ denotes the investment-equity ratio. For a unit of equity capital invested, the entrepreneur can borrow $(\lambda_t^i - 1)$ units of loan in period t. In period $t+1$, it receives the net return from the leveraged investment, $(R_{t+1}^i - R_t^{i,h})(\lambda_t^i - 1)$, in addition to the marginal product of its equity capital, R_{t+1}^i. Iff $R_{t+1}^i > R_t^{i,h}$, the entrepreneur borrows to the limit defined by (6) to fully explore the leverage effect; after repaying the debt in period $t+1$, it gets $(1-\theta^i)R_{t+1}^i i_t^i$ and the equity rate is $R_t^{i,e} = \frac{(1-\theta^i)R_{t+1}^i i_t^i}{d_t^{i,e}} = \frac{(1-\theta^i)R_{t+1}^i i_t^i}{i_t^i - d_t^{i,h}} = \frac{(1-\theta^i)R_{t+1}^i}{1-\frac{\theta^i R_{t+1}^i}{R_t^{i,h}}} > R_t^{i,h}$. If $R_t^{i,h} = R_{t+1}^i$, the entrepreneur does not borrow to the limit; after repaying the debt in period $t+1$, it gets $R_{t+1}^i d_t^{i,e}$ and the equity rate is $R_t^{i,e} = R_{t+1}^i$. The non-negative leverage effect ensures that the equity rate is no less than the loan rate and inequality (7) thus marks the entrepreneur's participation constraint.

In the follow, the social rate of return refers to the MPK, while the private rates of return refer to the loan rate and the equity rate.

The markets for credit capital, equity capital, and the final goods clear simultaneously,

$$S_t^{i,h} = (1-\eta)s_t^{i,h} = D_t^{i,h} = \eta d_t^{i,h}, \text{ and } S_t^{i,e} = \eta s_t^{i,e} = D_t^{i,e} = \eta d_t^{i,e}, \tag{8}$$

$$K_{t+1}^i = \eta i_t^i = D_t^{i,h} + D_t^{i,e}, \text{ and } C_t^i + K_{t+1}^i = Y_t^i \tag{9}$$

where $S_t^{i,h}$ and $D_t^{i,h}$ denote the aggregate credit supply and demand, $S_t^{i,e}$ and $D_t^{i,e}$ denote the aggregate equity supply and demand, and $C_t^i \equiv \eta(c_{y,t}^{i,e} + c_{o,t}^{i,e}) + (1-\eta)(c_{y,t}^{i,h} + c_{o,t}^{i,h})$ denotes aggregate consumption in country i and period t.

Definition 1. *Given the level of financial development θ^i, a market equilibrium in country $i \in \{1, 2, ..., N\}$ under IFA is a set of allocations of households, $\{c_{y,t}^{i,h}, s_t^{i,h}, c_{o,t}^{i,h}\}$, entrepreneurs, $\{i_t^i, c_{y,t}^{i,e}, s_t^{i,e}, c_{o,t}^{i,e}\}$, and aggregate variables, $\{Y_t^i, K_t^i, \omega_t^i, R_t^i, R_t^{i,h}, R_t^{i,e}\}$, satisfying equations* (1)-(2), (4)-(9),

2.1 The Model Solution

For notational convenience, we define some auxiliary parameters, $\rho \equiv \frac{\alpha}{1-\alpha}$, $\mathbb{m} \equiv \frac{(1-\beta)\epsilon}{(1+\epsilon)\rho}$, $\mathbb{R} \equiv \frac{(1+\epsilon)\rho}{\beta}(1+\mathbb{m})$, $\bar{\theta} \equiv 1-\eta$, $\mathbb{A}^i \equiv 1 - \frac{\bar{\theta}-\theta^i}{1-\eta}$, $\mathbb{B}^i \equiv 1 + \frac{\bar{\theta}-\theta^i}{\eta}$.

Given the Cobb-Douglas preference, the income effect and the substitution effect of interest rates cancel out so that an individual saves a fraction $(1-\beta)$ of its lifetime wealth when young. $\epsilon > 0$ makes its lifetime wealth interest-elastic through the human wealth effect. Thus, iff $\epsilon > 0$ and $\beta < 1$, its consumption when young is interest-elastic and so is its saving. $\mathbb{m}$ captures the joint impacts of the human wealth effect ($\epsilon > 0$) and impatience ($\beta < 1$) on the interest elasticity of saving. See Lemma 1 for the relationship between $\mathbb{m}$ and the interest elasticity of saving.

$\bar{\theta}$ is a critical value. As shown below, for $\theta^i \geq \bar{\theta}$, the borrowing constraint is slack so that the social and the private rates of return are equal to $\mathbb{R}$ in the steady state. For $\theta^i \in [0, \bar{\theta})$, the borrowing constraint is binding, $\mathbb{A}^i$ and $\mathbb{B}^i$ measure the wedge between the private and the social rates of return with $0 < \mathbb{A}^i < 1 < \mathbb{B}^i$ and $\frac{\partial \mathbb{A}^i}{\partial \theta^i} > 0 > \frac{\partial \mathbb{B}^i}{\partial \theta^i}$.

The aggregate rewards to capital in period $t+1$ is distributed to individuals as the returns to their savings, $(1-\eta)s_t^{i,h}R_t^{i,h} + \eta s_t^{i,e}R_t^{i,e} = R_{t+1}^i K_{t+1}^i$, where $R_{t+1}^i K_{t+1}^i = \rho L \omega_{t+1}^i$ according to equations (2). Use equation (5) to substitute away $s_t^{i,j}$, we get

$$(1-\eta)R_t^{i,h} + \eta R_t^{i,e} = \frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}, \tag{10}$$

which is called as *the reward splitting rule.*

In the following, we first show the model solution in the case of the binding borrowing constraints and then discuss the condition under which it is true.[2] Let X_{IFA} denote the steady-state value of variable X_t under IFA. The model solution is,

$$K_{t+1}^i = \frac{\beta\omega_t^i}{\mathbb{m}+1}\left[1 - \frac{\mathbb{m}(1-\mathbb{A}^i)(\mathbb{B}^i-1)}{(\mathbb{m}+\mathbb{A}^i)(\mathbb{m}+\mathbb{B}^i)}\right], \tag{11}$$

$$R_t^{i,e} = \frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}\left(1 + \frac{\mathbb{B}^i - 1}{\mathbb{m}+1}\right), \tag{12}$$

$$R_t^{i,h} = \frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}\left(1 - \frac{1-\mathbb{A}^i}{\mathbb{m}+1}\right), \tag{13}$$

[2] See the proof of Proposition 1 in the appendix for technical derivations of the model solution.

$$R_{t+1}^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}\left[1 + \frac{\mathbb{m}(1-\mathbb{A}^i)(\mathbb{B}^i-1)}{(\mathbb{m}+1)(\mathbb{m}+\mathbb{A}^i\mathbb{B}^i)}\right], \tag{14}$$

$$\psi_t^i \equiv \frac{R_t^{i,h}}{R_{t+1}^i} = \psi_{IFA}^i = 1 - \frac{(1-\mathbb{A}^i)\mathbb{B}^i}{\mathbb{m}+\mathbb{B}^i}, \tag{15}$$

$$\omega_{t+1}^i = \left(\frac{\Lambda_{IFA}^i}{\mathbb{R}}\omega_t^i\right)^\alpha, \quad \text{where} \quad \Lambda_t^i = \Lambda_{IFA}^i = \frac{(\mathbb{m}+\mathbb{A}^i\mathbb{B}^i)(\mathbb{m}+1)}{(\mathbb{m}+\mathbb{A}^i)(\mathbb{m}+\mathbb{B}^i)}, \tag{16}$$

$$\frac{\partial \ln \Lambda_{IFA}^i}{\partial \theta^i} = \frac{\mathbb{m}(\mathbb{B}^i-1)}{(\mathbb{m}+\mathbb{A}^i\mathbb{B}^i)(\mathbb{m}+\mathbb{A}^i)}\frac{\partial \mathbb{A}^i}{\partial \theta^i} - \frac{\mathbb{m}(1-\mathbb{A}^i)}{(\mathbb{m}+\mathbb{A}^i\mathbb{B}^i)(\mathbb{m}+\mathbb{B}^i)}\frac{\partial \mathbb{B}^i}{\partial \theta^i} \geq 0. \tag{17}$$

ψ_t^i denotes the relative loan rate and Λ_t^i denotes the aggregate efficiency indicator. Both are time-invariant. As output is proportional to wage, $Y_t^i = \frac{(1+\epsilon)\omega_t^i}{(1-\alpha)}$, the model dynamics are characterized by the dynamic equation of wages (16). Given $\alpha \in (0,1)$, there exists a unique and stable steady state with the wage at $\omega_{IFA}^i = \left(\frac{\Lambda_{IFA}^i}{\mathbb{R}}\right)^\rho$.

Now, we show intuitively that $\bar{\theta}$ is the critical value for the borrowing constraints to be binding. If $\theta^i = \bar{\theta}$, $\mathbb{A}^i = \mathbb{B}^i = 1$ and thus, $R_t^{i,h} = R_{t+1}^i = \frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}$, so that the borrowing constraints are weakly binding. In this case, the aggregate credit demand is strong enough to push the loan rate equal to the social rate of return, $\psi^i = 1$; according to equation (7), the zero spread implies that $R_t^{i,e} = R_{t+1}^i = R_t^{i,h} = \frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R} = \mathbb{R}^{1-\alpha}\rho^{\alpha(1-\alpha)}(\frac{K_t^i}{L})^{-\alpha(1-\alpha)}$. Intuitively, in the country with a lower capital-labor ratio $\frac{K_t^i}{L}$, the growth rate $\frac{\omega_{t+1}^i}{\omega_t^i}$ is higher and so are the interest rates. We call this the *neoclassical* effect, as it arises from the concavity of the neoclassical production function with respect to the capital-labor ratio. For $\theta^i > \bar{\theta}$, entrepreneurs do not have an incentive to borrow to the limit and the equilibrium allocation is identical as in the case of $\theta^i = \bar{\theta}$. In both cases, aggregate savings $\frac{\beta\omega_t^i}{1+\mathbb{m}}$ is transformed by entrepreneurs into capital so that the aggregate efficiency indicator is $\Lambda_{IFA}^i = 1$. In the steady state, the wage is $\omega_{IFA}^i = \mathbb{R}^{-\rho}$, and the interest rates are $R_{IFA}^{i,j} = R_{IFA}^i = \mathbb{R}$. Iff $\theta^i < \bar{\theta}$, it holds that $\mathbb{A}^i < 1 < \mathbb{B}^i$. According to equations (13) and (14), $R_t^{i,h} < \frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R} < R_{t+1}^i$ so that the borrowing constraints are strictly binding.

In subsection 2.2 and 2.3, we focus on the case of $\theta^i \in [0, \bar{\theta})$ and analyze the distortions of financial frictions in the presence of inelastic saving ($\mathbb{m} = 0$) and elastic saving ($\mathbb{m} > 0$), respectively. The individuals' saving rates are,

$$\frac{s_t^{i,h}}{\omega_t^i} = \beta\left[1 - \frac{(1-\beta)\epsilon}{\beta}\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,h}}\right] = \frac{\beta\mathbb{A}^i}{\mathbb{m}+\mathbb{A}^i}, \quad \text{and iff } \mathbb{m} > 0, \quad \frac{\partial \frac{s_t^{i,h}}{\omega_t^i}}{\partial \theta^i} > 0; \tag{18}$$

$$\frac{s_t^{i,e}}{\omega_t^i} = \beta\left[1 - \frac{(1-\beta)\epsilon}{\beta}\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,e}}\right] = \frac{\beta\mathbb{B}^i}{\mathbb{m}+\mathbb{B}^i}, \quad \text{and iff } \mathbb{m} > 0, \quad \frac{\partial \frac{s_t^{i,e}}{\omega_t^i}}{\partial \theta^i} < 0. \tag{19}$$

Define the aggregate saving rate as the ratio of aggregate saving $S_t^i \equiv (1-\eta)s_t^{i,h} + \eta s_t^{i,e}$ over aggregate labor income of young individuals in country i,

$$\frac{S_t^i}{\omega_t^i} = \beta - (1-\beta)\epsilon\frac{\omega_{t+1}^i}{\omega_t^i}\left(\frac{1-\eta}{R_t^{i,h}} + \frac{\eta}{R_t^{i,e}}\right) = \frac{\beta(\mathbb{m}+\mathbb{A}^i\mathbb{B}^i)}{(\mathbb{m}+\mathbb{A}^i)(\mathbb{m}+\mathbb{B}^i)}; \text{ iff } \mathbb{m}>0, \quad \frac{\partial\frac{S_t^i}{\omega_t^i}}{\partial\theta^i} > 0. \tag{20}$$

2.2 The Equilibrium with Inelastic Savings

$\mathbb{m} = 0$ if individuals are fully patient ($\beta = 1$) or if there is no human wealth effect ($\epsilon = 0$). According to equations (18)-(20), the individual's and aggregate saving rates are constant at $\frac{s_t^{i,j}}{\omega_t^i} = \frac{S_t^i}{\omega_t^i} = \beta$. The binding borrowing constraints depress aggregate credit demand and the loan rate falls below the social rate of return to clear the credit market. According to equation (7), the positive spread makes the equity rate higher than the social rate of return. Thus, financial frictions create a wedge between the private and the social rates of return, $\psi_t^i = \frac{R_t^{i,h}}{R_{t+1}^i} = \mathbb{A}^i < 1 < \frac{R_t^{i,e}}{R_{t+1}^i} = \mathbb{B}^i$. The smaller θ^i, the larger the interest rate wedge. We call this the *financial-underdevelopment* effect and measure it by $1 - \psi_t^i$.

Being interest inelastic, aggregate saving is not affected by financial frictions. Thus, aggregate investment is efficient $K_{t+1}^i = \frac{\beta\omega_t^i}{\mathbb{m}+1}$ and so is aggregate output, $\Lambda_{IFA}^i = 1$.

2.3 The Equilibrium with Elastic Savings

$\mathbb{m} > 0$ if individuals are impatient ($\beta < 1$) or if there is the human wealth effect ($\epsilon > 0$). According to equations (18)-(20), the saving rates are interest elastic. Besides distorting the interest rates through the financial-underdevelopment effect, financial frictions also distort aggregate saving, investment, and output.

According to equations (18)-(19), the distorted interest rates depress household saving and raise entrepreneurial saving through the individuals' human wealth channel. According to equation (20), a lower θ^i leads to a lower aggregate saving rate, implying that inefficiently low household saving must dominate inefficiently high entrepreneurial saving. What is the economic intuition behind that?

Let $R^{i,j} \equiv \frac{\omega_t^i}{\omega_{t+1}^i}R_t^{i,j}$ denote the interest rate normalized by the the gross growth rate of wage. Define an auxiliary function, $\mathbb{M}(x_1, x_2, p) \equiv (1-\eta)x_1^p + \eta x_2^p$. The aggregate saving rate is rewritten as $\frac{S_t^i}{\omega_t^i} = \beta - (1-\beta)\epsilon\mathbb{M}(R^{i,h}, R^{i,e}, -1)$, where $\epsilon\mathbb{M}(R^{i,h}, R^{i,e}, -1)$ captures the aggregate human wealth effect.

Without loss of generality, we assume that country N is more financially developed, $0 < \theta^S < \theta^N < \bar{\theta}$. As discussed above, the loan rate is higher but the equity rate is lower in country N than in country S. According to the reward splitting rule (10), the normalized interest rates are linearly related, $(1-\eta)R^{i,h} + \eta R^{i,e} = \mathbb{R}$. Points S and N in figure 1 represent the interest rates in the two countries, which are on the same reward splitting line (the downward-sloping solid line). $\mathbb{M}(R^{i,h}, R^{i,e}, -1)$ is shown by the convex isoquant. According to the Jensen's inequality theorem, the lower the isoquant, the larger

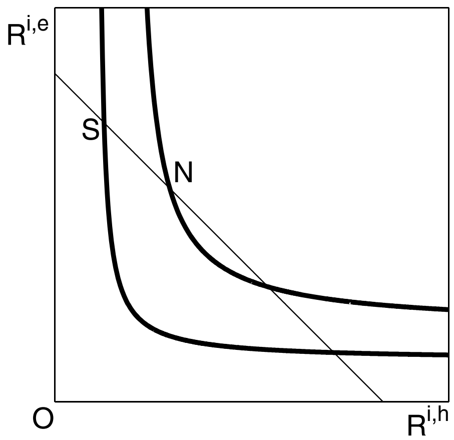

Figure 1: Graphic Illustration of the Aggregate Human Wealth Effect

the aggregate human wealth effect and the lower the aggregate saving rate. Thus, financial frictions reduce the aggregate saving rate through the interest rates channel. We call this the *elastic saving* effect.

Let $\upsilon_t^{i,j} \equiv \frac{\partial \ln s_t^{i,j}}{\partial \ln R_t^{i,j}}$ denote the interest elasticity of saving for individual j and $\Upsilon_t^i \equiv \frac{\partial \ln S_t^i}{\partial \ln R_t^{i,h}}$ denote the elasticity of aggregate saving with respect to the loan rate under IFA.

Lemma 1. *$\upsilon_t^{i,h} = \frac{\mathbb{m}}{A^i}$ and $\upsilon_t^{i,e} = \frac{\mathbb{m}}{B^i}$ are linear in $\mathbb{m}$. Iff $\theta^i < \bar{\theta}$, $\Upsilon_t^i > 0$ and rises in $\mathbb{m}$.*

In a country with a higher ϵ or a lower β, $\mathbb{m}$ is larger and, according to equation (5), individuals save less when young. Changes in the interest rates tend to have larger impacts on aggregate savings. Thus, $\mathbb{m}$ is a key parameter affecting the interest elasticity of saving and crucial for the aggregate implications of capital mobility in section 3.

Since aggregate investment is financed by domestic saving under IFA, financial frictions distort aggregate investment and output. According to equation (17), $\frac{\partial \Lambda_{IFA}^i}{\partial \theta^i} > 0$ and Λ_{IFA}^i reaches its maximum of one, when the borrowing constraints are weakly binding at $\theta^i = \bar{\theta}$. According to equation (15), the same pattern exists for the relative loan rate, ψ_{IFA}^i. Thus, we can use ψ_{IFA}^i to measure the distortions on the interest rates and output.[3]

Proposition 1. *For $\theta^i \in [0, \bar{\theta})$, the borrowing constraint is binding and there is a unique and stable steady state in country i with the wage at $\omega_{IFA}^i = \left(\frac{\Lambda_{IFA}^i}{\mathbb{R}}\right)^\rho$.*

There is a wedge between the private and social rates of return, $R_t^{i,h} < R_{t+1}^i < R_t^{i,e}$. In the steady state, the loan rate rises and the equity rate falls in θ^i.

If $\beta = 1$ or $\epsilon = 0$, aggregate output is independent of θ^i. If $\beta < 1$ and $\epsilon > 0$, aggregate output is below the efficient level and rises in θ^i.

[3] von Hagen and Zhang (2009, 2011) develop a model with heterogenous projects and show that financial frictions distort aggregate investment among projects with different productivity and thus, aggregate output is inefficiently low. Although output is distorted through different channels in the current paper and in von Hagen and Zhang (2009, 2011), the implications of capital mobility are identical.

3 International Capital Mobility

Under full capital mobility, individuals are allowed to lend and make direct investments globally. Without loss of generality, we assume that the borrowing constraints are binding in both countries under IFA and country N is more financially developed, $0 \leq \theta^S < \theta^N \leq \bar{\theta}$. We first solve the equilibrium allocation analytically and show that the steady-state patterns of international capital flows under full capital mobility in our model are consistent with the three empirical facts mentioned in the introduction.

Let Φ_t^i and Ω_t^i denote the aggregate outflows of financial capital and FDI from country i in period t, respectively, with negative values indicating capital inflows. Financial capital outflows reduce the aggregate credit capital used for domestic investment, $D_t^{i,h} = (1-\eta)s_t^{i,h} - \Phi_t^i$, while FDI outflows reduce the aggregate equity capital used for domestic investment, $D_t^{i,e} = \eta s_t^{i,e} - \Omega_t^i$. Therefore, FDI flows raise the aggregate credit demand in the host country and reduce that in the parent country.[4] With these changes, the analysis in section 2 carries through for the cases of capital mobility, due to the (log-)linearity of preferences, projects, and borrowing constraints. Financial capital flows equalize loan rates and FDI flows equalize equity rates in the two countries. Credit and equity markets clear in each country as well as globally. To summarize,

$$\Phi_t^S + \Phi_t^N = \Omega_t^S + \Omega_t^N = 0, \quad R_t^{S,h} = R_t^{N,h} = R_t^{*,h}, \quad R_t^{S,e} = R_t^{N,e} = R_t^{*,e},$$
$$K_{t+1}^i = (1-\eta)s_t^{i,h} + \eta s_t^{i,e} - (\Phi_t^i + \Omega_t^i) = \lambda_t^i(\eta s_t^{i,e} - \Omega_t^i) \ .$$

The remaining conditions for market equilibrium in each country are same as under IFA.

At the world level, aggregate revenue of capital in period $t+1$ is distributed to households and entrepreneurs as the returns to their respective savings,

$$(1-\eta)R_t^{*,h} \sum_{i\in\{N,S\}} s_{t+1}^{i,h} + \eta R_t^{*,e} \sum_{i\in\{N,S\}} s_{t+1}^{i,e} = \sum_{i\in\{N,S\}} R_{t+1}^i K_{t+1}^i = \rho(1+\epsilon) \sum_{i\in\{N,S\}} \omega_{t+1}^i.$$

Using equation (5) to substitute away $s_t^{i,j}$, we get

$$(1-\eta)R_t^{*,h} + \eta R_t^{*,e} = \frac{\omega_{t+1}^w}{\omega_t^w}\mathbb{R}, \quad \text{where} \quad \omega_t^w \equiv \frac{\omega_t^S + \omega_t^N}{2}. \tag{21}$$

We call this the reward splitting rule at the world level.

Lemma 2. *Under full capital mobility, there is a unique and stable steady state.*

[4]In the case of debt default, the project liquidation value depends on the efficiency of the legal institution, the law enforcement, and the asset market in the host country. Thus, we assume that entrepreneurs making FDI borrow only from the host country and are subject to the borrowing constraints there. Alternatively, we can assume that entrepreneurs may borrow only in their parent country no matter where they invest, since the financial institutions in their parent country have better information on the credit record, social network, and business activities of the entrepreneurs. The realistic case should be a hybrid of these two. Our results hold under the two alternative assumptions.

Let X_{FCM} denote the steady-state value of variable X under full capital mobility. Define a time-invariant auxiliary variable $\mathcal{Z}^i_{FCM} \equiv \frac{(\psi^i_{FCM}-\psi^i_{IFA})\frac{\mathbb{m}+\mathbb{B}^i}{\mathbb{m}+1}}{(\psi^i_{FCM}-\psi^i_{IFA})\frac{\mathbb{m}+\mathbb{B}^i}{\mathbb{m}+1}+\mathbb{B}^i\frac{\eta}{(1-\eta)}} R^{i,e}_{IFA}$. The solution to the equilibrium allocation is,

$$R^{i,e}_t = \frac{\omega^w_{t+1}}{\omega^w_t}(R^{i,e}_{IFA} - \mathcal{Z}^i_{FCM}), \tag{22}$$

$$R^{i,h}_t = \frac{\omega^w_{t+1}}{\omega^w_t}\left(R^{i,h}_{IFA} + \frac{\eta}{1-\eta}\mathcal{Z}^i_{FCM}\right), \tag{23}$$

$$\psi^i_t = \psi^i_{FCM} = \frac{(1-\theta^i)R^{*,h}_{FCM}}{R^{*,e}_{FCM}} + \theta^i, \tag{24}$$

$$\Phi^i_t = (1-\eta)\beta\omega^i_t\left[1 - \frac{\omega^i_{t+1}}{\omega^i_t}\frac{R^{i,h}_{IFA}}{R^{*,h}_t}\right], \tag{25}$$

$$\Omega^i_t = \eta\beta\omega^i_t\left[1 - \frac{\omega^i_{t+1}}{\omega^i_t}\frac{R^{i,e}_{IFA}}{R^{*,e}_t}\right], \tag{26}$$

$$\Omega^i_t + \Phi^i_t = \beta\omega^i_t\left\{1 - \frac{\omega^i_{t+1}}{\omega^i_t}\left[\eta\frac{R^{i,e}_{IFA}}{R^{*,e}_t} + (1-\eta)\frac{R^{*,h}_{IFA}}{R^{*,h}_t}\right]\right\}, \tag{27}$$

$$\omega^i_{t+1} = \left(\frac{1-\theta^i}{R^{*,e}_t} + \frac{\theta^i}{R^{*,h}_t}\right)^\rho. \tag{28}$$

Under full capital mobility, the steady-state interest rates and capital flows are,

$$R^{i,e}_{FCM} = R^{i,e}_{IFA} - \mathcal{Z}^i_{FCM}, \quad R^{i,h}_{FCM} = R^{i,h}_{IFA} + \frac{\eta}{1-\eta}\mathcal{Z}^i_{FCM}, \tag{29}$$

$$\Phi^i_{FCM} = (1-\eta)\beta\omega^i_{FCM}\left(1 - \frac{R^{i,h}_{IFA}}{R^{*,h}_{FCM}}\right) = \eta\beta\omega^i_{FCM}\frac{\mathcal{Z}^i_{FCM}}{R^{*,h}_{FCM}}, \tag{30}$$

$$\Omega^i_{FCM} = \eta\beta\omega^i_{FCM}\left(1 - \frac{R^{i,e}_{IFA}}{R^{*,e}_{FCM}}\right) = -\eta\beta\omega^i_{FCM}\frac{\mathcal{Z}^i_{FCM}}{R^{*,e}_{FCM}}, \tag{31}$$

$$\Phi^i_{FCM} + \Omega^i_{FCM} = \eta\beta\omega^i_{FCM}\mathcal{Z}^i_{FCM}\frac{(R^{*,e}_{FCM} - R^{*,h}_{FCM})}{R^{*,e}_{FCM}R^{*,h}_{FCM}}. \tag{32}$$

Proposition 2. *In the steady state under full capital mobility, the world interest rates are* $R^{*,h}_{FCM} \in (R^{S,h}_{IFA}, R^{N,h}_{IFA})$ *and* $R^{*,e}_{FCM} \in (R^{N,h}_{IFA}, R^{S,h}_{IFA})$*, implying the partial convergence in the relative loan rate,* $\psi^S_{IFA} < \psi^S_{FCM} < \psi^N_{FCM} < \psi^N_{IFA}$*. Aggregate output is higher in country N than in country S. The gross and net capital flows are* $\Phi^S_{FCM} > 0 > \Phi^N_{FCM}$*,* $\Omega^S_{FCM} < 0 < \Omega^N_{FCM}$*, and* $\Phi^S_{FCM} + \Omega^S_{FCM} > 0 > \Phi^N_{FCM} + \Omega^N_{FCM}$*. The gross international investment return sums up to zero in each country,* $\Phi^i_{FCM}R^{*,h}_{FCM} + \Omega^i_{FCM}R^{*,e}_{FCM} = 0$.

With a higher level of financial development, country N imports financial capital, exports FDI, and receives net capital inflows. Since the rate of return on its foreign asset (FDI outflow) exceeds the interest rate paid for its foreign liability (financial capital inflow), $R^{*,e}_{FCM} > R^{*,h}_{FCM}$, country N receives the positive net international investment incomes, $\Phi^N_{FCM}(R^{*,h}_{FCM}-1)+\Omega^N_{FCM}(R^{*,e}_{FCM}-1)=\Phi^N_{FCM}R^{*,h}_{FCM}+\Omega^N_{FCM}R^{*,e}_{FCM}-(\Phi^N_{FCM}+\Omega^N_{FCM})=-(\Phi^N_{FCM}+\Omega^N_{FCM})>0$, despite its negative international investment positions, $\Phi^N_{FCM}+\Omega^N_{FCM}<0$. Thus, our model predictions are consistent with the three empirical evidences mentioned in the introduction.

In the following, we use this analytical framework to address the aggregate implications of capital mobility. Subsections 3.1 and 3.2 focus on the output and welfare implications, if both countries are initially in the steady state under IFA before capital mobility is allowed from period $t=0$ on. Subsection 3.3 analyzes how the patterns of capital flows may change or even reverse along its convergence path if country S is initially below its steady state under IFA. Subsection 3.4 shows that Matsuyama's symmetry-breaking property depends critically on the fact that aggregate investment takes place in the extensive margin.

3.1 The Output Implications of Capital Mobility

Let us start with the case of inelastic saving ($\mathbb{m}=0$), which results from either $\epsilon=0$ or $\beta=1$. Since the output implications of capital mobility are qualitatively identical in the case of either $\epsilon=0$ or $\beta=1$, we focus on the case of $\epsilon=0$ as follows. Individuals save a fraction β of the labor income when young and financial frictions do not affect output under IFA, $Y^i_{IFA}=\frac{1}{1-\alpha}\mathbb{R}^{-\rho}$. Upon full capital mobility in period $t=0$, aggregate saving is same as under IFA, $S^i_0=\beta\omega^i_0=\beta\omega^i_{IFA}$, and net capital flows directly reallocate the funds for investment from country S to country N, which has two consequences on output. First, output in country S (N) is lower (higher) in period $t=1$ than before; second, given the concave aggregate production with respect to the capital-labor ratio at the country level, world output is lower than under IFA, because net capital flows are in equilibrium from country S where the MPK is higher to country N where the MPK is lower.

Corollary 1. *In the case of inelastic saving, from period $t=1$ on, world output is lower than its steady-state value under IFA.*

Under IFA, financial frictions do not production and steady-state output is same in the two countries, even though the two countries differ in the level of financial development. Capital mobility breaks the initial symmetry in the two countries in the sense that capital, in the net term, flows "uphill" from the poor to the rich country in the new steady state, leading to world output losses, which is also present in Matsuyama (2004). This is a typical result of the theory of second best. In the presence of domestic financial frictions, capital account liberalization causes capital to flow to the country with the higher interest rates rather than to the country with the higher MPK. The output responses at the country and the world level depends on the size of net capital flows, $|\Omega^i_t+\Phi^i_t|$.

In the case of elastic saving ($\mathbb{m}>0$), besides the direct impact on output through cross-country capital reallocation, full capital mobility also has an indirect impact on output through aggregate saving. Take country S as an example. Financial capital outflows reduce the domestic credit supply and FDI inflows raise the domestic credit demand. Both forces push up the loan rate and induces domestic households to save more. Net capital outflows reduces the domestic credit supply and the rising competition from foreign entrepreneurs reduces the MPK. Both forces push down the equity rate and induce domestic entrepreneurs to save less. The opposite applies for country N. Thus, changes in the individual's saving depends on the size of gross capital flows, $|\Omega_t^i| + |\Phi_t^i|$. The aggregate saving rate in country i is,

$$\frac{S_t^i}{\omega_t^i} = \frac{(1-\eta)s_t^{i,h} + \eta s_t^{i,h}}{\omega_t^i} = \beta - (1-\beta)\epsilon \frac{\omega_{t+1}^i}{\omega_t^i}\left(\frac{1-\eta}{R_t^{*,h}} + \frac{\eta}{R_t^{*,e}}\right). \tag{33}$$

As shown in subsection 2.3, a higher level of financial development gives rise to a higher aggregate saving rate under IFA, $\frac{S_{IFA}^N}{\omega_{IFA}^N} > \frac{S_{IFA}^S}{\omega_{IFA}^S}$. Under full capital mobility, the cross-country equalization of the loan rate as well as the equity rate leads to the cross-country equalization of the aggregate saving rate in the steady state, i.e., the aggregate saving rate rises (declines) in country S (N), $\frac{S_{IFA}^S}{\omega_{IFA}^S} < \frac{S_{FCM}^S}{\omega_{FCM}^S} = \frac{S_{FCM}^N}{\omega_{FCM}^N} < \frac{S_{IFA}^N}{\omega_{IFA}^N}$. Thus, by raising (reducing) aggregate saving and hence, the total funds available for domestic investment, full capital mobility indirectly affect output in country S (N). Lemma 3 summarizes the overall effect on steady-state output in the case of elastic saving $\mathbb{m} > 0$.

Lemma 3. *If $\eta \in (0, 0.5)$, define $\kappa \equiv \frac{1-\sqrt{1-4\mathbb{m}^2(1-\eta)\eta}}{2} < \frac{1}{2}$ and there are three scenarios:*

1. *if $\mathbb{m} \in (0,1)$, $Y_{FCM}^S > Y_{IFA}^S$ holds for $\theta^S \in (0,\kappa)$, and $Y_{FCM}^N > Y_{IFA}^N$ holds for $\theta^N \in (\kappa, \bar{\theta})$;*
2. *if $\mathbb{m} \in (1, \frac{1}{2\sqrt{\eta(1-\eta)}})$, $Y_{FCM}^S > Y_{IFA}^S$ holds for $\theta^S \in (0,\kappa) \cup (1-\kappa, \bar{\theta})$, and $Y_{FCM}^N > Y_{IFA}^N$ holds for $\theta^N \in (\kappa, 1-\kappa)$;*
3. *if $\mathbb{m} > \frac{1}{2\sqrt{\eta(1-\eta)}}$, $Y_{FCM}^S > Y_{IFA}^S$ always holds.*

If $\eta \in (0.5, 1)$, there are two scenarios:

1. *if $\mathbb{m} \in (0,1)$, $Y_{FCM}^S > Y_{IFA}^S$ holds for $\theta^S \in (0,\kappa)$, and $Y_{FCM}^N > Y_{IFA}^N$ holds for $\theta^N \in (\kappa, \bar{\theta})$;*
2. *if $\mathbb{m} > 1$, $Y_{FCM}^S > Y_{IFA}^S$ always holds.*

Put it plainly, a larger $\mathbb{m}$ implies that aggregate saving is more interest-elastic. The rise in domestic saving in country S is more likely to exceed net capital outflows so that domestic investment is higher than under IFA and so is output.

Full capital mobility affects world output also through the direct and indirect channels. First, "uphill" net capital flows directly lead to cross-country capital reallocation, which widens the cross-country output gap. The direct effect on world output is negative, depending on the size of net capital flows. Second, both financial capital and FDI flows indirectly affect aggregate saving at the country level. Given $\theta^S < \theta^N$, the rise in aggregate saving of country S dominates the decline in country N so that world saving rises and so does world output. The indirect effect on world output is positive, depending on the size of gross capital flows. Two-way capital flows imply that gross flows significantly exceed net flows. Thus, it is possible that full capital mobility raises world output, despite "uphill" net capital flows. Since the indirect effect essentially results from the elastic saving, the size of the indirect effect naturally depends on the interest elasticity of aggregate saving. According to Lemma 1, the higher $\mathbb{m}$, the more elastic the aggregate saving, the larger the indirect effect, the more likely full capital mobility raise world output.

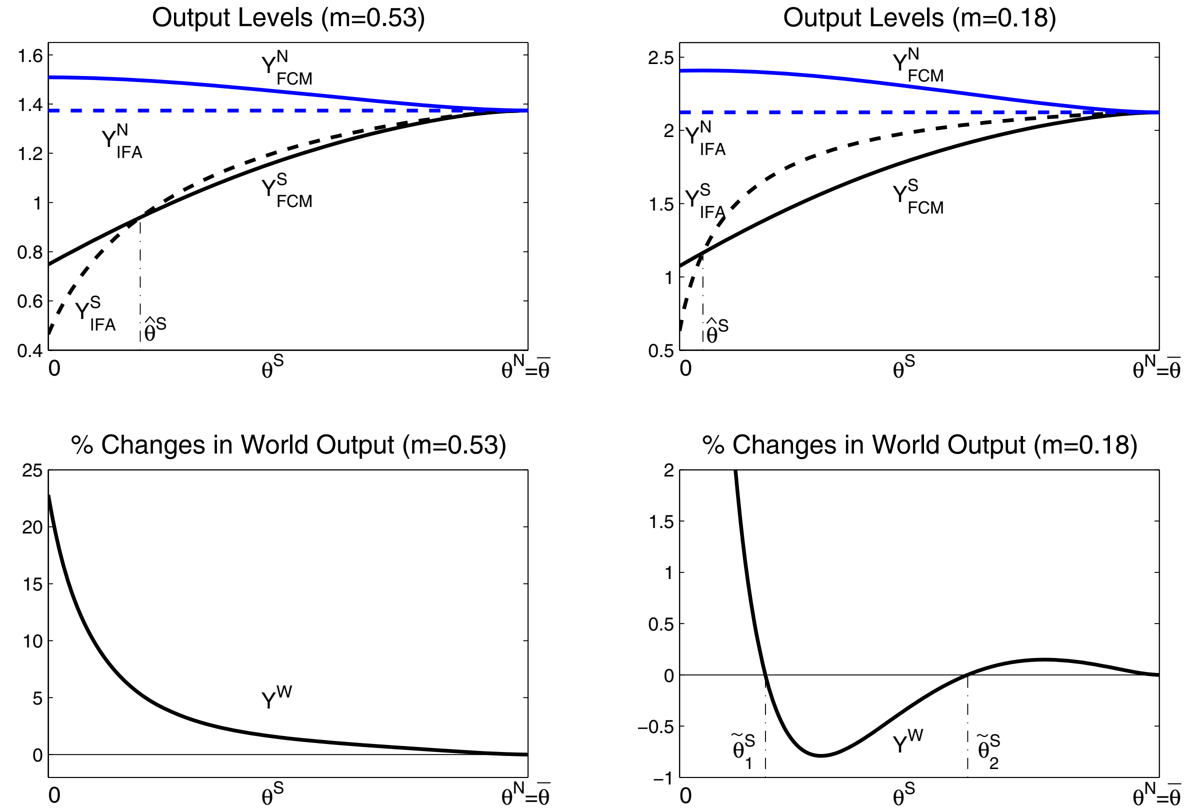

Figure 2: Comparing Steady-State Output under IFA and under Full Capital Mobility

For illustration purpose, we set up a numerical example and show that the world output implications of capital mobility depend on $\mathbb{m}$ as well as the cross-country difference in financial development. We set the population share of entrepreneurs at $\eta = 10\%$, the share of labor income in aggregate output, $1 - \alpha = 64\%$, and individuals put more weights on consumption when young, $1 - \beta = 0.6 > \beta = 0.4$. We consider two alternative cases with $\epsilon \in \{1, 0.2\}$ and correspondingly, $\mathbb{m} \in \{0.53, 0.18\}$.

The upper-left and upper-right panels of figure 2 show the steady-state output levels in the two countries under full capital mobility versus under IFA, with $\theta^S \in [0, \bar{\theta})$ on the horizontal axes, given $\theta^N = \bar{\theta}$. Given the parameter values, full capital mobility strictly raises steady-state output in country N, while it raises steady-state output in country S if θ^S is below a threshold value $\hat{\theta}^S$, confirming our results in Lemma 3. The bottom-left and bottom-right panels show the percentage changes of steady-state world output under full capital mobility versus under IFA, $\left(\frac{Y^w_{FCM}}{Y^w_{IFA}} - 1\right) 100$. If $\mathbb{m}$ is sufficiently high, e.g., $\mathbb{m} = 0.53$ in our example, the output gains in country N always exceed the output losses (if any) in country S so that world output is higher than under IFA; if $\mathbb{m}$ is small, e.g., $\mathbb{m} = 0.18$ in our example, there exists two threshold values $\tilde{\theta}^S_1$ and $\tilde{\theta}^S_2$ such that, for $\theta^S \in (\tilde{\theta}^S_1, \tilde{\theta}^S_2)$, full capital mobility reduces steady-state world output, while, for $\theta^S \in (0, \tilde{\theta}^S_1) \cup (\tilde{\theta}^S_2, \bar{\theta})$, it raises steady-state world output.

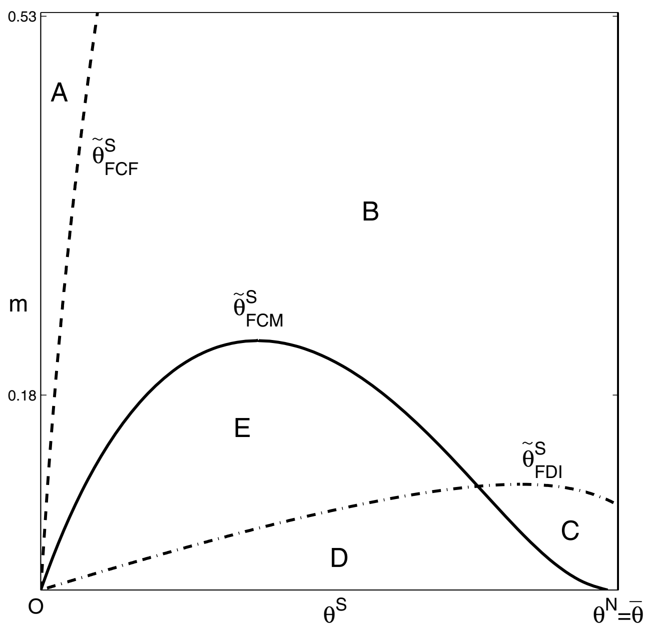

Figure 3: Threshold Values under Three Scenarios of Capital Mobility

Given $\mathbb{m} \in (0, 0.53)$ and $\theta^N = \bar{\theta}$, we compute $\tilde{\theta}^S$ for world output under full capital mobility as well as under the two alternative scenarios, i.e., *free mobility of financial capital* under which individuals are allowed to lend abroad but entrepreneurs are not allowed to make direct investments abroad, and *free mobility of FDI* under which entrepreneurs are allowed to make direct investments abroad but individuals are not allowed to lend abroad.[5] Figure 3 shows these threshold values in the parameter space $(\theta^S, \mathbb{m})$, where the solid curve denoted by $\tilde{\theta}^S_{FCM}$, the dash curve denoted by $\tilde{\theta}^S_{FCF}$, and the dash-dot line denoted by $\tilde{\theta}^S_{FDI}$ refer to the threshold values under the scenarios of full capital mobility, free mobility of financial capital, and free mobility of FDI, respectively. In each scenario, capital mobility raises the steady-state world output if the parameters are in the region above the respective curve. As mentioned above, the indirect effect, which contributes positively to world output, depends crucially on elastic saving. Given θ^N and θ^S, a larger ϵ leads to a larger interest elasticity of savings, represented by a larger $\mathbb{m}$. In this case,

[5] See the technical analysis of the two scenarios in appendix A and B, respectively.

the output distortion of financial frictions under IFA is more severe. By ameliorating the output distortion, capital mobility generates a stronger indirect effect through elastic saving and world output is more likely to be higher than under IFA.

Let us first compare the scenarios of full capital mobility and free mobility of financial capital. Under free mobility of financial capital, financial capital flows from country S to country N. "Uphill" capital flows directly widen cross-country output gap, leading to world output losses; by equalizing the loan rate across the border, financial capital flows indirectly induce households in country S (N) to save more (less) and aggregate saving at the world level is higher, leading to world output gains. The cross-country difference in θ^i has to be sufficiently large so that the indirect effect can be strong enough to override the direct effect. In our example, the parameters need to be in region A. Under full capital mobility, two-way capital flows imply that gross flows are significantly larger than net flows. Thus, even if the cross-country difference in θ^i is small, as in region B and C, the indirect effect may still dominate the direct effect. Thus, full capital mobility dominates free mobility of financial capital in generating world output gains.

Turning to free mobility of FDI alone, for parameters in region C, full capital mobility raises world output, while free mobility of FDI reduces world output. However, for parameters in region E, the opposite applies. Thus, full capital mobility does not necessarily dominate free mobility of FDI in generating world output gains. Consider parameters in region C. Since the cross-country output gap under IFA is small in this case, free mobility of FDI reverses the output gap through cross-country capital reallocation and the direct effect on world output is negative. The indirect effect, which depends on gross capital flows, is small here. Under full capital mobility, gross flows are significantly larger than net flows so that the indirect effect easily dominates the direct effect and world output is higher. Consider parameters in region E where two countries differ modestly in θ^i. Given the relatively large initial cross-country output gap under IFA, free mobility of FDI directly narrows the cross-country output gap through cross-country capital reallocation, implying a positive direct effect on world output. Thus, free mobility of FDI strictly raises world output. In contrast, under full capital mobility, "uphill" net capital flows imply that the direct effect is always negative and full capital mobility reduces world output.

Here, elastic saving is a critical channel through which full capital mobility may raise output in the less financially developed country as well as globally. Shutting down either financial capital or FDI flows may undermine such world output gains.

3.2 The Welfare Implications of Full Capital Mobility

As shown before, β and ϵ are two key parameters affecting the interest elasticity of saving and the output implications of capital mobility. We address here the welfare implications of full capital mobility in the cases of inelastic and elastic saving, respectively.

Case I: $\epsilon = 0$ and $\beta \leq 1$.

With no labor endowment when old ($\epsilon = 0$), an individual's lifetime wealth is simply its labor income when young, $\mathbb{W}_t^{i,j} = \omega_t^i$. If the individual is fully patient ($\beta = 1$), it does not consume when young but saves its entire labor income. In this case, the lifetime welfare only depends on its consumption when old, funded fully by its financial income, $u_t^{i,j} = c_{o,t+1}^{i,j} = \omega_t^i R_t^{i,j}$. If it is impatient ($\beta < 1$), it consumes a fraction $(1-\beta)$ of its labor income when young and save the rest. In this case, its welfare depends on its consumption in both periods of life and the financial income has smaller welfare impacts. In this sense, *impatience reduces the welfare impacts of interest rates*, $u_t^{i,j} = \omega_t^i (R_t^{i,j})^\beta$.

As a sufficient condition for $\mathbb{m} = 0$, $\epsilon = 0$ leads to interest-inelastic saving so that capital mobility reduces (raises) output and wage in country S (N) and world output is lower than under IFA. $\beta = 1$ or $\beta < 1$ does not change this result qualitatively. For generation $t = 0$, given the predetermined labor income, $\omega_0^i = \omega_{IFA}^i$, capital mobility makes households better (worse) off and entrepreneurs worse (better) off in country S (N) through the interest rate channel. For generation $t \to \infty$, the declines (rises) in labor income and the equity rate make entrepreneurs in country S (N) worse (better) off than under IFA; as labor income and the loan rate move in the opposite direction, the welfare implications to households are ambiguous. Intuitively, patience (a larger β) enhances the welfare impacts of interest rates and the interest rate effect is more likely to dominate the labor income effect. Using equation (28) to substituting away ω_{FCM}^i, we rewrite the long-run household welfare as

$$u_{FCM}^{i,h} = \omega_{FCM}^i (R_{FCM}^{*,h})^\beta = \left[(1-\theta^i)\frac{R_{FCM}^{*,h}}{R_{FCM}^{*,e}} + \theta^i\right]^\rho (R_{FCM}^{*,h})^{\beta-\rho}. \tag{34}$$

The loan rate converges across the border and so does the equity rate, i.e., $R_{IFA}^{S,h} < R_{FCM}^{*,h} < R_{IFA}^{N,h}$, and $\frac{R_{IFA}^{S,h}}{R_{IFA}^{S,e}} < \frac{R_{FCM}^{*,h}}{R_{FCM}^{*,e}} < \frac{R_{IFA}^{N,h}}{R_{IFA}^{N,e}}$. Thus, $\beta \geq \rho$ is a sufficient condition for households in country S (N) to be better (worse) off in the long run than under IFA.

Figure 4 shows the percentage differences in welfare under full capital mobility versus under IFA in the case of $\epsilon = 0$ and $\beta = 1$. The dashed lines show the welfare changes for generation $t = 0$, $\left(\frac{u_0^{i,j}}{u_{IFA}^{i,j}} - 1\right) 100$, and the solid lines for generation $t \to \infty$, $\left(\frac{u_{FCM}^{i,j}}{u_{IFA}^{i,j}} - 1\right) 100$. The upper (bottom) panels show the relevant variables in country S (N) and the horizontal axes denote $\theta^S \in (0, \bar{\theta})$. The parameter values are same as in the numerical example in subsection 3.1, except $\beta = 1$ and $\epsilon = 0$. Changes in the welfare of generation $t = 0$ ($t \to \infty$) reflect the short-run (long-run) welfare implications. Figure 5 shows the welfare changes in the case of $\epsilon = 0$ and $\beta = 0.4$.

Given $\alpha = 0.36$, if $\beta = 1$, $\beta > \rho$ so that households in country S (N) are strictly better (worse) off in the long run, as shown in figure 4; if $\beta = 0.4$, $\beta < \rho$ so that households in country S (N) may be worse (better) off in the long run, as shown in figure 5. The other measures of individuals' welfare have the qualitatively same responses in the two cases.

The social welfare of generation t is defined as the weighted sum of the welfare of

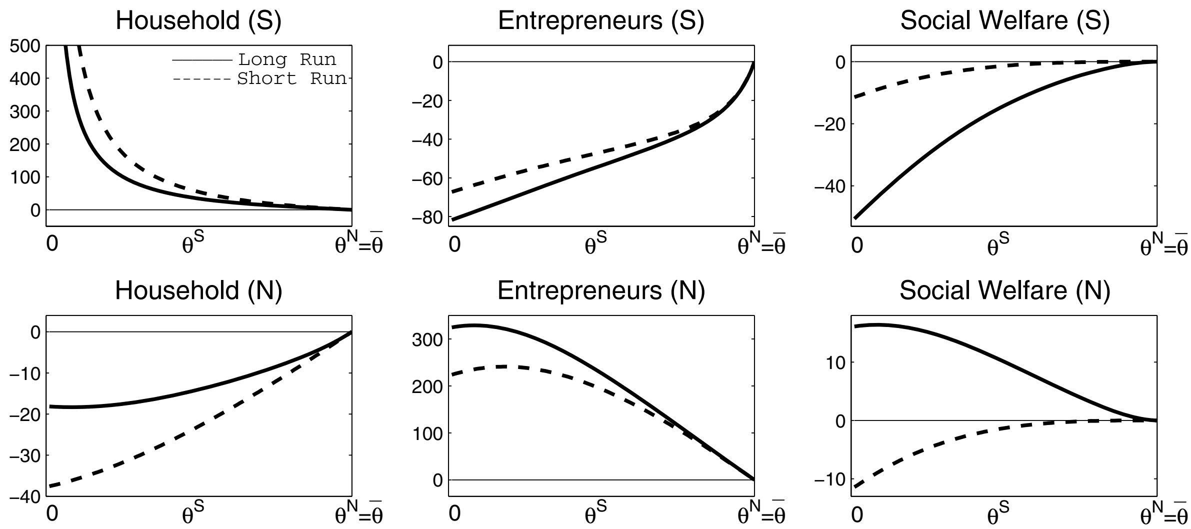

Figure 4: Percentage Changes in the Short-Run and Long-Run Welfare: $\epsilon = 0$ and $\beta = 1$

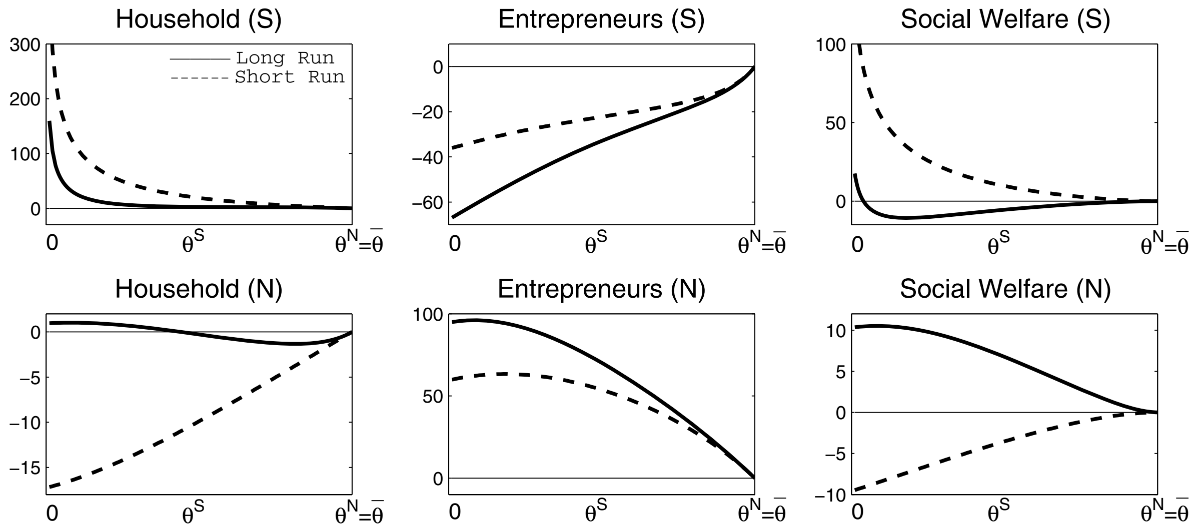

Figure 5: Percentage Changes in the Short-Run and Long-Run Welfare: $\epsilon = 0$ and $\beta = 0.4$

individuals born in period t, $U_t^i \equiv (1-\eta)u_t^{i,h} + \eta u_t^{i,e} = \omega_t^i \mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta)$, where $\mathbb{M}(x_1, x_2, p)$ is the auxiliary function defined in subsection 2.3. Full capital mobility affects social welfare through the labor income, ω_t^i, and a composite of interest rates in the form of the weighted average with the power β, $\mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta)$. Upon capital mobility, the responses in labor income are unambiguous, while the responses in the composite of interest rates depend on β, which is analyzed as follows.

Figure 6 shows the composite of interest rates in the space of $(R^{i,h}, R^{i,e})$. Point S (N) denotes the interest rate combination in country S (N) in the steady state under IFA, point A denotes that in period $t = 0$, and point L denotes that in period $t \to \infty$, i.e., in the steady state under full capital mobility.[6] According to equations (10) and (21), the

[6]Under full capital mobility, the loan rate converges across the border and so does the equity rate.

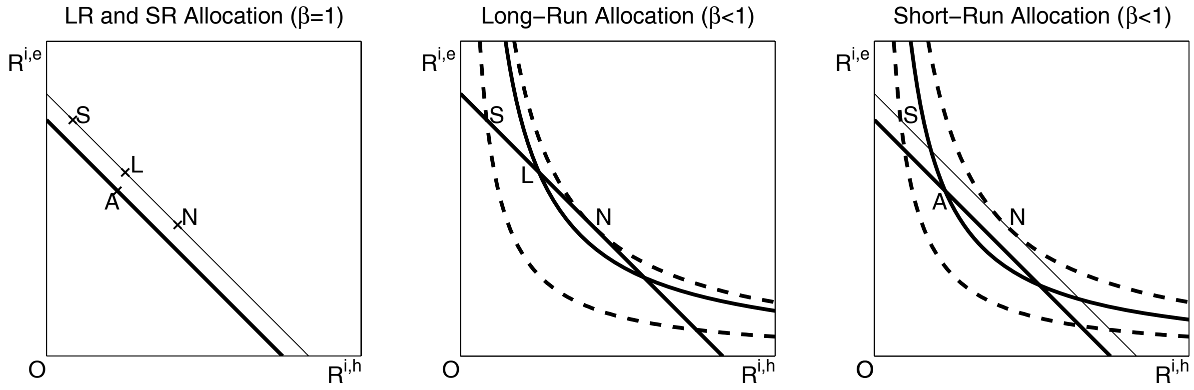

Figure 6: Graphic Illustration of $\mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta)$ under Full Capital Mobility versus IFA

reward splitting rules in the steady state under IFA and under full capital mobility are $(1-\eta)R_{IFA}^{i,h} + \eta R_{IFA}^{i,e} = \mathbb{R} = (1-\eta)R_{FCM}^{*,h} + \eta R_{FCM}^{*,e}$. Thus, point S, N, and L are on the same isoquant (the thin solid straight line). As capital mobility reduces world output, the world-average wage in period $t = 1$ falls. Given the reward-splitting rule (21) in period $t = 0$, $(1-\eta)R_0^{*,h} + \eta R_0^{*,e} = \frac{\omega_1^w}{\omega_0^w}\mathbb{R} < \mathbb{R}$, point A is on an isoquant (the thick solid straight line) below the previous one.

$\mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta)$ can be shown as the isoquant in the space of $(R^{i,h}, R^{i,e})$. Let us start with the case of $\beta = 1$ where the isoquant of $\mathbb{M}(R_t^{i,h}, R_t^{i,e}, 1)$ is a downward-sloping straight line and coincides with the one representing the reward splitting rule. See the left panel of figure 6. In period $t = 0$, $\mathbb{M}(R_0^{i,h}, R_0^{i,e}, 1) = \frac{\omega_1^w}{\omega_0^w}\mathbb{R} < \mathbb{R}$, while in the steady state under IFA and under full capital mobility, $\mathbb{M}(R_{IFA}^{i,h}, R_{IFA}^{i,e}, 1) = \mathbb{M}(R_{FCM}^{i,h}, R_{FCM}^{i,e}, 1) = \mathbb{R}$. Thus, the composite of interest rates declines in period $t = 0$ and converges back to its previous level in the long run, which is purely driven by the world-average growth effect.

Let us then consider the case of $\beta < 1$ where the isoquant of $\mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta)$ is convex and downward-sloping. The dashed curves and the solid curve in the middle panel of figure 6 are the isoquants of $\mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta)$ in the steady state under IFA and under full capital mobility. Due to the Jensen's inequality theorem, $\mathbb{M}(R_{IFA}^{S,h}, R_{IFA}^{S,e}, \beta) < \mathbb{M}(R_{FCM}^{i,h}, R_{FCM}^{i,e}, \beta) < \mathbb{M}(R_{IFA}^{N,h}, R_{IFA}^{N,e}, \beta)$. The dashed curves and the solid curve in the right panel of figure 6 show the isoquants of $\mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta)$ before and in period $t = 0$, respectively. The world-average growth effect reduces $\mathbb{M}(R_0^{i,h}, R_0^{i,e}, \beta)$, while the Jensen's inequality effect reduces $\mathbb{M}(R_{IFA}^{S,h}, R_{IFA}^{S,e}, \beta)$. If β is sufficiently small, the Jensen's inequality effect dominates so that $\mathbb{M}(R_0^{i,h}, R_0^{i,e}, \beta) > \mathbb{M}(R_{IFA}^{S,h}, R_{IFA}^{S,e}, \beta)$; if β is sufficiently close to one, the world-average growth effect dominates so that $\mathbb{M}(R_0^{i,h}, R_0^{i,e}, \beta) < \mathbb{M}(R_{IFA}^{S,h}, R_{IFA}^{S,e}, \beta)$. Nevertheless, $\mathbb{M}(R_0^{i,h}, R_0^{i,e}, \beta) < \mathbb{M}(R_{IFA}^{N,h}, R_{IFA}^{N,e}, \beta)$ always holds.

Now, we are ready to analyze the responses of social welfare. For generation $t = 0$, given the predetermined labor income, $\omega_0^i = \omega_{IFA}^i$, social welfare is driven purely by the

Thus, the interest rates in period $t = 0$ and in period $t \to \infty$ must be in the region to the bottom-right of point S and to the upper-left of point N.

composite of interest rates $\mathbb{M}(R_0^{i,h}, R_0^{i,e}, \beta)$. Thus, the social welfare in country N declines while the responses of social welfare in country S depends on β. For generation $t \to \infty$, since the changes in the labor income and the composite of interest rates are opposite, the social welfare implications are ambiguous, depending on β.

Let us compare the social welfare responses in the cases of $\beta = 0.4$ versus $\beta = 1$ (the third columns of figures 5 and 4). For a decline in β from 1 to 0.4, the short-run social welfare responses in country S changes from negative to positive and so does the long-run social welfare responses for θ^S close to zero. Thus, *(im)patience is an important factor affecting the welfare implications of capital mobility.*

Case II: $\epsilon > 0$ and $\beta < 1$.

An individual's lifetime welfare is

$$u_t^{i,j} = \mathbb{W}_t^{i,j}(R_t^{i,j})^{\beta} = \omega_t^i \left[1 + \epsilon \frac{\omega_{t+1}^i}{\omega_t^i} \frac{1}{R_t^{i,j}}\right] (R_t^{i,j})^{\beta}. \tag{35}$$

Compared with case I, allowing $\epsilon > 0$ introduces the human wealth component, $\epsilon \frac{\omega_{t+1}^i}{\omega_t^i} \frac{1}{R_t^{i,j}}$. An increase in the relevant interest rate affects the individual's welfare positively through the financial income channel as mentioned in case I and negatively through the human wealth channel. A larger ϵ or a larger β amplifies the welfare impacts of interest rate. Furthermore, a rise in the wage growth rate positively affect the individual's welfare through the human wealth channel and a larger ϵ magnifies its welfare impacts.

As shown in subsection 3.1, allowing either a positive human wealth ($\epsilon > 0$) or impatience ($\beta < 1$) does not change qualitatively the output implications of capital mobility as saving is interest-inelastic; combining them makes savings interest-elastic so that full capital mobility may generate output gains in country S and globally. Here, we focus on the welfare implications in the case of output gain in country S, i.e., θ^S is small.

Take entrepreneurs in country S as an example. Upon full capital mobility in period $t = 0$, and the decline in the equity rate, $R_0^{S,e} < R_{IFA}^{S,e}$, together with the positive wage growth, $\frac{\omega_1^S}{\omega_0^S} > 1$, raise the human wealth, partially offsetting its negative welfare effect through the financial income channel. Compared with case I, the welfare decline of entrepreneurs is much smaller. In period $t \to \infty$, the wage growth vanishes $\frac{\omega_{t+1}^S}{\omega_t^S} \to 1$ and the decline in the equity rate raises the human wealth, $\frac{\epsilon}{R_{FCM}^{i,j}}$, partially offsetting its negative welfare effect through the financial income channel, $(R_{FCM}^{i,j})^{\beta}$. Furthermore, in the presence of long-run output gains, $\omega_{FCM}^S > \omega_{IFA}^S$ has a positive welfare effect. For a sufficiently small β, the financial income effect can be dominated by the human wealth effect so that entrepreneurs of generation $t \to \infty$ can be better off than under IFA, in contrast to case I.

Let us consider social welfare of generation t. Rewrite the social welfare as

$$U_t^i = (1-\eta)u_t^{i,h} + \eta u_t^{i,e} = \omega_t^i \left[\mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta) + \epsilon \frac{\omega_{t+1}^i}{\omega_t^i} \mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta - 1)\right]. \tag{36}$$

Compared with case I, allowing $\epsilon > 0$ introduces the term $\epsilon \frac{\omega_{t+1}^i}{\omega_t^i} \mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta - 1)$. Take generation $t \to \infty$ in country S as an example. As discussed in case I, if individuals are impatient ($\beta < 1$), $\mathbb{M}(R_{FCM}^{S,h}, R_{FCM}^{S,e}, \beta) > \mathbb{M}(R_{IFA}^{S,h}, R_{IFA}^{S,e}, \beta)$. We can use the Jensen's inequality theorem to prove that $\mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta)$ and $\mathbb{M}(R_t^{i,h}, R_t^{i,e}, \beta - 1)$ move in the opposite direction. Thus, $\mathbb{M}(R_{FCM}^{S,h}, R_{FCM}^{S,e}, \beta - 1) < \mathbb{M}(R_{IFA}^{S,h}, R_{IFA}^{S,e}, \beta - 1)$. The wage growth vanishes in the long run. The overall responses of the term in the square bracket of equation (36) are ambiguous. Nevertheless, in the case of output gains, $\omega_{FCM}^S > \omega_{IFA}^S$, the positive labor income effect may dominate so that the social welfare is higher.

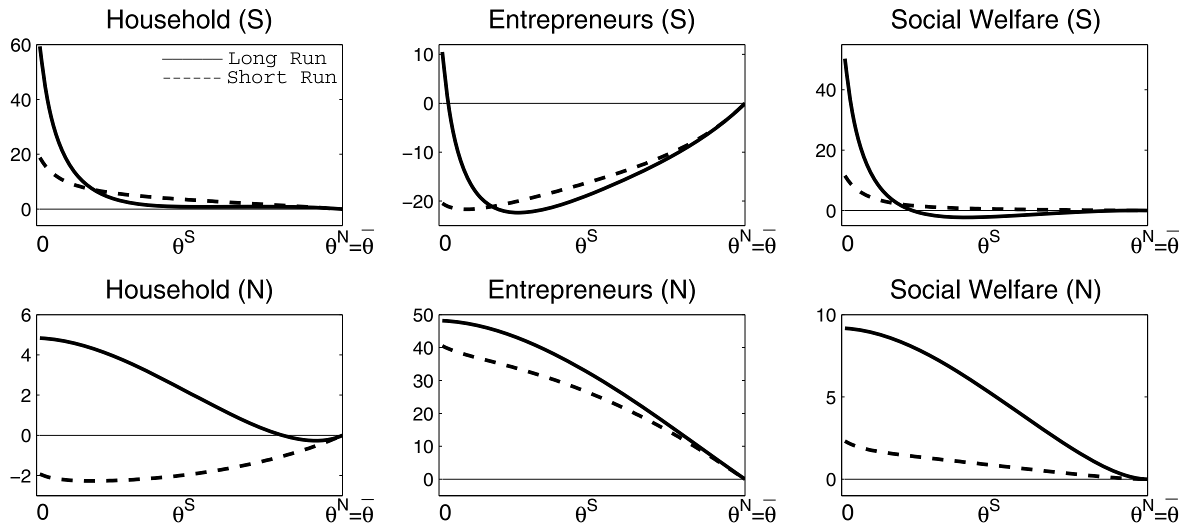

Figure 7: Percentage Changes in the Short-Run and Long-Run Welfare: $\epsilon = 1$ and $\beta = 0.4$

Figure 5 shows the welfare changes in the case of $\epsilon = 1$ and $\beta = 0.4$. Compared with case I, the most distinctive welfare responses are the welfare gains of entrepreneurs and the social welfare gains in the long run in country S, mainly due to output gains and the positive labor income effect. This way, *combining* $\epsilon > 0$ *and* $\beta < 1$ *creates the possibility of output gains which can make almost everyone better off.*

3.3 Full Capital Mobility and Economic Convergence

The analysis in subsections 3.1 and 3.2 is based on the assumption that both countries are initially in the steady state under IFA before capital mobility is allowed in period $t = 0$. In this subsection, we assume that country N is initially in the steady state, $K_0^N = K_{IFA}^N$, but country S is below the steady state under IFA, $K_0^S < K_{IFA}^S$. We address the interactions between international capital flows and domestic capital accumulation along the convergence path of country S. For simplicity, we assume that $0 < \theta^S < \theta^N = \bar{\theta}$. We focus on the case of elastic saving, i.e., $\epsilon > 0$ and $\beta < 1$.

As shown in subsection 2.2, a lower capital-labor ratio $K_0^S < K_0^N$ tends to keep the interest rates higher in country S through the neoclassical effect; a lower level of financial development $\theta^S < \theta^N$ tends to keep the loan rate lower and the equity rate higher in country S through the financial-underdevelopment effect. As a result, the equity rate is initially higher in country S so that FDI flows "downhill" in period $t = 0$. Depending on the relative magnitude of the neoclassical effect and the financial-underdevelopment effect, the loan rate in country S can be initially higher or lower than in country N so that financial capital flows can be "downhill" or "uphill", accordingly. In the following, we consider these two effects on the loan rate and the patterns of capital flows.

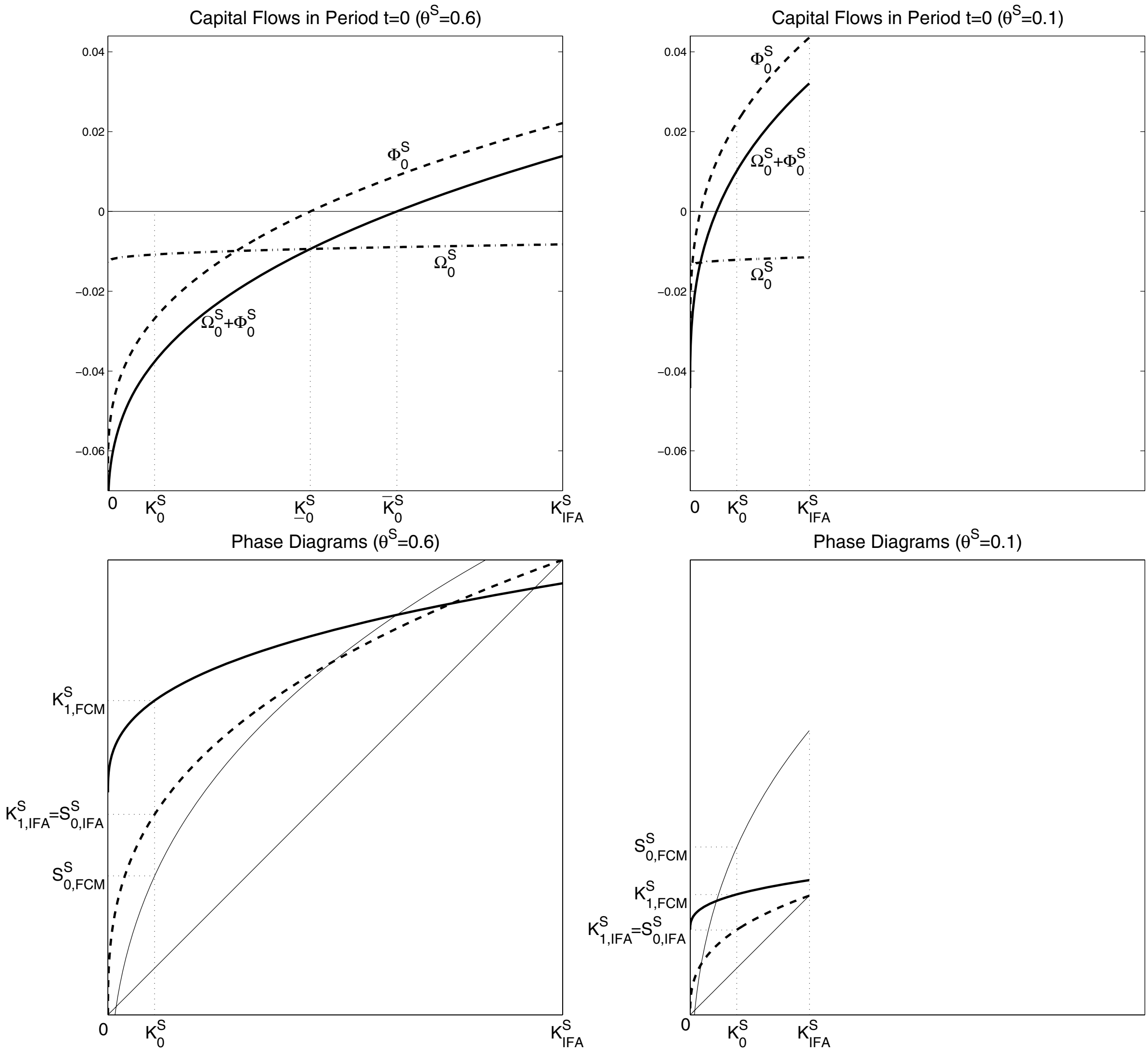

Figure 8: Patterns of Capital Flows and Economic Convergence

Let us start with the case of $\theta^S = 0.6$. The dashed line, the dash-dot line, and the solid line in the left panel of figure 8 show financial capital flows, FDI flows, and net capital flows in period $t = 0$ as the functions of K_0^S, respectively. The parameter values are identical as in the numerical exercise in subsection 3.1, except that $\epsilon = 0$.

If K_0^S is smaller than the first threshold value $\underline{K}_0^S$, the neoclassical effect dominates the financial-underdevelopment effect so that the loan rate is initially higher in country S.

Besides FDI inflows, country S also receives financial capital inflows in period $t = 0$, i.e., $\Phi_0^S, \Omega_0^S < 0$. Thus, net capital flows are "**D**ownhill" and gross flows are "**O**ne-way". The thick solid line and the dashed line in the right panel of figure 8 are the phase diagrams of the capital-labor ratio under full capital mobility and under IFA, respectively; the thin solid line is the 45 degree line. Net capital inflows speed up capital accumulation, $K_{1,FCM}^S - K_{1,IFA}^S = -(\Phi_0^S + \Omega_0^S) > 0$, so that country S converges faster to its steady state than otherwise under IFA.

If K_0^S is larger than the second threshold value $\bar{K}_0^S$, the neoclassical effect is significantly dominated by the financial-underdevelopment effect so that the loan rate is much lower in country S than in country N. Country S exports financial capital and imports FDI in period $t = 0$, $\Phi_0^S > 0 > \Omega_0^S$. As financial capital outflows dominate FDI inflows, country S has net capital outflows, $\Phi_0^S + \Omega_0^S > 0$. Thus, net capital flows are "**U**phill" and gross flows are "**T**wo-way". Net capital outflows slow down capital accumulation in country S in the short run and the capital-labor ratio converges in the long run to the level lower than under IFA.

If K_0^S is between the two threshold values, the neoclassical effect is slightly dominated by the financial-underdevelopment effect so that the loan rate is initially a bit lower in country S. In period $t = 0$, country S exports financial capital and imports FDI, $\Phi_0^S > 0 > \Omega_0^S$, while financial capital outflow is dominated by FDI inflow. Thus, net capital flows are still "**D**ownhill" but gross flows become "**T**wo-way". Net capital inflows speed up capital accumulation and economic convergence in country S.

The dash-dotted line and the dashed line in figure 9 show the two threshold values as the functions of θ^S in the space of (K^S, θ^S). Due to inelastic saving, K_{IFA}^S is independent of θ^S and shown as the vertical line at the right boundary. K_{FCM}^S increases in θ^S and is shown as the upward-sloping solid curve. Given $\theta^S = 0.6$ and the initial value of K_0^S as represented by point A, K_t^S rises over time along the flat path and sequentially crosses the two threshold values. Financial capital flows and net capital flows change directions when the capital-labor ratio moves from region **D-O** to **D-T** and **U-T**, respectively.

China's patterns of capital flows in the last two decades are consistent with our model predictions. The upper-left and the upper-right panels of figure 10 show China's patterns of international capital flows and investment positions in percentage of GDP in 1982-2011.[7] As is well known, Deng Xiaoping's southern tour in 1992 marked the beginning of China's dramatic economic opening represented by the policies encouraging FDI inflows and exports. Since then, China has received annual FDI inflows over 3% of GDP on average, leading to a negative position of FDI around 21% GDP in 2009.[8] Meanwhile, China's financial capital outflows are predominantly driven by official reserve accumulation, due to its fixed exchange rate. In order to distinguish between private and public flows of

[7]See Appendix D for data sources and computation.

[8]Prasad and Wei (2007) provide extensive description on China's FDI policy and data.

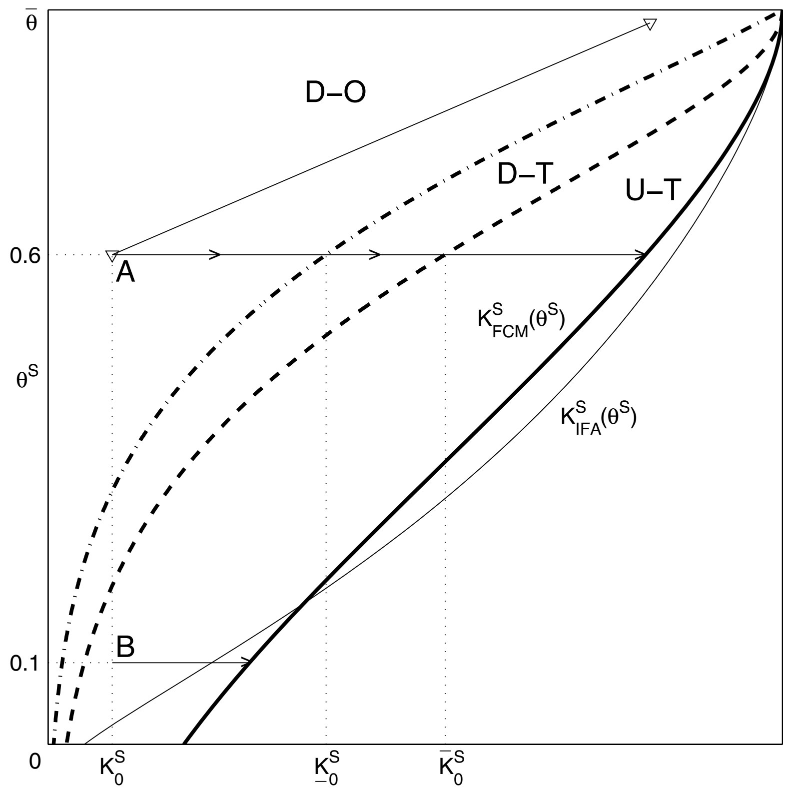

Figure 9: Threshold Values under Full Capital Mobility

financial capital, the bottom-left panel of figure 10 shows China's private foreign indirect investment (FII) flows and the annual changes in foreign reserve (FR) in percentage of GDP, while the bottom-right panel shows the positions of China's private foreign indirect investment (FII) and reserve assets (RA). Until the early 1990s, China received net inflows of foreign indirect investment (including portfolio investment and bank lending), leading to a negative position of FII around 9% of GDP. Since 1995, China witnessed outflows of indirect investment, leading to a positive FII position in 1998. The net international investment position turned from negative in 1990s to positive in 1998 and reached the peak at 34% of GDP in 2007. China's reversing patterns of capital flows over the past two decades are consistent with our theoretical predictions along the flat convergence path in figure 9.

So far, θ^S is assumed to be time invariant. Suppose that θ^S rises together with the capital-labor ratio, i.e., country S converges along the upward-sloping path starting from point A in figure 9.[9] For sufficiently dramatic improvements in the level of financial development, the entire convergence path has a high slope so that it stays in region **D-O**. In this case, country S receives the continuous inflows of FDI and financial capital, which speeds up the convergence process and leads to a higher steady-state capital-labor ratio than in the case of the time-invariant θ^S. If the improvement in financial development is

[9]The convergence path does not have to be a straight line in the case of time-variant θ^S.

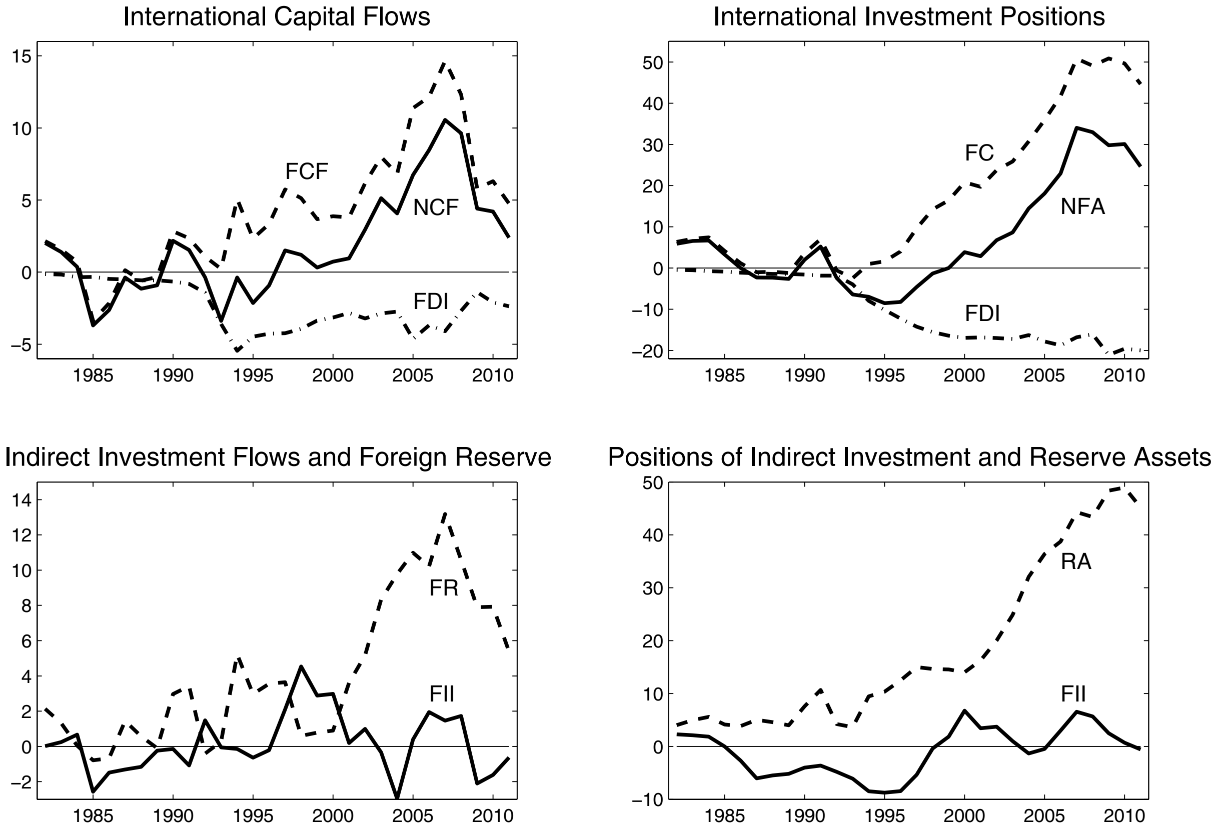

Figure 10: China's International Capital Flows and Investment Positions in 1982-2011

less dramatic, the slope of the convergence path is smaller so that it may still cross the threshold values but later than in the case of time-invariant θ^S.

Lane and Milesi-Ferretti (2007a) show that Central and East European countries (CEECs) ran on average current account deficits of over 5.5% of GDP, while Emerging Asian Economies (EAEs) ran on average a current account surplus of over 3% of GDP in 1995-2004. Abiad, Leigh, and Mody (2009) obtain similar results. Figure 11 shows the patterns of international capital flows and investment positions in 1994-2011 in percentage of GDP for ten CEECs vs. seven EAEs.[10] The ten CEECs as a whole have received financial capital and FDI inflows since 1997, expanding their negative international investment positions; the seven EAEs as a whole witnessed financial capital and net capital outflow after the 1997-98 Asian financial crisis, pushing their international investment positions to the positive range. Comparing the patterns in CEECs and EAEs, we could raise a hypothesis that

[10]The ten CEECs are Bulgaria, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Romania, Slovakia, Slovenia, which are currently the member states of European Union, while the seven EAEs are Indonesia, South Korea, Malaysia, Philippines, Taiwan, Thailand, and Vietnam. We focus on CEECs' post-transition period, i.e., 1994-2011. As the regional financial centers, Hong Kong and Singapore are left out of our sample. Adding these two economies further strengthens our results. Data source: CEIC. Variables are calculated in the same way as those for China. See appendix D for details.

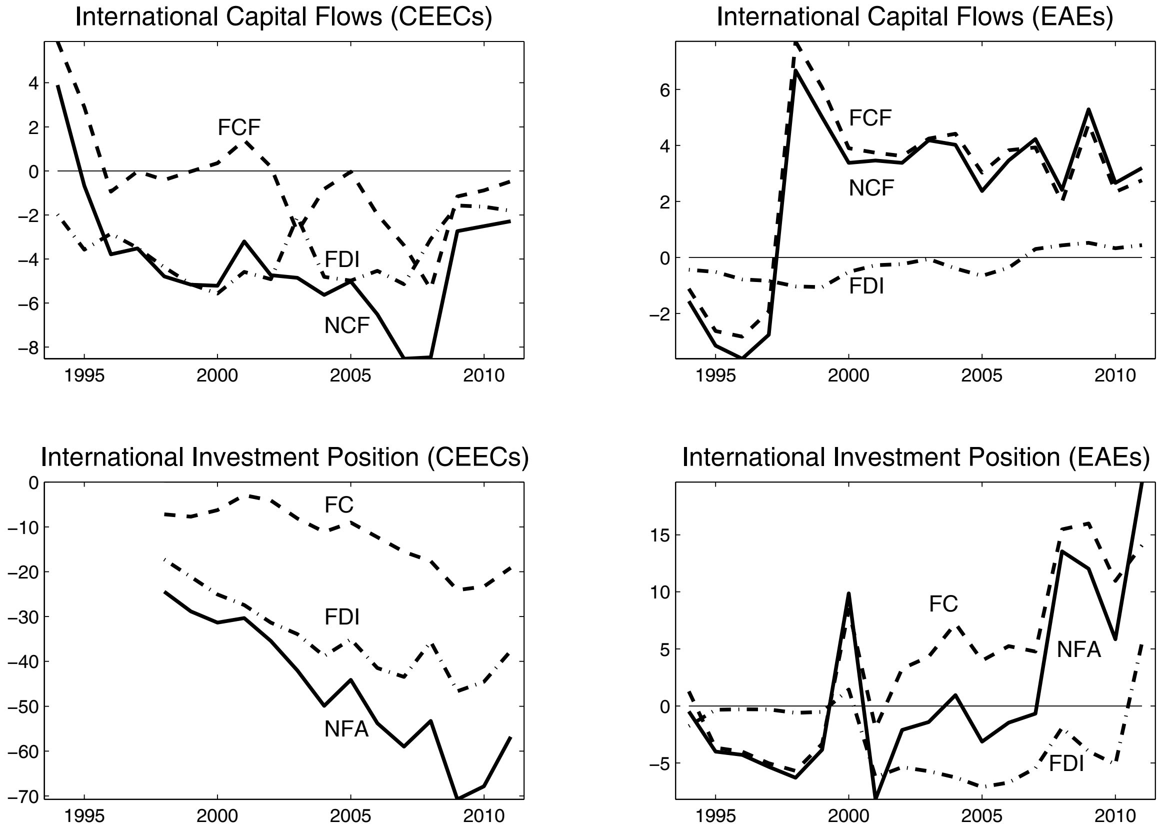

Figure 11: International Capital Flows and Investment Positions in 1994-2011

the levels of financial development in CEECs vs. EAEs are crucial for understanding their opposite patterns of current accounts.

Now, we can analyze the welfare implications of full capital mobility for a developing country along its convergence path to the steady state. Suppose that $\theta^S = 0.6$ is time invariant and country S starts from point A in figure 9. Under IFA, it would converge horizontally to the steady state with the capital labor ratio at K^S_{IFA}. Under full capital mobility, the inflows of financial capital and FDI reduce the interest rates in period $t = 0$. Given $u_0^{S,j} = \omega_0^S (R_0^{S,j})^\beta$ and the predetermined ω_0^S, the declines in the interest rates make both households and entrepreneurs of generation $t = 0$ worse off. However, net capital inflows speed up capital accumulation so that the labor income rises faster than under IFA. Additionally, if the capital-labor ratio exceeds the first threshold value, financial capital flows out of country S. Thus, the rises in labor income and the loan rate make households of later generations better off. As country S always receives FDI inflows, the decline in the equity rate dominates the rise in labor income so that entrepreneurs are worse off in the short run and in the long run. The dashed curve, dash-dot curve and the solid curve in figure 12 show the percentage differences in lifetime utility of households, entrepreneurs, and social welfare of generation $t = 0$ under full capital mobility versus under IFA, respectively. To sum up, full capital mobility has opposite welfare implications to individuals in the intra- and intergenerational dimension; at the country level, there is

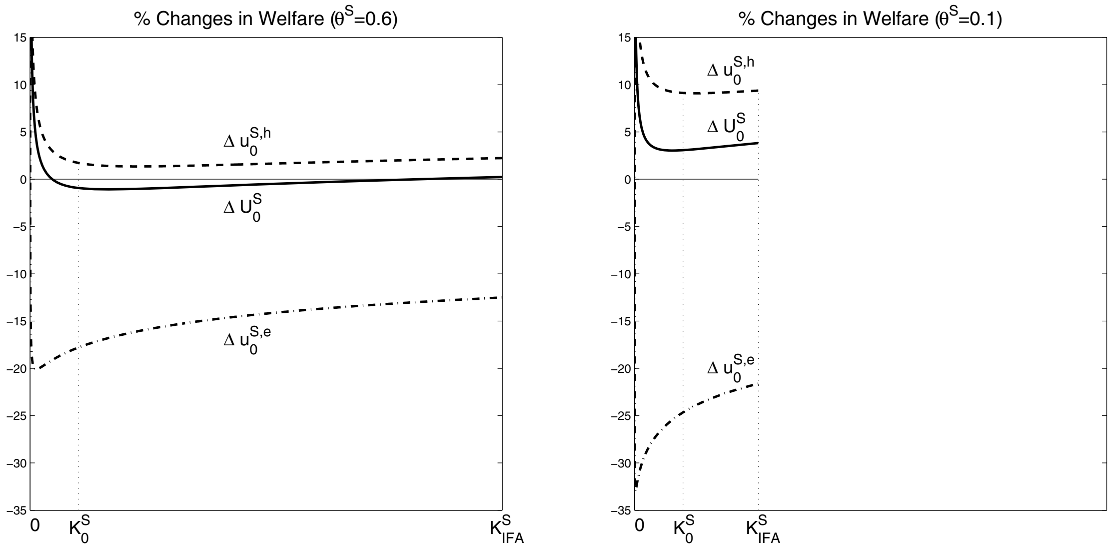

Figure 12: Welfare Comparison under Full Capital Mobility versus under IFA

a tradeoff between faster economic convergence in the short run and a lower output level in the long run.

As discussed in subsection 3.1, in the case of elastic saving ($\epsilon > 0$ and $\beta < 1$) and a sufficiently low θ^S, full capital mobility raises steady-state output in country S. Thus, the negative welfare implications due to lower steady-state output mentioned above are weakened so that full capital mobility may bring both short-run and long-run benefits to country S. Furthermore, if capital account liberalization is accompanied with the policies raising the level of financial development as observed in CEECs, output in the developing country can be higher both in the short run and in the long run.

3.4 Financial Integration and Symmetry Breaking

In a similar but simplified setting, i.e., $\beta = 1$ and $\epsilon = 0$, Matsuyama (2004) assumes that the size of every production project is fixed at $i_t^i = 1$, while the mass of individuals in a country who become entrepreneurs, η_t, is endogenously determined. He shows that, at a given world loan rate, free mobility of financial capital may lead to an equilibrium with multiple steady states. In contrast, we assume that the mass of entrepreneurs in a country is fixed at η, while the investment size of the entrepreneurial project i_t^i is endogenously determined. Since his model and ours differ essentially in this one aspect, it is straightforward to illustrate Matsuyama's result in the current framework.

Under free mobility of financial capital, the equilibrium conditions are almost identical as under full capital mobility except that FDI flows are set at zero, $\Omega_t^S = \Omega_t^N = 0$, and the equity rate is determined domestically rather than equalized across the border.[11] Given $\beta = 1$ and $\epsilon = 0$, individuals save their entire labor income, $s_t^{i,j} = \omega_t^i$. The borrowing

[11]See appendix A for detailed analysis of free mobility of financial capital.

constraints, if binding, take the same form under financial integration in Matsuyama (2004) and in our model,

$$R_t^{*,h}(1 - \frac{\omega_t^i}{i_t^i}) = \theta^i R_{t+1}^i = \theta^i(\omega_{t+1}^i)^{-\frac{1}{\rho}}. \tag{37}$$

Lemma 4. *Given the world loan rate $R_t^{*,h}$, for $\omega_t^i \in [0, 1-\theta^i]$, the phase diagram of wages in Matsuyama (2004) described by $R_t^{*,h}(1-\omega_t^i) = \theta^i(\omega_{t+1}^i)^{-\frac{1}{\rho}}$ is strictly convex, and ω_{t+1}^i increases monotonically in ω_t^i with an intercept on the vertical axis at $\omega_{t+1}^i = \left(\frac{\theta^i}{R_t^{*,h}}\right)^\rho$; for $\omega_t^i > 1-\theta^i$, the phase diagram of wages is flat with $\omega_{t+1}^i = \left(\frac{1}{R_t^{*,h}}\right)^\rho$.*

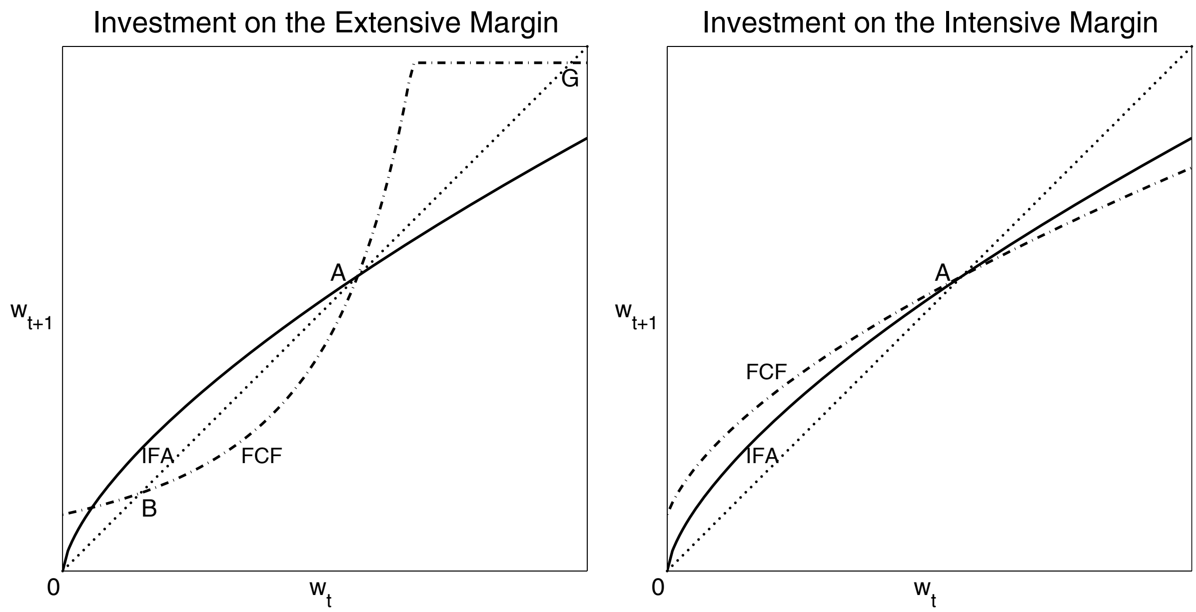

Figure 13: Phase Diagrams of Wage in Matsuyama (2004) and Our Model

The solid line in the left panel of figure 13 shows the phase diagram of wages under IFA in Matsuyama (2004), which gives rise to a unique and stable steady state at point A. Given a fixed world loan rate $R_t^{*,h} = R_{IFA}^{*,h}$, the dash-dotted line shows the phase diagram under free mobility of financial capital, which is convex for wages below a threshold value. Since the slope of the phase diagram at point A is now larger than one, the initial steady state at point A becomes unstable in the sense that any small perturbation at point A moves the allocation permanently away from point A. There are two stable steady states at points B and G. It implies that countries with the identical fundamentals (including θ) and, thus, the same steady state under IFA may end up with different levels of income under financial integration. Thus, Matsuyama (2004) claims that, in the presence of financial frictions, financial globalization may result in symmetry breaking.

The mechanism behind the symmetry breaking is as follows. According to equation (37), given the fixed size of project investment $i_t^i = 1$, a marginal increase in ω_t^i (hence, the entrepreneurial net worth) reduces the debt-investment ratio, $\frac{d_t^{i,h}}{i_t^i} = 1 - \frac{\omega_t^i}{i_t^i} = 1 - \omega_t^i$.

Given the fixed world loan rate, if the current wage ω_t^i exceeds the level corresponding to point A, the decline in the debt-investment ratio reduces the effective tightness of borrowing constraint, which allows increasingly more individuals to produce as entrepreneurs. Thus, even though all countries have the same level of financial development, the effective tightness of the borrowing constraints in a country depends critically on its initial capital stock in Matsuyama (2004), given the exogenously determined world interest rate. The higher the current wage ω_t^i, the easier to become entrepreneurs, the larger the expansion of aggregate investment and, consequently, the larger the increase in aggregate output and the wage in the next period. The opposite applies to the case where the current wage is below the level corresponding to point A. This explains the convexity of the phase diagram of wages in Matsuyama's model.

Lemma 5. *There is a unique and stable steady state under financial integration in our model.*

Given an exogenous world loan rate and a fixed mass of entrepreneurs, a marginal increase in the current wage enables entrepreneurs to borrow and invest more in our model. The negative effect of a marginal increase in the current wage ω_t^i on the debt-investment ratio, $\frac{d_t^{i,h}}{i_t^i} = (1 - \frac{\omega_t^i}{i_t^i})$, is partially offset by the increase in i_t^i, which is absent in Matsuyama (2004). According to equation (37), the positive impact on ω_{t+1}^i, is also smaller. The higher the current wage, the smaller the investment expansion and, consequently, the smaller the increase in aggregate output and the wage in the next period. This explains the concavity of the phase diagram of wages and the uniqueness of the steady state in our model. The solid line and the dash-dotted line in the right panel of figure 13 show the respective phase diagrams of wages under IFA and under free mobility of financial capital in our model, given a fixed world loan rate at $R_t^{*,h} = R_{IFA}^{*,h}$. In our model, wages converge monotonically and globally to a unique and stable steady state (point A) under IFA and under financial integration.

To sum up, given the fixed project size and borrowing constraints, aggregate investment responds to financial integration along the extensive margin in Matsuyama (2004), which may generate symmetry breaking; given the fixed mass of entrepreneurs and borrowing constraints, aggregate investment responds to financial integration along the intensive margin in our model, which preserves the model's stability property.

4 Conclusion

We develop a tractable, two-country, overlapping-generations model and show that cross-country differences in financial development can explain three recent empirical facts of international capital flows. International capital mobility may raise output at the country and the global level even when the less financially developed countries experience net capital outflows. The reason is that international capital flows not only lead directly to cross-country reallocation of aggregate saving but also trigger indirectly the adjustment along the consumption-saving margin. Under certain conditions, the indirect effect may override the prediction of conventional models in this literature, i.e., that net capital outflows from less financially developed countries raise output in these countries and globally. As it turns out, output gains are more likely, the larger are gross compared to net capital flows and the larger the difference in the levels of financial development among the countries under consideration. An obvious question then is whether the patterns of international capital flows observed in recent years are indeed output improving. Our model suggests two empirical indicators to consider. The first is the development of labor productivity after a less financially developed country opens up to international capital flows. Our model suggests that output gains come with the gains of labor productivity and, hence, real wages in this country. The second is that output gains come with a narrowing of the gap between the rate of return on equity and the rate of return on financial assets (equity premium) in the less financially developed country.

We also show that capital account liberalization may offer a developing country the short-run benefit of faster capital accumulation but possibly at the long-run cost of a lower level of output. In order to reduce the cost and exploit the benefit, the developing country should promote its level of financial development when liberalizing capital account.

We take the level of financial development as given and analyze how its differences affect capital flows. For future research, we plan to address how economic growth and various forms of capital flows reshape the level of financial development.

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Appendices

A Free Mobility of Financial Capital

Financial capital flows equalize the loan rate across the border and the credit markets clear in each country and globally.

$$R_t^{S,h} = R_t^{N,h} = R_t^{*,h}, \quad (1-\eta)s_t^{i,h} - \Phi_t^i = (\lambda_t^i - 1)\eta s_t^{i,e}, \quad \Phi_t^S + \Phi_t^N = 0.$$

The remaining conditions for market equilibrium in each country are same as under IFA. The solution to the equilibrium allocation is[12]

$$R_t^{i,e} = \frac{\omega_{t+1}^i}{\omega_t^i} R_{IFA}^{i,e}, \tag{38}$$

$$R_t^{i,h} = \frac{\omega_{t+1}^i}{\omega_t^i} R_{IFA}^{i,h} \frac{\psi_t^i - \theta^i}{\psi_{IFA}^i - \theta^i}, \tag{39}$$

$$\Phi_t^i = (1-\eta)\beta\omega_t^i \left(1 - \frac{\omega_{t+1}^i}{\omega_t^i} \frac{R_{IFA}^{i,h}}{R_t^{i,h}}\right) = (1-\eta)\beta\omega_t^i \frac{\psi_t^i - \psi_{IFA}^i}{\psi_t^i - \theta^i}, \tag{40}$$

$$\omega_{t+1}^i = \left(\frac{\Lambda_t^i}{\mathbb{R}}\omega_t^i\right)^{\alpha}, \text{ where } \Lambda_t^i = \Lambda_{IFA}^i \frac{1 - \frac{\theta^i}{\psi_{IFA}^i}}{1 - \frac{\theta^i}{\psi_t^i}}, \tag{41}$$

$$\frac{\partial \ln \Lambda_t^i}{\partial \psi_t^i} = -\frac{\theta^i}{\psi_t^i(\psi_t^i - \theta^i)} < 0 \tag{42}$$

The relative loan rate ψ_t^i is key to understand the model mechanism. Since the loan rate is initially lower in country S, $R_{IFA}^{S,h} < R_{IFA}^{N,h}$, financial capital flows from country S to country N in period $t = 0$, $\Phi_0^S > 0 > \Phi_0^N$, implying that $\psi_0^S > \psi_{IFA}^S$ and $\psi_0^N > \psi_{IFA}^N$, according to equation (40). Given $\psi_{IFA}^S < \psi_{IFA}^N$, financial integration leads to the (partial) convergence of the relative loan rate.

Let X_{FCF} denote the steady-state value of variable X under free mobility of financial capital. In the steady state, $\frac{\omega_{t+1}^i}{\omega_t^i} = 1$ and substitute it into the solution (38)-(41),

$$R_{FCF}^{i,e} = R_{IFA}^{i,e}, \quad R_{FCF}^{i,h} = R_{IFA}^{i,h} + R_{IFA}^{i,h} \frac{\psi_{FCF}^i - \psi_{IFA}^i}{\psi_{IFA}^i - \theta^i}, \tag{43}$$

$$\Phi_{FCF}^i = (1-\eta)\omega_{FCF}^i \frac{\psi_{FCF}^i - \psi_{IFA}^i}{\psi_{FCF}^i - \theta^i}, \quad \omega_{FCF}^i = \left(\frac{1-\theta^i}{R_{FCF}^{i,e}} + \frac{\theta^i}{R_{FCF}^{i,h}}\right)^{\rho}. \tag{44}$$

Proposition 3. *In the steady state, the world loan rate is $R_{FCF}^{*,h} \in (R_{IFA}^{S,h}, R_{IFA}^{N,h})$, implying that $\psi_{IFA}^S < \psi_{FCF}^S < \psi_{FCF}^N < \psi_{IFA}^N$; the equity rate in each country is same as under IFA, $R_{FCF}^{i,e} = R_{IFA}^{i,e}$; financial capital flows from country S to country N, $\Phi_{FCF}^S > 0 > \Phi_{FCF}^N$.*

If $\mathbb{m} > 0$, the household saving rate responds positively to the changes in the relative loan rate; if $\mathbb{m} = 0$, the household saving rate is time invariant and same as under IFA. The entrepreneurial saving rate is time invariant and same as under IFA.

$$\frac{s_t^{i,h}}{\omega_t^i} = \frac{s_{IFA}^{i,h}}{\omega_{IFA}^i}\left[1 + \frac{\mathbb{m}}{\mathbb{A}^i}\frac{\psi_t^i - \psi_{IFA}^i}{\psi_t^i - \theta^i}\right], \text{ and } \frac{s_t^{i,e}}{\omega_t^i} = \frac{s_{IFA}^{i,e}}{\omega_{IFA}^i} \tag{45}$$

B Free Mobility of FDI

The analysis for free mobility of FDI yields a mirror image of that for free mobility of financial capital and the main results are summarized as follows.[13]

[12] See the proof of Lemma 5 for technical derivations.

[13] See von Hagen and Zhang (2010) for detailed proofs and analysis.

Under free mobility of FDI, there exists a unique and stable steady state. Let X_{FDI} denote the steady-state value of variable X under free mobility of FDI. The loan rate is $R_t^{i,h} = \frac{\omega_{t+1}^i}{\omega_t^i} R_{IFA}^{i,h}$ with the same steady-state value as under IFA, $R_{FDI}^{i,h} = R_{IFA}^{i,h}$. FDI outflow from country i is $\Omega_t^i = \eta\beta\omega_t^i \left(1 - \frac{\omega_{t+1}^i}{\omega_t^i} \frac{R_{IFA}^{i,e}}{R_t^{*,e}}\right) = -\beta\eta\omega_t^i \left(\frac{\psi_t^i - \psi_{IFA}^i}{\psi_{IFA-\theta^i}^i}\right)$. Given the initial equity rate differential, $R_{IFA}^{S,e} > R_{IFA}^{N,e}$, FDI flows from country N to country S, $\Omega_t^N > 0 > \Omega_t^S$, implying the partial convergence of the relative loan rate, $\psi_{IFA}^S < \psi_t^S < \psi_t^N < \psi_{IFA}^N$. The equity rate responds negatively to the changes in the relative loan rate, $R_t^{i,e} = \frac{\omega_{t+1}^i}{\omega_t^i} R_{IFA}^{i,e} - \frac{\omega_{t+1}^i}{\omega_t^i} R_{IFA}^{i,e} \left[\frac{\psi_t^i - \psi_{IFA}^i}{\psi_t^i - \theta^i}\right]$. The dynamic equation of wage is $\omega_{t+1}^i = \left(\omega_t^i \frac{\Lambda_t^i}{\mathbb{R}}\right)$, with the aggregate efficiency indicator, $\Lambda_t^i = \Lambda_{IFA}^i \frac{\psi_t^i}{\psi_{IFA}^i}$, increasing in the relative loan rate. In the steady state, the world equity rate is $R_{FDI}^{*,e} \in (R_{IFA}^{N,e}, R_{IFA}^{S,e})$, FDI flows from country N to country S, $\Omega_{FDI}^N > 0 > \Omega_{FDI}^S$, where $\Omega_{FDI}^i = \eta\beta\omega_{FDI}^i \frac{(R_{FDI}^{*,e} - R_{IFA}^{i,e})}{R_{FDI}^{*,e}}$, and the wage rate is $\omega_{FDI}^S > \omega_{IFA}^S$ and $\omega_{FDI}^N < \omega_{IFA}^N$.

The household saving rate is time invariant and same as under IFA. If $\mathbb{m} > 0$, the entrepreneurial saving rate responds negatively to the changes in the relative loan rate; if $\mathbb{m} = 0$, the entrepreneurial saving rate is time invariant and same as under IFA.

$$\frac{s_t^{i,h}}{\omega_t^i} = \frac{s_{IFA}^{i,h}}{\omega_{IFA}^i}, \text{ and } \frac{s_t^{i,e}}{\omega_t^i} = \frac{s_{IFA}^{i,e}}{\omega_{IFA}^i} \left[1 - \frac{\mathbb{m}}{\mathbb{B}^i} \frac{\psi_t^i - \psi_{IFA}^i}{\psi_{IFA}^i - \theta^i}\right]. \tag{46}$$

C Proof

Proof of Lemma 1

Proof. Without loss of generality, we suppress the country index for simplicity. According to equation (5), $\upsilon_t^j = \frac{1}{\frac{\omega_t R_t^j}{\omega_{t+1}} \frac{\beta}{(1-\beta)\epsilon} - 1}$. Using equations (12) and (13) to substitute away R_t^j, we get $\upsilon_t^h = \frac{\mathbb{m}}{A}$ and $\upsilon_t^e = \frac{\mathbb{m}}{B}$ which are linear in $\mathbb{m}$.

According to the revenue splitting rule, $(1-\eta)R_t^h + \eta R_t^e = \frac{\omega_{t+1}}{\omega_t}\mathbb{R}$. The aggregate saving under IFA is rewritten as $S_t = \beta\omega_t - (1-\beta)\epsilon\omega_{t+1}\left[\frac{1-\eta}{R_t^h} + \frac{\eta}{R_t^e}\right]$. Let $\Upsilon_t \equiv \frac{\partial \ln S_t}{\partial \ln R_t^h}$ denote the elasticity of aggregate saving with respect to the loan rate.

$$\Upsilon_t = \frac{\partial S_t}{\partial R_t^h} \frac{R_t^h}{S_t} = (1-\beta)\epsilon\omega_{t+1}(1-\eta)\left[\frac{1}{(R_t^h)^2} - \frac{1}{(R_t^e)^2}\right]\frac{R_t^h}{S_t} \tag{47}$$

$$= \frac{(1-\beta)\epsilon\omega_{t+1}(1-\eta)}{\beta\omega_t - (1-\beta)\epsilon\omega_{t+1}\left(\frac{1-\eta}{R_t^h} + \frac{\eta}{R_t^e}\right)}\left[\frac{1}{R_t^h} - \frac{1}{R_t^e}\frac{R_t^h}{R_t^e}\right]. \tag{48}$$

Use equations (12) and (13) to substitute away R_t^j,

$$\Upsilon_t = \frac{(1-\beta)\epsilon(1-\eta)}{(1+\epsilon)\rho - (1-\beta)\epsilon\left(\frac{1-\eta}{\mathbb{m}+\mathbb{A}} + \frac{\eta}{\mathbb{m}+\mathbb{B}}\right)}\left[\frac{1}{\mathbb{m}+\mathbb{A}} - \frac{1}{\mathbb{m}+\mathbb{B}}\frac{\mathbb{m}+\mathbb{A}}{\mathbb{m}+\mathbb{B}}\right] \tag{49}$$

$$= \frac{\mathbb{m}(1-\eta)}{1 - \mathbb{m}\left(\frac{1-\eta}{\mathbb{m}+\mathbb{A}} + \frac{\eta}{\mathbb{m}+\mathbb{B}}\right)}\left[\frac{(\mathbb{m}+\mathbb{B})^2 - (\mathbb{m}+\mathbb{A})^2}{(\mathbb{m}+\mathbb{A})(\mathbb{m}+\mathbb{B})^2}\right] \tag{50}$$

$$= \frac{\mathbb{m}(1-\eta)}{(\mathbb{m}+\mathbb{A}\mathbb{B})(\mathbb{m}+\mathbb{B})}(2\mathbb{m}+\mathbb{B}+\mathbb{A})(\mathbb{B}-\mathbb{A}). \tag{51}$$

Iff $\theta < \bar{\theta}$, $\mathbb{B} > \mathbb{A}$ and $\Upsilon_t > 0$, implying that aggregate saving rises in the loan rate.

$$\frac{\partial \ln \Upsilon}{\partial \mathbb{m}} = \frac{1}{\mathbb{m}} - \frac{1}{\mathbb{m}+\mathbb{B}} - \frac{1}{\mathbb{m}+\mathbb{A}\mathbb{B}} + \frac{2}{(2\mathbb{m}+\mathbb{B}+\mathbb{A})} \tag{52}$$

$$= \frac{\mathbb{A}\mathbb{B}^2 - \mathbb{m}^2}{\mathbb{m}(\mathbb{m}+\mathbb{A}\mathbb{B})(\mathbb{m}+\mathbb{B})} + \frac{2}{(2\mathbb{m}+\mathbb{B}+\mathbb{A})} \tag{53}$$

$$= \frac{(\mathbb{B}-\mathbb{A})\mathbb{m}^2 + \mathbb{A}\mathbb{B}(2\mathbb{m}^2 + 4\mathbb{B}\mathbb{m} + \mathbb{A}\mathbb{B} + \mathbb{B}^2)}{\mathbb{m}(\mathbb{m}+\mathbb{A}\mathbb{B})(\mathbb{m}+\mathbb{B})(2\mathbb{m}+\mathbb{B}+\mathbb{A})} > 0. \tag{54}$$

Thus, Υ_t is positively related to $\mathbb{m}$. □

Proof of Proposition 1

Proof. If the borrowing constraints are binding, $R_t^{i,h} < R_{t+1}^i$, according to equation (7). We prove that equations (11)-(17) are the model solution in this case.

At the aggregate level, $\theta^i R_{t+1}^i K_{t+1}^i$ and $(1-\theta^i) R_{t+1}^i K_{t+1}^i$ are paid to households and entrepreneurs as the rewards to their respective contributions in the form of credit capital $D_t^{i,h}$ and equity capital $D_t^{i,e}$,

$$K_{t+1}^i = D_t^{i,h} + D_t^{i,e} = \frac{\theta^i R_{t+1}^i K_{t+1}^i}{R_t^{i,h}} + \frac{(1-\theta^i) R_{t+1}^i K_{t+1}^i}{R_t^{i,e}} \;\Rightarrow\; \frac{\theta^i}{R_t^{i,h}} + \frac{(1-\theta^i)}{R_t^{i,e}} = \frac{1}{R_{t+1}^i}. \tag{55}$$

We call it *the investment sharing rule.*

Aggregate capital stock consists of aggregate savings of households and entrepreneurs,

$$K_{t+1}^i = (1-\eta)s_t^{i,h} + \eta s_t^{i,e} = \beta\omega_t^i - (1-\beta)\epsilon\omega_{t+1}^i \left(\frac{\eta}{R_t^{i,e}} + \frac{1-\eta}{R_t^{i,h}}\right). \tag{56}$$

According to equations (2), the aggregate reward to capital is $R_{t+1}^i K_{t+1}^i = \rho(1+\epsilon)\omega_{t+1}^i$. Combine it with equation (56), we get *the aggregate capital reward rule*

$$\frac{\rho(1+\epsilon)\omega_{t+1}^i}{R_{t+1}^i} = \beta\omega_t^i - (1-\beta)\epsilon\omega_{t+1}^i \left(\frac{\eta}{R_t^e} + \frac{1-\eta}{R_t^h}\right). \tag{57}$$

Let $r_{t+1}^i \equiv \frac{R_{t+1}^i}{\frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}}$, $r_t^{i,e} \equiv \frac{R_t^{i,e}}{\frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}}$, and $r_t^{i,h} \equiv \frac{R_t^{i,h}}{\frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}}$ denote the social and the private interest rates normalized by $\frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}$. The aggregate capital reward rule (57), the reward splitting rule (10), and the investment sharing rule (55) are simplified as,

$$\frac{1}{r_{t+1}^i} = 1 + \mathbb{m} - \mathbb{m}\left(\frac{1-\eta}{r_t^{i,h}} + \frac{\eta}{r_t^{i,e}}\right)$$

$$\frac{1}{r_{t+1}^i} = \frac{\theta}{r_t^{i,h}} + \frac{1-\theta}{r_t^{i,e}}$$

$$1 = (1-\eta)r_t^{i,h} + \eta r_t^{i,e}.$$

Given the parameters θ, η, and $\mathbb{m}$, there exists a unique and time-invariant solution to the normalized interest rates, $r_t^{i,h} = \frac{\mathbb{m}+\mathbb{A}^i}{\mathbb{m}+1}$, $r_t^{i,e} = \frac{\mathbb{m}+\mathbb{B}^i}{\mathbb{m}+1}$, and $r_{t+1}^i = \frac{(\mathbb{m}+\mathbb{A}^i)(\mathbb{m}+\mathbb{B}^i)}{(\mathbb{m}+1)(\mathbb{m}+\mathbb{A}^i\mathbb{B}^i)}$. Thus, equations (12)-(14) are the solutions to interest rates. Using equation (14) to substitute away R_{t+1}^i from the factor reward equation $K_{t+1}^i = \rho(1+\epsilon)\frac{\omega_{t+1}^i}{R_{t+1}^i}$, we get the solution to aggregate capital stock (11). Combining equations (1)-(2), the factor prices are

$$Y_{t+1}^i = \left(\frac{\frac{\alpha Y_{t+1}^i}{R_{t+1}^i}}{\alpha}\right)^{\alpha}\left(\frac{\frac{(1-\alpha)Y_{t+1}^i}{\omega_{t+1}^i}}{1-\alpha}\right)^{1-\alpha} = \frac{Y_{t+1}^i}{(R_{t+1}^i)^{\alpha}(\omega_t^i)^{1-\alpha}} \Rightarrow (R_{t+1}^i)^{\alpha}(\omega_{t+1}^i)^{1-\alpha} = 1. \tag{58}$$

Using equation (14) to substitute away R_{t+1}^i, we get the dynamic equation of wages (16) with the aggregate efficiency indicator Λ^i.

□

Proof of Lemma 2

Proof. The proof consists of three steps. First, we prove that equation (22) is the solution to the equity rate. Define $\Delta\psi_t^i \equiv \psi_t^i - \psi_{IFA}^i$. Given the binding borrowing constraints, use $\psi_t^i = \frac{R_t^{i,h}}{R_{t+1}^i}$ and $\psi_{IFA}^i = \frac{R_{IFA}^{i,h}}{R_{IFA}^i}$ to rewrite the investment sharing rule (55) under IFA and under full capital mobility,

$$\frac{\psi_t^i}{1-\theta^i} - \frac{R_t^{i,h}}{R_t^{i,e}} = \frac{\theta^i}{1-\theta^i} = \frac{\psi_{IFA}^i}{1-\theta^i} - \frac{R_{IFA}^{i,h}}{R_{IFA}^{i,e}}, \quad \Rightarrow \quad \frac{\Delta\psi_t^i}{1-\theta^i} = \frac{R_t^{i,h}}{R_t^{i,e}} - \frac{R_{IFA}^{i,h}}{R_{IFA}^{i,e}}. \tag{59}$$

Substituting $R_t^{i,h}$ and $R_{IFA}^{i,h}$ with $R_t^{i,e}$ and $R_{IFA}^{i,e}$ using the reward splitting rules (21) and (10), we solve the equity rate from equation (59). Plug the solution to the equity rate into the reward splitting rule (21) to solve for $R_t^{i,h}$. Using the approach in the proof of Lemma 5, we can prove the solutions to financial capital and FDI flows (25) and (26).

Second, we prove that ψ_t^i is constant under full capital mobility. Suppose that ψ_t^i is time variant and so is $\mathcal{Z}_t^i$ defined in section 3. According to equation (22), the international equalization of the equity rate equalization implies that,

$$R_{IFA}^{S,e} - \mathcal{Z}_t^S = R_{IFA}^{N,e} - \mathcal{Z}_t^N, \tag{60}$$

$$\Delta\psi_t^S = \frac{\mathbb{B}^S}{\mathbb{B}^N}\Delta\psi_t^N + \frac{\mathbb{R}\mathbb{B}^S\eta}{1-\eta}\left(\frac{1}{R_{IFA}^{N,e}} - \frac{1}{R_{IFA}^{S,e}}\right). \tag{61}$$

Using equations (22), (26), and (59), we rewrite the condition, $\Omega_t^S + \Omega_t^N = 0$, into

$$\frac{R_{IFA}^{i,e}}{R_t^{i,e}} = \left(1 + \frac{R_{IFA}^{i,e}}{\mathbb{R}}\frac{1-\eta}{\eta}\frac{\Delta\psi_t^i}{\mathbb{B}^i}\right)\frac{\omega_t^w}{\omega_{t+1}^w}, \Rightarrow \omega_{t+1}^S\Delta\psi_t^S\frac{\mathbb{m}+\mathbb{B}^S}{\mathbb{B}^S} + \omega_{t+1}^N\Delta\psi_t^N\frac{\mathbb{m}+\mathbb{B}^N}{\mathbb{B}^N} = 0.$$

Given the international equalization of the loan rate, $R_t^{i,h} = R_t^{*,h}$, substitute away ω_{t+1}^i using equation (58) and the definition of the relative loan rate,

$$\mathcal{K}_t^S + \mathcal{K}_t^N = 0, \text{ where } \mathcal{K}_t^i \equiv (\Delta\psi_t^i + \psi_{IFA}^i)^{\rho}\Delta\psi_t^i\frac{\mathbb{m}+\mathbb{B}^i}{\mathbb{B}^i}, \tag{62}$$

$$\frac{\partial\mathcal{K}_t^i}{\partial\Delta\psi_t^i} = [(\rho+1)\Delta\psi_t^i + \psi_t^i](\Delta\psi_t^i + \psi_{IFA}^i)^{\rho-1}\frac{\mathbb{m}+\mathbb{B}^i}{\mathbb{B}^i} > 0. \tag{63}$$

According to equations (62)-(63), $\Delta\psi_t^S$ is an implicit function of $\Delta\psi_t^N$, which is downward sloping and cross the origin point; according to equation (61), $\Delta\psi_t^S$ is an implicit function of $\Delta\psi_t^N$, which is upward sloping and has an intercept on the vertical axis. Thus, there must exists a unique and, hence, time-invariant, solution with $\Delta\psi_t^S > 0 > \Delta\psi_t^N$.

Finally, we prove the existence of a unique and stable steady state under full capital mobility. ψ_t^i is time-invariant and so is $\mathcal{Z}_t^i$. Let $R_{FCM}^{i,h} \equiv R_{IFA}^{i,h} + \frac{\eta}{1-\eta}\mathcal{Z}_{FCM}^i$. It is same across countries, $R_{FCM}^{i,h} = R_{FCM}^{*,h}$. Thus, according to equation (23), the loan rate depends on the dynamics of the world-average wages. So is the wage in country i,

$$\omega_{t+1}^i = (R_{t+1}^i)^{-\rho} = (\frac{R_t^{i,h}}{\psi_t^i})^{-\rho} = (\frac{\omega_{t+1}^w}{\omega_t^w} R_{IFA}^{*,h})^{-\rho}(\psi_t^i)^\rho.$$

Given the time-invariant relative loan rate, the dynamics of world-average wages are

$$\omega_{t+1}^w = \frac{\omega_{t+1}^S + \omega_{t+1}^N}{2} = (\frac{\omega_{t+1}^w}{\omega_t^w} R_{IFA}^{*,h})^{-\rho}\frac{(\psi_{FCM}^S)^\rho + (\psi_{FCM}^N)^\rho}{2},$$

$$\omega_{t+1}^w = \left(\frac{\omega_t^w}{R_{FCM}^{*,h}}\right)^\alpha \left[\frac{(\psi_{FCM}^S)^\rho + (\psi_{FCM}^N)^\rho}{2}\right]^{1-\alpha}$$

Given $\alpha \in (0,1)$, the phase diagram of the world-average wage is concave. Thus, there exists a unique and stable steady state. Proportional to wage, aggregate output in country i is determined by the world output dynamics. □

Proof of Proposition 2

Proof. According to equation (30), the world credit market clearing condition, $\Phi_{FCM}^S + \Phi_{FCM}^N = 0$ implies that $\left(1 - \frac{R_{IFA}^{S,h}}{R_{FCM}^{*,h}}\right)\left(1 - \frac{R_{IFA}^{N,h}}{R_{FCM}^{*,h}}\right) < 0$. Given $R_{IFA}^{S,h} < R_{IFA}^{N,h}$, the world loan rate must be $R_{FCM}^{*,h} \in (R_{IFA}^{S,h}, R_{IFA}^{N,h})$. By analogy, we can prove $R_{FCM}^{*,e} \in (R_{IFA}^{N,e}, R_{IFA}^{S,e})$.

According to equation (29), $R_{FCM}^{*,h} \in (R_{IFA}^{S,h}, R_{IFA}^{N,h})$ implies $\mathcal{Z}_{FCM}^S > 0 > \mathcal{Z}_{FCM}^N$, which then implies that $\psi_{FCM}^S > \psi_{IFA}^S$ and $\psi_{FCM}^N < \psi_{IFA}^N$. Use the same approach as in the proof of Lemma 5, we can prove the partial convergence of the relative loan rate, $\psi_{IFA}^S < \psi_{FCM}^S < \psi_{FCM}^N < \psi_{IFA}^N$.

According to equations (30) and (31), the changes in the interest rates imply that $\Phi_{FCM}^S > 0 > \Phi_{FCM}^N$ and $\Omega_{FCM}^S < 0 < \Omega_{FCM}^N$. Since $R_{FCM}^{*,e} > R_{FCM}^{*,h}$, the steady-state net capital flows have the same sign as $\mathcal{Z}_{FCM}^i$, according to equation (32). Thus, $\mathcal{Z}_{FCM}^S > 0 > \mathcal{Z}_{FCM}^N$ implies that $\Phi_{FCM}^S + \Omega_{FCM}^S > 0 > \Phi_{FCM}^N + \Omega_{FCM}^N$.

According to equations (30) and (31), we get,

$$R_{FCM}^{*,h}\Phi_{FCM}^i + R_{FCM}^{*,e}\Omega_{FCM}^i = \beta\eta\omega_{FCM}^i(\mathcal{Z}_{FCM}^i - \mathcal{Z}_{FCM}^i) = 0.$$

□

Proof of Corolarry 1

Proof. Let $a_t \equiv \frac{\omega_t^N + \omega_t^S}{2\omega_{IFA}}$ and $b_t \equiv \frac{\omega_t^N - \omega_t^S}{2\omega_{IFA}} + \frac{\Phi_t^S + \Omega_t^S}{\beta\omega_{IFA}}$, where $t = 0, 1, 2, 3, ...$ According to the aggregate resource constraint in country S, net capital outflows cannot exceed aggregate saving, $0 < \Phi_t^S + \Omega_t^S < \beta\omega_t^S$, we get $b_t \in (0, a_t)$. In period $t \geq 0$, the aggregate investment in the two countries are $I_t^S = \beta\omega_t^S - (\Phi_t^S + \Omega_t^S) = (a_t - b_t)\beta\omega_{IFA}^S$ and $I_t^N = \beta\omega_t^N + (\Phi_t^S + \Omega_t^S) = (a_t + b_t)\beta\omega_{IFA}^N$, respectively. Given $\alpha \in (0,1)$, $b_t \in (0, a_t)$, and $\omega_{IFA} = \left(\frac{\beta}{\rho}\right)^{\rho}$, the world-average wage is reformulated into a condensed form,

$$\frac{\omega_{t+1}^S + \omega_{t+1}^N}{2} = \left(\frac{1}{\rho}\right)^{\alpha}\left[\frac{(I_t^S)^{\alpha} + (I_t^N)^{\alpha}}{2}\right] \Leftrightarrow a_{t+1} = \frac{(a_t - b_t)^{\alpha} + (a_t + b_t)^{\alpha}}{2} < (a_t)^{\alpha}, \tag{64}$$

where the last inequality sign results from the Jensen's Inequality. The wage in period $t = 0$ is same in the two countries, $\omega_0^S = \omega_0^N = \omega_{IFA}$, and, thus, $a_0 = 1$. From period 0 on, full capital mobility is allowed. According to the inequality in equation (64), $a_1 < 1$. For $t = 1, 2, 3, \ldots$, given $b_t \in (0, a_t)$, we have $a_{t+1} < (a_t)^{\alpha}$ and, thus, the time series of a_t is below 1, or equivalently, $\frac{\omega_t^S + \omega_t^N}{2} < \omega_{IFA}$. Thus, the world output is smaller than before period $t = 0$, $Y_t^S + Y_t^N = \frac{\omega_t^S + \omega_t^N}{1-\alpha} < \frac{2\omega_{IFA}}{1-\alpha} = Y_{IFA}^S + Y_{IFA}^N$. □

Proof of Lemma 3

Proof. The factor price equation (58), the reward splitting rule, (10), and the investment sharing rule (55) hold for the steady-state interest rates and wages under full capital mobility and under IFA,

$$\omega_j^i = \left(\frac{R_j^{i,h}}{\psi_j^i}\right)^{-\rho}, \quad \psi_j^i = \frac{R_j^{i,h}}{R_j^{i,e}}(1-\theta^i) + \theta^i, \quad \eta R_j^{i,e} + (1-\eta)R_j^{i,h} = \mathbb{R}. \tag{65}$$

where $j \in \{IFA, FCM\}$ refers to the scenarios of IFA and full capital mobility, respectively. Under full capital mobility, the international loan rate equalization and the partial convergence of the relative loan rate, $\psi_{FCM}^S < \psi_{FCM}^N$, implies that $\omega_{FCM}^S < \omega_{FCM}^N$, or equivalently, $Y_{FCM}^S < Y_{FCM}^N$.

Define $r_j^{i,h} \equiv \frac{R_j^{i,h}}{\mathbb{R}}$ and $r_j^{i,e} \equiv \frac{R_j^{i,e}}{\mathbb{R}}$. According to equations (65), the steady-state wage under IFA and under full capital mobility is a function of $r_j^{i,e}$,

$$\omega_j^i = \frac{1}{\mathbb{R}^{\rho}}\left[\frac{(1-\theta^i)r_j^{i,h}}{r_j^{i,e}} + \theta^i\right]^{\rho}(r_j^{i,h})^{-\rho} \quad \text{and} \quad r_j^{i,h} = \frac{1-\eta r^{i,e}}{1-\eta}. \tag{66}$$

Given θ^i, if full capital mobility affects the equity rate in country i, the wage and hence output in this country change accordingly. Define $\mathcal{T}_j^i \equiv \frac{\partial\omega_j^i}{\partial r_j^{i,e}}$ as the first derivative of ω_j^i with respect to $r_j^{i,e}$,

$$\mathcal{T}_j^i \equiv \frac{\partial\omega_j^i}{\partial r_j^{i,e}} = \frac{\rho\omega_j^i\mathcal{N}_j^i}{[(1-\theta^i)r_j^{i,h} + \theta^i r_j^{i,e}]r_j^{i,e}r_j^{i,h}}, \tag{67}$$

$$\text{where, } \mathcal{N}_j^i \equiv \theta^i\left[\frac{(1-r_j^{i,h})^2}{\eta} + \frac{1}{1-\eta}\right] - (r_j^{i,h})^2. \tag{68}$$

Thus, $\mathcal{T}_j^i$ has the same sign as $\mathcal{N}_j^i$.

According to equations (12)-(13), $r^{i,e}_{IFA} = \frac{\mathbb{m}+\mathbb{B}^i}{\mathbb{m}+1}$ and $r^{i,h}_{IFA} = \frac{\mathbb{m}+\mathbb{A}^i}{\mathbb{m}+1}$. Evaluate $\mathcal{T}^i_j$ in the steady state under IFA by substituting $r^{i,e}_{IFA}$ and $r^{i,h}_{IFA}$ into equation (67)-(68),

$$\mathcal{T}^i_{IFA} = \rho\omega^i_{IFA}(1+\mathbb{m})\frac{\frac{(\bar{\theta}-\theta^i)}{1-\eta}\left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - \mathbb{m}^2\right]}{(\mathbb{m}+A^i)(\mathbb{m}+B^i)\left[\mathbb{m}+B\left(1-\frac{(\bar{\theta}-\theta^i)}{1-\eta}\right)\right]}, \tag{69}$$

$$\mathcal{N}^i_{IFA} = \left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - \mathbb{m}^2\right]\frac{(\bar{\theta}-\theta^i)}{(1-\eta)}\frac{1}{(1+\mathbb{m})^2}. \tag{70}$$

We take the following approach to provide the sufficient conditions on the output implications of full capital mobility. Consider country N. If θ^N can make $\mathcal{N}^N_{IFA} \geq 0$, full capital mobility reduces the steady-state loan rate so that $\mathcal{N}^N_{FCM} > \mathcal{N}^N_{IFA} \geq 0$. Thus, $\mathcal{T}^N_{FCM} > 0$ and $\mathcal{T}^N_{IFA} \geq 0$. As full capital mobility raises the steady-state equity rate for country N, we get $\omega^N_{FCM} > \omega^N_{IFA}$ or $Y^N_{FCM} > Y^N_{IFA}$. Consider country S. If θ^S can make $\mathcal{N}^S_{IFA} \leq 0$, full capital mobility raises the steady-state loan rate so that $\mathcal{N}^S_{FCM} < \mathcal{N}^S_{IFA} \leq 0$. Thus, $\mathcal{T}^S_{FCM} < 0$ and $\mathcal{T}^S_{IFA} \leq 0$. As full capital mobility reduces the steady-state equity rate for country S, we get $\omega^S_{FCM} > \omega^S_{IFA}$ or $Y^S_{FCM} > Y^S_{IFA}$.

It is trivial to prove the general results for $\theta^S = 0$ and $\theta^N = \bar{\theta}$. If $\theta^N = \bar{\theta}$, $\mathcal{N}^N_{IFA} = 0$ so that full capital mobility raises its steady-state output, $Y^N_{FCM} > Y^N_{IFA}$. If $\theta^S = 0$, $\mathcal{N}^S_{IFA} \leq 0$ so that full capital mobility raises its steady-state output, $Y^S_{FCM} > Y^S_{IFA}$.

For $\theta^i \in [0, \bar{\theta})$, the sign of $\mathcal{T}^i_{IFA}$ depends on that of $\mathcal{N}^i_{IFA}$, or, that of $\left[\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} - \mathbb{m}^2\right]$.

Figure 14 shows all possible cases on the relative size of $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)}$ and $\mathbb{m}^2$ where the three panels in the first row show the cases with $\eta \in (0, 0.5)$, the two panels in the second row show the cases with $\eta \in (0, 5, 1)$, and the horizontal axis shows $\theta^i \in (0, \bar{\theta})$.

Given $\eta \in (0, 0.5)$, $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} \in (0, \frac{1}{4\eta(1-\eta)})$ is a hump-shaped function of $\theta^i \in (0, \bar{\theta})$. Point H denotes its highest value $\frac{1}{4\eta(1-\eta)} > 1$. Define $\kappa \equiv \frac{1-\sqrt{1-4\mathbb{m}^2(1-\eta)\eta}}{2}$.

- If $\mathbb{m} \in (0, 1)$, there exists a threshold value $\tilde{\theta}_1 = \kappa$ such that, for $\theta^i \in (0, \tilde{\theta}_1)$, $\mathcal{N}^i_{IFA} < 0$ and, for $\theta^i \in (\tilde{\theta}_1, \bar{\theta})$, the opposite applies.
- If $\mathbb{m} \in (1, \frac{1}{2\sqrt{\eta(1-\eta)}})$, there exists two threshold values $\tilde{\theta}_1 = \kappa$ and $\tilde{\theta}_2 = 1-\kappa$ such that for $\theta^i \in (\tilde{\theta}_1, \tilde{\theta}_2)$, $\mathcal{N}^i_{IFA} > 0$ and, for $\theta^i \in (0, \tilde{\theta}_1) \cup (\tilde{\theta}_2, \bar{\theta})$, the opposite applies.
- If $\mathbb{m} > \frac{1}{2\sqrt{\eta(1-\eta)}}$, for $\theta^i \in (0, \bar{\theta})$, it holds that $\mathcal{N}^i_{IFA} < 0$.

Given $\eta \in (0.5, 1)$, $\frac{\theta^i(1-\theta^i)}{\eta(1-\eta)} \in (0, 1)$ is a monotonically increasing function of $\theta^i \in (0, \bar{\theta})$.

- If $\mathbb{m} \in (0, 1)$, there exists a threshold value $\tilde{\theta}_1 = \kappa$ such that, for $\theta^i \in (0, \tilde{\theta}_1)$, $\mathcal{N}^i_{IFA} > 0$ and, for $\theta^i \in (\tilde{\theta}_1, \bar{\theta})$, the opposite applies.
- If $\mathbb{m} > 1$, for $\theta^i \in (0, \bar{\theta})$, $\mathcal{N}^i_{IFA} < 0$.

Using the approach mentioned above, we can provide the sufficient conditions as summarized in Lemma 3. □

Proof of Lemma ??

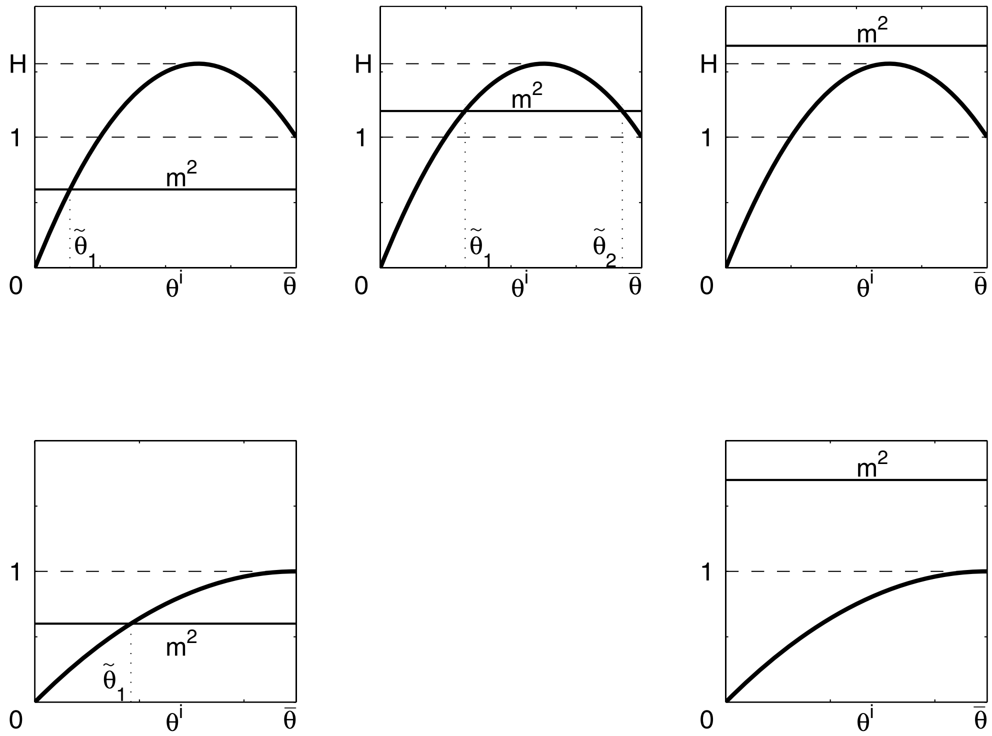

Figure 14: Threshold Values under Various Scenarios

Proof. Use equation (28) to substitute away the wage in the steady state,

$$u^{i,h}_{FCM} = \omega^i_{FCM}(R^{i,h}_{FCM} + \epsilon) = \left[(1-\theta^i)\frac{R^{*,h}_{FCM}}{R^{*,e}_{FCM}} + \theta^i\right]^\rho [(R^{*,h}_{FCM})^{1-\rho} + \epsilon(R^{*,h}_{FCM})^{-\rho}]. \tag{71}$$

Consider country S first. Compared with the scenario under IFA, $\frac{R^{*,h}_{FCM}}{R^{*,e}_{FCM}} > \frac{R^{*,h}_{IFA}}{R^{*,e}_{IFA}}$. Thus, a sufficient condition for $u^{S,h}_{FCM} > u^{S,h}_{IFA}$ is $[(R^{*,h}_{FCM})^{1-\rho} + \epsilon(R^{*,h}_{FCM})^{-\rho}] > [(R^{S,h}_{IFA})^{1-\rho} + \epsilon(R^{S,h}_{IFA})^{-\rho}]$, or equivalently to prove the function $Y = x^{1-\rho} + \epsilon x^{-\rho}$ is an increasing function of x for $x \in (R^{S,h}_{IFA}, R^{S,h}_{FCM})$. A sufficient condition for the latter is $(1-\rho)x^{-\rho} - \epsilon\rho x^{-\rho-1} > 0$ or $\frac{x(1-\rho)}{\rho} > \epsilon$. If $\frac{R^{S,h}_{IFA}(1-\rho)}{\rho} > \epsilon$ holds, Y is an increasing function of x for $x \in (R^{S,h}_{IFA}, R^{S,h}_{FCM})$. Use equation (13) to plug in the analytical solution of $R^{S,h}_{IFA} = \frac{\theta^i}{1-\eta}(1+\epsilon)\rho$, we get $\frac{\epsilon}{1+\epsilon} \leq \frac{\theta^i}{1-\eta}(1-\rho)$. □

Proof of Lemma 4

Proof. Take the world loan rate $R^{*,h}_t$ as given. For $\omega^i_t \in (0, 1-\theta^i]$ and $i^i_t = 1$, take the first and second derivatives of equation (37) with respect to ω^i_t,

$$\frac{d\omega^i_{t+1}}{d\omega^i_t} = \frac{\rho R^{*,h}}{\theta^i}(\omega^i_{t+1})^{\frac{1}{\alpha}} > 0, \quad \text{and,} \quad \frac{d\omega^i_{t+1}}{d^2\omega^i_t} = \frac{\rho R^{*,h}}{\theta^i}\frac{1}{\alpha}(\omega^i_{t+1})^{\frac{1}{\rho}}\frac{d\omega^i_{t+1}}{d\omega^i_t} > 0. \tag{72}$$

The phase diagram of wages is convex for $\omega^i_t \in (0, 1-\theta^i]$. By setting $\omega^i_t = 0$ in equation (37), we get the vertical intercept of the phase diagram of wages at $\omega^i_{t+1} = \left[\frac{\theta^i}{R^{*,h}_t}\right]^\rho$. For $\omega^i_t > 1-\theta^i$, the marginal return on investment is equal to the world loan rate, $R^i_{t+1} = R^{*,h}_t$, and, thus, entrepreneurs do not borrow to the limit. The phase diagram of wages $\omega_{t+1}^i = (R_{t+1}^i)^{-\frac{1}{\rho}} = \left(\frac{1}{R_t^{*,h}}\right)^{\rho}$ is flat and independent of ω_t^i. □

Proof of Lemma 5

Proof. The proof consists of three parts. First, we show that the model solution is characterized by equations (38)-(41). Under free mobility of financial capital, entrepreneurs invest their entire savings as equity in their projects. Use equation (5) and the investment-equity ratio λ_t^i to rewrite the aggregate domestic investment as

$$\eta s_t^{i,e}\lambda_t^i = \frac{\eta[\beta\omega_t^i - (1-\beta)\epsilon\frac{\omega_{t+1}^i}{R_t^{i,e}}]}{1-\frac{\theta^i R_{t+1}^i}{R_t^{i,h}}} = K_{t+1}^i = (1+\epsilon)\rho\frac{\omega_{t+1}^i}{R_{t+1}^i}.$$

Multiplying both sides with $\frac{1-\frac{\theta^i R_{t+1}^i}{R_t^{i,h}}}{\eta\beta\omega_t^i}$ and using the investment sharing rule (55), we get the solution to the equity rate (38),

$$1 - \frac{(1-\beta)\epsilon}{\beta}\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,e}} = \frac{(1+\epsilon)\rho}{\beta}\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1-\theta^i}{\eta R_t^{i,e}}, \Rightarrow R_t^{i,e} = \frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}\frac{\mathbb{m}+\mathbb{B}}{\mathbb{m}+1} = \frac{\omega_{t+1}^i}{\omega_t^i}R_{IFA}^{i,e}.$$

Combining equations (38), (55), (12), (13) and using the definition of the relative loan rate, we get the solution to the loan rate (39),

$$\begin{aligned} R_t^{i,h} = R_t^{i,e}\frac{(\psi_t^i - \theta^i)}{1-\theta^i} &= \frac{\omega_{t+1}^i}{\omega_t^i}\mathbb{R}\frac{\mathbb{m}+\mathbb{A}}{\mathbb{m}+1}\frac{\mathbb{m}+\mathbb{B}}{\mathbb{m}+\mathbb{A}^i}\frac{(\psi_t^i-\theta^i)}{1-\theta^i} \\ &= \frac{\omega_{t+1}^i}{\omega_t^i}R_{IFA}^{i,h} + \frac{\omega_{t+1}^i}{\omega_t^i}R_{IFA}^{i,h}\left[\frac{\psi_t^i-\theta^i}{\psi_{IFA}^i-\theta^i} - 1\right] \\ &= \frac{\omega_{t+1}^i}{\omega_t^i}R_{IFA}^{i,h} + \frac{\omega_{t+1}^i}{\omega_t^i}R_{IFA}^{i,h}\left[\frac{\psi_t^i-\psi_{IFA}^i}{\psi_{IFA}^i-\theta^i}\right]. \end{aligned}$$

In equilibrium, financial capital outflows are the excess domestic saving, $\Phi_t^i = (1-\eta)s_t^{i,h} - (\lambda_t^i - 1)\eta s_t^{i,e}$. Using equation (5), the investment-equity ratio, $\lambda_t^i = \frac{1}{1-\theta^i\frac{R_{t+1}^i}{R_t^{i,h}}}$, and the definition of the relative loan rate, we get the solution to financial capital outflows, (40).

$$
\begin{aligned}
\Phi_t^i &= (1-\eta)\beta\omega_t^i\left[1-\frac{(1-\beta)\epsilon}{\beta}\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,h}}\right] - \eta\beta\omega_t^i\left[1-\frac{(1-\beta)\epsilon}{\beta}\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,e}}\right]\frac{\theta^i}{\psi_t^i-\theta^i} \\
&= (1-\eta)\beta\omega_t^i\left\{1-\frac{(1-\beta)\epsilon}{\beta}\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,h}} - \frac{\eta}{1-\eta}\left[1-\frac{(1-\beta)\epsilon}{\beta}\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,e}}\right]\frac{\theta^i}{\psi_t^i-\theta^i}\right\} \\
&= (1-\eta)\beta\omega_t^i\left\{1-\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,h}}\left[\frac{(1-\beta)\epsilon}{\beta}+\frac{\eta}{1-\eta}\frac{\theta^i}{\psi_t^i-\theta^i}\frac{R_t^{i,h}\omega_t^i}{\omega_{t+1}^i}-\frac{(1-\beta)\epsilon}{\beta}\frac{\eta}{1-\eta}\frac{\theta^i}{(1-\theta^i)}\right]\right\} \\
&= (1-\eta)\beta\omega_t^i\left\{1-\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,h}}\left[\frac{(1-\beta)\epsilon}{\beta}\left(1-\frac{\mathbb{A}}{\mathbb{B}}\right)+\frac{\eta}{1-\eta}\frac{\theta^i}{1-\theta^i}\frac{R_t^{i,e}\omega_t^i}{\omega_{t+1}^i}\right]\right\} \\
&= (1-\eta)\beta\omega_t^i\left\{1-\frac{\omega_{t+1}^i}{\omega_t^i}\frac{1}{R_t^{i,h}}\left[\frac{\mathbb{R}\mathbb{m}(\mathbb{B}-\mathbb{A})}{(\mathbb{m}+1)\mathbb{B}}+\frac{\mathbb{A}}{\mathbb{B}}R_{IFA}^{i,e}\right]\right\} \\
&= (1-\eta)\beta\omega_t^i\left(1-\frac{\omega_{t+1}^i}{\omega_t^i}\frac{R_{IFA}^{i,h}}{R_t^{i,h}}\right).
\end{aligned}
$$

Using the definition of the relative loan rate, the investment sharing rule, and equations (12), (13), (38), we get the solution to the social rate of return,

$$
R_{t+1}^i = \frac{R_t^{i,h}}{\psi_t^i} = R_t^{i,e}\frac{1-\frac{\theta^i}{\psi_t^i}}{1-\theta^i} = \frac{\omega_{t+1}^i}{\omega_t^i}R_{IFA}^i\frac{1-\frac{\theta^i}{\psi_t^i}}{1-\frac{\theta^i}{\psi_{IFA}^i}}.
$$

Substitute away R_{t+1}^i in equation (58), we get the dynamic equation of wages (41),

$$
\omega_{t+1}^i = \left(\frac{\omega_{t+1}^i}{R_{t+1}^i}\right)^\alpha = \left(\frac{\omega_t^i}{R_{IFA}^i}\frac{1-\frac{\theta^i}{\psi_{IFA}^i}}{1-\frac{\theta^i}{\psi_t^i}}\right)^\alpha = \left(\omega_t^i\frac{\Lambda_t^i}{\mathbb{R}}\right)^\alpha, \quad \text{with } \Lambda_t^i = \Lambda_{IFA}^i\frac{1-\frac{\theta^i}{\psi_{IFA}^i}}{1-\frac{\theta^i}{\psi_t^i}}.
$$

Second, prove the uniqueness and stability of the model economy. Given $R_t^{*,h}$, we use equations (39), (41), (58) to rewrite the dynamic equation of wages as,

$$
\ln\left(\frac{\omega_t^i}{\omega_{t+1}^i}R_t^{*,h}\frac{\psi_{IFA}^i-\theta^i}{R_{IFA}^{i,h}}+\theta^i\right) = \ln\psi_t^i = \ln R_t^{*,h} - \ln R_{t+1}^i = \ln R_t^{*,h} + \frac{1}{\rho}\ln\omega_{t+1}^i \tag{73}
$$

$$
\Rightarrow \quad \ln\omega_{t+1}^i = -\rho\ln R_t^{*,h} + \rho\ln\left(\frac{\omega_t^i}{\omega_{t+1}^i}R_t^{*,h}\frac{\psi_{IFA}^i-\theta^i}{R_{IFA}^{i,h}}+\theta^i\right). \tag{74}
$$

Let $\mathcal{W}^i \equiv \frac{\partial\ln\omega_{t+1}^i}{\partial\ln\omega_t^i}$. The first and the second derivatives of ω_{t+1}^i with respect to ω_t^i are

$$
\begin{aligned}
&\frac{\partial\omega_{t+1}^i}{\partial\omega_t^i} = \frac{\omega_{t+1}^i}{\omega_t^i}\frac{\rho}{\rho+\frac{\psi_{t+1}^i}{\psi_{t+1}^i-\theta^i}}, \quad \Rightarrow \quad \mathcal{W}^i \equiv \frac{\partial\ln\omega_{t+1}^i}{\partial\ln\omega_t^i} = \frac{\rho}{\rho+\frac{\psi_{t+1}^i}{\psi_{t+1}^i-\theta^i}} \in (0,1), \\
&\frac{\partial^2\omega_{t+1}^i}{\partial(\omega_t^i)^2} = -(1-\mathcal{W}^i)(\mathcal{W}^i)^2\frac{\omega_{t+1}^i}{(\omega_t^i)^2}\frac{(1+\rho)}{\rho}
\end{aligned}
$$

Since $\mathcal{W}^i \in (0,1)$, we get $\frac{\partial^2\omega_{t+1}^i}{\partial(\omega_t^i)^2} < 0$. Thus, the phase diagram of wages is a concave function under free mobility of financial capital if the borrowing constraints are binding.

According to equation (74), for $\omega_t^i = 0$, the phase diagram has a positive intercept on the vertical axis at $\omega_{t+1}^i = (R_t^{*,h})^{-\rho}(\theta^i)^\rho$. Define a threshold value $\bar{\omega}_t^i = R_{IFA}^{i,e}(R_t^{*,h})^{-\frac{1}{1-\alpha}}$. For $\omega_t^i \in (0, \bar{\omega}_t^i)$, the phase diagram of wages is monotonically increasing and concave. For $\omega_t^i > \bar{\omega}_t^i$, aggregate saving and investment are so high that the relative loan rate is equal to one, or equivalently, $R_{t+1}^i = R_t^{*,h}$. Thus, the borrowing constraints are slack and the phase diagram is flat with $\omega_{t+1}^i = \bar{\omega}_{t+1}^i = (R_t^{*,h})^{-\rho}$. Given $R_t^{*,h} < \mathbb{R} < R_{IFA}^{i,e}$, we get $\bar{\omega}_{t+1}^i < \bar{\omega}_t^i$. In other words, the kink point is below the 45 degree line.

Thus, the phase diagram of wages crosses the 45 degree line once and only once from the left, and the intersection is in its concave part. Thus, the model economy has a unique and stable steady state under free mobility of financial capital.

Finally, we prove the (partial) convergence of the relative loan rate. As the loan rate is initially lower in country S, $R_{IFA}^{S,h} < R_{IFA}^{N,h}$, financial capital flows from country S to N, $\Phi_t^S > 0 > \Phi_t^N$, implying that $\psi_t^S > \psi_{IFA}^S$ and $\psi_t^N > \psi_{IFA}^N$, according to equation (40). In the steady state, given the world loan rate $R_{FCF}^{*,h}$, the relative loan rate under financial integration is,

$$R_{FCF}^{*,h} = R_{IFA}^{i,h}\frac{\psi_{FCF}^i - \theta^i}{\psi_{IFA}^i - \theta^i} = R_{IFA}^{i,e}\frac{\psi_{FCF}^i - \theta^i}{1-\theta^i} = \mathbb{R}\frac{\mathbb{m}+\mathbb{B}^i}{\mathbb{m}+1}\frac{\psi_{FCF}^i - \theta^i}{\eta\mathbb{B}^i}, \tag{75}$$

$$\Rightarrow \quad \psi_{FCF}^i = \frac{R_{FCF}^{*,h}}{\mathbb{R}}\eta\mathbb{B}^i\frac{\mathbb{m}+1}{\mathbb{m}+\mathbb{B}^i} + \theta^i \tag{76}$$

In order to prove $\psi_{FCF}^S < \psi_{FCF}^N$, we just need to prove that

$$\frac{R_{FCF}^{*,h}}{\mathbb{R}}\eta(\mathbb{m}+1)\left(\frac{\mathbb{B}^S}{\mathbb{m}+\mathbb{B}^S} - \frac{\mathbb{B}^N}{\mathbb{m}+\mathbb{B}^N}\right) < \theta^N - \theta^S \tag{77}$$

$$\Leftarrow \frac{R_{FCF}^{*,h}}{\mathbb{R}}\eta(\mathbb{m}+1)\frac{\mathbb{m}(\mathbb{B}^S - \mathbb{B}^N)}{(\mathbb{m}+\mathbb{B}^S)(\mathbb{m}+\mathbb{B}^N)} < \theta^N - \theta^S, \tag{78}$$

$$\Leftarrow \frac{R_{FCF}^{*,h}}{\mathbb{R}}\frac{(\mathbb{m}+1)\mathbb{m}}{(\mathbb{m}+\mathbb{B}^S)(\mathbb{m}+\mathbb{B}^N)} < 1, \tag{79}$$

$$\Leftarrow R_{FCF}^{*,h}\mathbb{m} < R_{IFA}^{N,h}(\mathbb{m}+\mathbb{B}^S). \tag{80}$$

Since $0 < R_{FCF}^{*,h} < R_{IFA}^{N,h}$ and $0 \leq \mathbb{m} < \mathbb{m}+\mathbb{B}^S$, the inequality (80) must hold. Thus, we prove the partial convergence of the relative loan rate, $\psi_{IFA}^S < \psi_{FCF}^S < \psi_{FCF}^N < \psi_{IFA}^N$.

□

D Data Description

Data source for China's patterns of capital flows: the annual data of financial account in the Balance of Payments from IMF International Financial Statistics.

The flows of direct investment is computed as the sum of the entries under direct investment abroad and direct investment in reporting economy.[14] The flows of indirect investment is computed as the sum of the entries under portfolio investment assets and liabilities, under net

[14]In practice, direct investment abroad is recorded as a negative value while direct investment in reporting economy is recorded as a positive value. Thus, if the sum of these two entries is positive, there is a net FDI inflow into the reporting economy.

financial derivatives, and under other investment assets and liabilities. China's foreign reserves are directly from the data series. China's net capital flows are the sum of these three components. We divide the four time series by China's nominal GDP in terms of USD. In practice, a positive value of capital flows in the balance of payments represents capital inflows while a positive value of capital flows in our model is defined as capital outflows. In order to be consistent with our model definition, the signs of the four time series computed above are reversed. In the bottom-left panel of figure 10, the solid line (FII) shows the flows of foreign indirect investment and the dashed line (FR) shows the changes in foreign reserves. In the upper-left panel, the dash-dotted line (FDI) shows FDI flows, the dashed line (FCF) shows financial capital flows as the sum of FII and FR, the solid line (NCF) shows net capital flows.

Data source for China's international investment positions: the annual data in 1982-2003 are from the data base of Lane and Milesi-Ferretti (2007c) and the annual data in 2004-2011 are from China State Administration of Foreign Exchange.

The net position of direct investment abroad is computed as the difference between direct investment abroad and direct investment in reporting economy. China's net position of indirect investment abroad is computed as the sum of the net portfolio investment abroad, net financial derivatives, and net other investment assets. China's reserve assets are directly from the data series. China's net international investment position is the sum of these three components. We divide the four series by China's nominal GDP in terms of USD. In the bottom-right panel of figure 10, the solid line (FII) shows the position of foreign indirect investment and the dashed line (RA) shows the position of reserve assets. In the upper-left panel, the dash-dotted line (FDI) shows the position of FDI, the dashed line (FC) shows the position of foreign financial capital as the sum of FII and RA, the solid line (NFA) shows net foreign asset position.