Time-Varying Wage Risk, Incomplete Markets, and Business Cycles*

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Abstract

This paper analyzes how cyclical variation in idiosyncratic earnings risk affects labor market dynamics. I construct a heterogeneous-agent incomplete asset markets model with time-varying idiosyncratic wage risk and indivisible labor. I calibrate the model's risk variation to micro-level wage data. When moved by shocks to idiosyncratic wage risk and aggregate total factor productivity, the model replicates two key features of the actual labor market dynamics: large fluctuations in the labor wedge and a weakly negative correlation between total hours worked and average labor productivity. In contrast, under constant risk, the labor wedge varies little, and hours and productivity comove strongly.

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1 Introduction

Idiosyncratic earnings risk exhibits cyclical fluctuations (Storesletten, Telmer, and Yaron (2004) and Heathcote, Perri, and Violante (2010)). Moreover, there is empirical evidence, as well as theoretical support, for the finding that an increase in wage uncertainty increases labor supply (Parker, Belghitar, and Barmby (2005) and Flodén (2006)). However, the quantitative implication for labor market fluctuations has not been studied. Are changes in wage uncertainty relevant for aggregate fluctuations or the business cycle? The present paper examines this question using a heterogeneous-agent dynamic stochastic general equilibrium model.

The model analyzed herein is built upon incomplete asset markets models used in recent labor market analyses (e.g., Chang and Kim (2006, 2007), Alonso-Ortiz and Rogerson (2010), and Krusell, Mukoyama, Rogerson, and Şahin (2010, 2011)). Individuals face idiosyncratic wage risk because person-specific labor productivity changes stochastically. Individuals cannot fully insure against this risk because there is only one asset, physical capital, in the economy. They partially self-insure by holding capital and make discrete labor supply choices each period. I introduce risk variation into this environment using uncertainty shocks in the spirit of Bloom (2009), i.e., time-varying volatility of idiosyncratic productivity shocks. Further, I calibrate these uncertainty shocks and the stochastic process for idiosyncratic productivity to individual wage data in the Panel Study of Income Dynamics (PSID). With persistent productivity and indivisible labor, the calibrated model generates inequality in wealth and labor earnings and the positive correlation of the two that are similar to those in the U.S. economy.

I find that uncertainty shocks are the key to accounting for salient features of the U.S. labor market dynamics. Specifically, two key statistics move closer to the U.S. data when introducing uncertainty shocks into the present model in addition to shocks to aggregate total factor productivity (TFP). The first is the volatility of the labor wedge. The labor wedge is computed by the ratio of average labor productivity (output per labor hour) to the marginal rate of substitution of leisure for consumption, assuming a representative individual. As shown by Chari, Kehoe, and McGrattan (2007) and Shimer (2010), the labor wedge is volatile in the U.S. With both uncertainty and aggregate TFP shocks, the present model generates the volatility of the labor wedge that is 95% of that in the U.S. economy. In contrast, the number is only 17% without uncertainty shocks. Importantly, in the model, the variation in the labor wedge arises solely from the variation in the gap between the real wage and the marginal rate of substitution, while average labor productivity is always equal to the real wage. This result is in line with Karabarbounis (2014)'s finding that the deviation of the household optimality condition is predominantly responsible for the variation in the labor wedge in the U.S.

The second improvement is seen in the correlation between total hours worked and average labor productivity. With shocks to both wage uncertainty and aggregate TFP, the model replicates the weakly negative correlation between total hours worked and average labor productivity found in the U.S. data (–0.40 in the model compared with –0.32 in the data).[1] In contrast, in the absence of uncertainty shocks, the model produces a counterfactually strong, positive correlation of 0.83. Hence, introducing time-varying idiosyncratic wage risk resolves the so-called hours-productivity puzzle typically present in equilibrium business cycle models driven by aggregate TFP shocks only (e.g., Kydland and Prescott (1982) and Hansen (1985)).

Fluctuations in idiosyncratic wage risk generate these improvements because even a temporary increase in risk produces persistent, negative comovement of total hours worked with average labor productivity. The main mechanism is the (ex-post) *distribution effect*. As the increased volatility

[1] The data on total hours worked is taken from Cociuba, Prescott, and Ueberfeldt (2009), and as Shimer (2010) argues, it is the most comprehensive data on hours worked. The reported correlation between total hours worked and average labor productivity is based on the data from 1947Q3 to 2009Q3. While there is a consensus that the correlation after 1984 is weakly negative, several papers, such as Gali and Gambetti (2009), find a slightly positive correlation before 1984. In contrast, the pre-1984 correlation is weakly negative (–0.30) in the data used here. Gali and Gambetti (2009) use data on the nonfarm business sector, while the data used here includes the farm, government, and military sectors. I use the most comprehensive data on hours because the data on consumption used to compute the labor wedge is consumption in the entire economy.

of shocks to idiosyncratic productivity realizes, the positive correlation between wealth and productivity across individuals is reduced. Accordingly, the wealth-productivity distribution shifts, generating a flow of low-productivity individuals towards larger wealth levels or nonemployment and a flow of high-productivity individuals towards smaller wealth levels or employment. Crucially, because of the shape of the distribution and the decrease in the equilibrium wage rate, even a small increase in risk leads to a large decrease in low-productivity employment and a much smaller increase in high-productivity employment. Thus, total hours worked decreases substantially. Average labor productivity increases significantly because the share of high-productivity employment increases. Since the wealth-productivity distribution gradually returns to its long-run distribution, hours and productivity slowly return to their pre-shock levels, exhibiting persistent, negative comovement. In contrast, output and consumption move only slightly because labor input, measured in efficiency units, changes little under the movements in low- and high-productivity employment described above. Hence, the distribution effect produces large fluctuations in the labor wedge.

The (ex-ante) *uncertainty effect* also generates labor market fluctuations. An increase in wage uncertainty increases incentives to self-insure, especially for individuals close to their borrowing limit. Since such individuals with high productivity were likely to have been working, only low-productivity groups actually increase their employment. Hence, total hours worked increases, but average labor productivity decreases because of the increase in low-productivity employment. However, quantitatively, the uncertainty effect plays a minor role in shaping the labor market dynamics in the present model. I solve the version of the model including only the uncertainty effect and excluding the distribution effect, i.e., the ex-post change in the dispersion in idiosyncratic productivity. I find that the variability of the labor wedge is 27% of that in the U.S. data and the correlation between hours and productivity is 0.58. These results indicate that the major impact of varying idiosyncratic wage risk arises from the distribution effect.

The concurrent improvements in the volatility of the labor wedge and the hours-productivity correlation are a distinguished feature of the varying risk model presented here. Previous studies, such as Benhabib, Rogerson, and Wright (1991) and Christiano and Eichenbaum (1992), also consider shocks that shift labor supply in an effort to resolve the hours-productivity puzzle. However, assuming a representative agent, those models imply a constant (zero) labor wedge. Further, without changes in the composition of workers with different productivities, a relatively strong positive correlation remains between total hours worked and average labor productivity in their models calibrated to the U.S. economy.[2] In contrast, incorporating realistic heterogeneity in productivity across individuals, the calibrated time-varying wage risk model simultaneously generates fluctuations in the labor wedge and the hours-productivity correlation that are close to the U.S. data.

In addition, the varying risk model herein is consistent with the following pattern of the labor market fluctuations in the U.S. In the U.S., total hours worked lagged average labor productivity following a recession in which idiosyncratic wage risk increased, whereas hours and labor productivity recovered together after a recession in which risk remained low. The varying risk model exhibits a similar pattern. An increase in idiosyncratic wage risk delays the recovery of hours relative to labor productivity following a decline in aggregate TFP. I take this finding as additional evidence that cyclical variation in idiosyncratic wage risk has a significant impact on labor market fluctuations.

The present paper contributes to the vast literature on the impact of varying idiosyncratic earnings risk by analyzing its impact on labor market dynamics. While existing studies analyze how time-varying income risk affect aggregate fluctuations (Krusell and Smith (1998)), the welfare cost of business cycles (Krusell and Smith (1999), Storesletten, Telmer, and Yaron (2001), Mukoyama and Şahin (2006), and Krusell, Mukoyama, Şahin, and Smith (2009)), and asset pricing (Krusell and Smith (1997), Pijoan-Mas (2007), and Storesletten,

[2] Specifically, Benhabib, Rogerson, and Wright (1991) include shocks to home-production technology. Their benchmark model generates a correlation of 0.49 between total hours worked and average labor productivity. Christiano and Eichenbaum (1992) introduce government spending shocks. When estimated using establishment hours data, their model with indivisible labor generates a correlation of 0.58.

Telmer, and Yaron (2007)), they do not analyze labor market fluctuations, assuming exogenous earnings or inelastic labor supply. One exception is Lopez (2010), which assumes *divisible* labor and a different borrowing constraint from that assumed herein. Crucially, his model generates counterfactually strong comovement of total hours worked with average labor productivity.[3]

The present paper is also related to recent studies on the relationship between changes in firm-specific risk and business cycles. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and Bachmann and Bayer (2013) investigate how uncertainty shocks interact with input adjustment costs. Arellano, Bai, and Kehoe (2012) consider financial frictions, while Schaal (2012) analyzes labor search frictions. In these two models, uncertainty shocks trigger heterogeneous changes in labor *demand* across firms, generating negative comovement of labor input with average labor productivity and a volatile labor wedge. However, in contrast to the aforementioned finding by Karabarbounis (2014), these models imply that the variation in the deviation of average labor productivity from the real wage contributes to the variation in the labor wedge.[4] In contrast, uncertainty shocks in the present model generate heterogeneous changes in labor *supply* among individuals with different wealth and productivity, leading to a result that is more consistent with Karabarbounis (2014)'s finding.

The remainder of the present paper proceeds as follows. Section 2 quantifies cyclical variation in idiosyncratic wage risk in the U.S. using the PSID wage data. Section 3 lays out the incomplete asset markets model with varying idiosyncratic wage risk, while Section 4 determines the parameter values. Section 5 analyzes the impact of varying idiosyncratic wage risk on the model's business cycle. Section 6 examines the implication of the cyclicality of wage

[3] The Lopez (2010) model produces a strong positive correlation between output and total hours worked (0.98) and a low volatility of hours relative to output (0.32). These values imply a correlation between total hours worked and average labor productivity of 0.96.

[4] Arellano, Bai, and Kehoe (2012) assume a representative household and the marginal rate of substitution of leisure for consumption is always equal to the real wage. Hence, the variation in the labor wedge solely arises from the variation in the deviation of average labor productivity from the real wage. In Schaal (2012)'s model, there are some fluctuations in the deviation of the marginal rate of substitution from the real wage.

risk. Section 7 concludes.

2 Cyclical Fluctuations in Idiosyncratic Wage Risk

This section analyzes the PSID data and provides some estimates for the cyclical variation in idiosyncratic wage risk in the U.S. economy.[5] Idiosyncratic wage risk is computed as the cross-sectional dispersion of residuals obtained by the wage regression and the cyclical variation in the identified risk is analyzed. This approach is similar to that taken by recent studies that estimate uncertainty shocks affecting firms (e.g., Bloom (2009) and Bachmann and Bayer (2013)).

Specifically, for each person-year observation of the PSID data, I compute the hourly wage dividing the annual labor income by the annual total labor hours. Next, for each year, I fit individual wages to the wage process assumed in the present paper, which is also widely used in the literature.[6] The process is derived as follows. First, an individual wage $w_{i,t}$ (i: individual and t: time) is equal to $w_t x_{i,t}$, where w_t is the equilibrium wage rate per efficiency unit of labor and $x_{i,t}$ is person-specific labor productivity:

$$\ln w_{i,t} = \ln w_t + \ln x_{i,t}. \tag{1}$$

Second, $x_{i,t}$ follows an AR(1) process:

$$\ln x_{i,t} = \rho_{x,t} \ln x_{i,t-1} + \varepsilon_{x,i,t},\ \varepsilon_{x,i,t} \sim N(0, \sigma^2_{\varepsilon_x,t}). \tag{2}$$

As shown by Chang and Kim (2006), (1) and (2) imply the following wage process:

$$\ln w_{i,t} = \rho_{x,t} \ln w_{i,t-1} + (\ln w_t - \rho_{x,t} \ln w_{t-1}) + \varepsilon_{x,i,t}. \tag{3}$$

[5] Appendix A1 explains the data.

[6] For example, see Chang and Kim (2006, 2007), Alonso-Ortiz and Rogerson (2010), and Krusell, Mukoyama, Rogerson, and Şahin (2010, 2011).

I conduct three types of regression to identify idiosyncratic wage risk. The first regression estimates (3) each year with ordinary least squares (OLS), replacing $(\ln w_t - \rho_{x,t} \ln w_{t-1})$ with a constant. The regression is done for the period between 1969 and 1991.

In practice, variables such as years of education influence individual wages (e.g., Card (1999), Heathcote, Perri, and Violante (2010)), and hence individuals could forecast their wage, at least partially. In order to isolate the pure risk that individuals face, the second regression controls for demographic variables and estimates the following equation:

$$\ln w_{i,t} = \rho_{x,t} \ln w_{i,t-1} + (\ln w_t - \rho_{x,t} \ln w_{t-1}) + Z_{i,t}\beta_t + \varepsilon_{x,i,t}, \tag{4}$$

where $Z_{i,t}$ includes education, experience (defined as age minus education minus six), experience-squared, and sex.[7] I estimate (4) each year using OLS, replacing $(\ln w_t - \rho_{x,t} \ln w_{t-1})$ with a constant. Since the data on education is discontinuous in 1974, the regression is done for the period between 1975 and 1991.

The third regression takes into account the selection effect. Specifically, following Chang and Kim (2006), I introduce the selection equation of

$$I_{i,t} = V_{i,t}\gamma_t + v_{i,t}, v_{i,t} \sim N(0, \sigma^2_{v,t}), \tag{5}$$

where $I_{i,t} = 1$ if the individual worked in both t and $t-1$ (i.e., both $w_{i,t}$ and $w_{i,t-1}$ are available). The variables $V_{i,t}$ include marital status, the number of children, education, experience, experience-squared, sex, and a constant. I conduct Heckman-type estimation using (4) and (5). The regression is done for each year between 1975 and 1991.

Figure 1 plots the estimated idiosyncratic wage risk $\hat{\sigma}_{\varepsilon_x,t} = std(\hat{\varepsilon}_{x,i,t})$ obtained by the three types of regression. Consistent with existing findings (e.g., Heathcote, Perri, and Violante (2010)), idiosyncratic wage risk exhibits an upward trend. In order to isolate its cyclical variation, I compute the per-

[7] I also controlled for occupation. The identified risk becomes slightly smaller, but its cyclical variation is virtually unchanged.

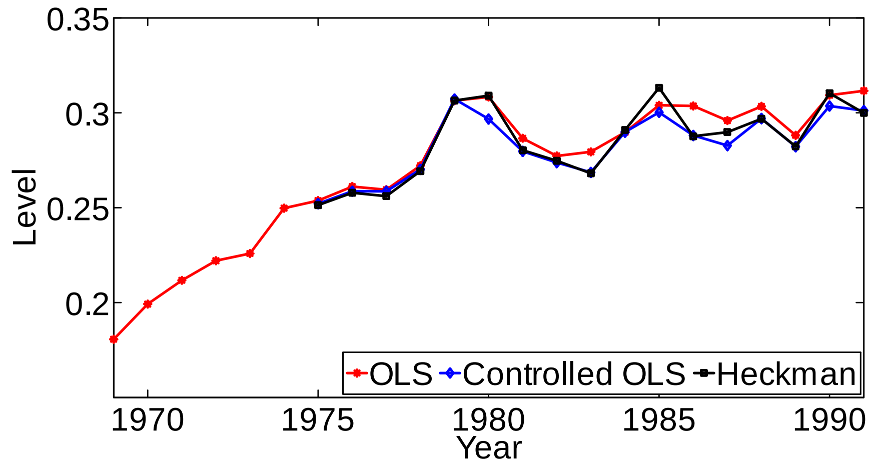

Figure 1: Estimated idiosyncratic wage risk.

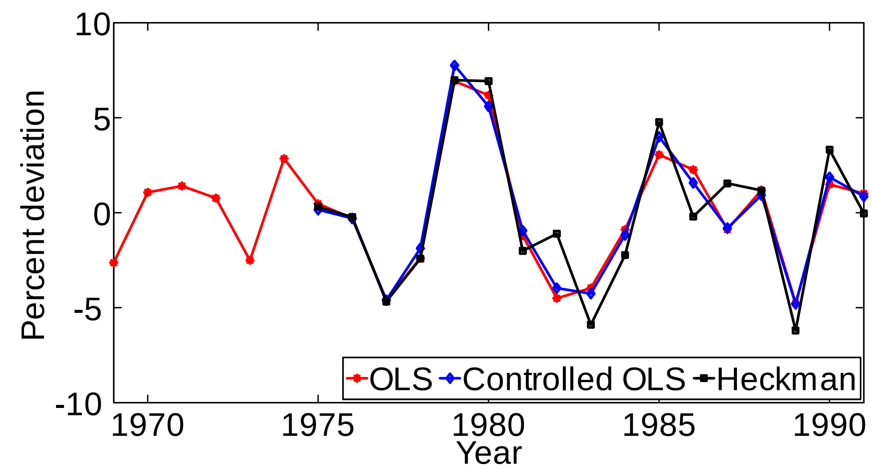

Figure 2: Cyclical components of estimated idiosyncratic wage risk.

	OLS	Controlled OLS	Heckman
$std(\hat{\sigma}_{\varepsilon_x,t})$	0.032	0.035	0.039
$corr(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1})$	0.185	0.236	0.056

Table 1: Cyclical moments of estimated idiosyncratic wage risk.

cent deviation from trend using the Hodrick-Prescott filter with a smoothing parameter of 10.[8] Figure 2 shows this detrended result. All the three cases show similar fluctuations in idiosyncratic wage risk and all the correlation coefficients exceed 0.90. As shown in the figure and also summarized in Table 1, four empirical regularities characterize the cyclical component of idiosyncratic wage risk. First, idiosyncratic wage risk varies over time. The largest deviation from trend is close to 8%, and 4% fluctuations are frequent. The standard deviation is 3.2–3.9%. Second, idiosyncratic wage risk exhibits some persistence, typically remaining above or below trend for approximately two years. However, its first-order autocorrelation coefficient is 0.056–0.236, and the hypothesis of no autocorrelation cannot be rejected. Third, risk variation is approximately symmetric. The size and persistence of idiosyncratic wage risk are similar when it is above and below trend. Fourth, consistent with the finding by Heathcote, Perri, and Violante (2010), idiosyncratic wage risk exhibits neither clear procyclicality nor countercyclicality.[9] Idiosyncratic wage risk remained low during the 1981–1982 recession, but it increased during the 1973–1975 and 1990–1991 recessions. Section 4 uses these findings to calibrate the model with varying idiosyncratic wage risk described below.

[8] The result did not change substantially when using a smoothing parameter of 6.25 or 100.

[9] Heathcote, Perri, and Violante (2010) show that the cross-sectional dispersion of *hourly wages* does not exhibit clear cyclicality, while the cross-sectional disperion of *labor income* is countercyclical, as consistent with the finding by Storesletten, Telmer, and Yaron (2004). Since previous macro analyses typically assume countercyclical risk, I analyze the impact of countercyclical idiosyncratic wage risk in Section 6.

3 Model

The model analyzed here is built upon that of Chang and Kim (2006, 2007), Alonso-Ortiz and Rogerson (2010), and Krusell, Mukoyama, Rogerson, and Şahin (2010, 2011). Individuals make consumption-saving and employment choices each period in the presence of idiosyncratic wage/productivity risk. I introduce risk variation into this environment using uncertainty shocks in the sense of Bloom (2009), i.e., assuming a time-varying second moment for idiosyncratic productivity shocks. In the following, I describe individuals, firms, and equilibrium.

3.1 Individuals

There is a continuum of individuals of measure one. Individuals differ in labor productivity x, which follows an AR(1) process. Individuals have the same momentary utility function $u(c, h)$, where c is consumption and h is labor hours. Labor is indivisible, as in Hansen (1985) and Rogerson (1988), and individuals choose whether to work for a fixed number of hours or not to work at all: $h \in \{\overline{h}, 0\}$. Individuals earn labor income of wxh, where w is the equilibrium wage rate per efficiency unit of labor.

Individuals face time-varying idiosyncratic wage risk because shocks to person-specific productivity x have a time-varying standard deviation σ_{ϵ_x}, which follows a Markov process. Following the convention of the literature on uncertainty shocks, such as Bloom (2009), Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012), and Bachmann and Bayer (2013), individuals learn of the size of σ_{ϵ_x} one period ahead.[10] This timing assumption captures the concept of risk. In what follows, σ_{ϵ_x} represents the volatility of shocks not to x, but to $x^{'}$, where a prime denotes next-period values hereinafter.

Since asset markets are incomplete, individuals are unable to insure themselves fully against varying idiosyncratic wage risk. As in Aiyagari (1994) and

[10] The business cycle results presented in Section 5 are largely unchanged under the assumption that individuals learn of σ_{ϵ_x} contemporaneously.

others, individuals partially self-insure by holding physical capital k, which is the only asset. Borrowing is allowed, but there is a borrowing limit: $k \geq \overline{k}$ ($\overline{k} < 0$).

Define $V(k, x; z, \mu, \sigma_{\epsilon_x})$ as the beginning-of-period value of an individual characterized by (k, x) under the aggregate state $(z, \mu, \sigma_{\epsilon_x})$, where z is aggregate TFP, which follows an AR(1) process, and μ denotes the individual distribution over k and x. This beginning-of-period value reflects the individual's current employment choice:

$$V(k, x; z, \mu, \sigma_{\epsilon_x}) = \max \left\{ V^E(k, x; z, \mu, \sigma_{\epsilon_x}), V^N(k, x; z, \mu, \sigma_{\epsilon_x}) \right\}. \tag{6}$$

The individual's within-period value conditional on working is $V^E(k, x; z, \mu, \sigma_{\epsilon_x})$, which satisfies

$$\begin{aligned} V^E(k, x; z, \mu, \sigma_{\epsilon_x}) &= \max_{c, k'} \left\{ \begin{array}{c} u(c, \overline{h}) \\ +\beta E\left[V(k', x'; z', \mu', \sigma'_{\epsilon_x}) | x, z, \mu, \sigma_{\epsilon_x} \right] \end{array} \right\}, \\ \text{subject to } c &= w(z, \mu, \sigma_{\epsilon_x}) x \overline{h} + [1 + r(z, \mu, \sigma_{\epsilon_x})]k - k' \\ k' &\geq \overline{k} \\ c &\geq 0 \\ \mu' &= \Gamma(z, \mu, \sigma_{\epsilon_x}), \end{aligned} \tag{7}$$

where β is the discount factor, E is the conditional expectation, r is the equilibrium rental rate of capital, and Γ is the law of motion for μ.

Similarly, $V^N(k, x; z, \mu, \sigma_{\epsilon_x})$ is the individual's within-period value conditional on not working, which satisfies

$$
\begin{aligned}
V^N(k,x;z,\mu,\sigma_{\epsilon_x}) &= \max_{c,k'}\left\{\begin{array}{c} u(c,0) \\ +\beta E\left[V(k',x';z',\mu',\sigma'_{\epsilon_x})|x,z,\mu,\sigma_{\epsilon_x}\right]\end{array}\right\}, \quad (8) \\
\text{subject to } c &= [1+r(z,\mu,\sigma_{\epsilon_x})]k-k' \\
k' &\geq \overline{k} \\
c &\geq 0 \\
\mu' &= \Gamma(z,\mu,\sigma_{\epsilon_x}).
\end{aligned}
$$

3.2 Representative Firm

A representative firm produces the final good Y using capital K and labor L. The production function is $Y = zF(K,L)$ and exhibits constant returns to scale. Given $r(z,\mu,\sigma_{\epsilon_x})$ and $w(z,\mu,\sigma_{\epsilon_x})$, the firm chooses $K(z,\mu,\sigma_{\epsilon_x})$ and $L(z,\mu,\sigma_{\epsilon_x})$, maximizing static profits. The first-order conditions are

$$r(z,\mu,\sigma_{\epsilon_x}) = zF_K(K,L)-\delta, \quad (9)$$

and

$$w(z,\mu,\sigma_{\epsilon_x}) = zF_L(K,L), \quad (10)$$

where δ is the capital depreciation rate.

3.3 General Equilibrium

A recursive competitive equilibrium is a set of functions,

$$\left(w,r,V^E,V^N,V,c,k',h,K,L,\Gamma\right), \quad (11)$$

that satisfy the following conditions.

1. Individuals' Optimization:

   $V(k,x;z,\mu,\sigma_{\epsilon_x}), V^E(k,x;z,\mu,\sigma_{\epsilon_x})$, and $V^N(k,x;z,\mu,\sigma_{\epsilon_x})$ satisfy (6), (7), and (8), while $c(k, x; z, \mu, \sigma_{\epsilon_x})$, $k^{'}(k, x; z, \mu, \sigma_{\epsilon_x})$, and $h(k, x; z, \mu, \sigma_{\epsilon_x})$ are the associated policy functions.

2. Firms' Optimization:

   The representative firm chooses $K(z, \mu, \sigma_{\epsilon_x})$ and $L(z, \mu, \sigma_{\epsilon_x})$ to satisfy (9) and (10).

3. Labor Market Clearing:

   $L(z, \mu, \sigma_{\epsilon_x}) = \int xh(k, x; z, \mu, \sigma_{\epsilon_x})\mu([dk \times dx])$

4. Capital Market Clearing:

   $K(z, \mu, \sigma_{\epsilon_x}) = \int k\mu([dk \times dx])$

5. Goods Market Clearing:

   $\int \left\{k^{'}(k, x; z, \mu, \sigma_{\epsilon_x}) + c(k, x; z, \mu, \sigma_{\epsilon_x})\right\} \mu([dk \times dx])$

   $= zF\left(K(z, \mu, \sigma_{\epsilon_x}), L(z, \mu, \sigma_{\epsilon_x})\right) + (1 - \delta)\int k\mu([dk \times dx])$

6. Evolution of Individual Distribution:

   $\Gamma(z, \mu, \sigma_{\epsilon_x})$ is consistent with individual decisions and the laws of motion for x, z, and σ_{ϵ_x}. Specifically, for all $B \subseteq K$,

   $\mu^{'}(B, x^{'}) = \int_{\left\{(k,x)|k^{'}(k,x;z,\mu,\sigma_{\epsilon_x})\in B\right\}} \pi_x(x^{'}|x, \sigma_{\epsilon_x})\mu([dk^{'} \times dx^{'}])$,

   where $\pi_x(x^{'}|x, \sigma_{\epsilon_x})$ is the transition probability from x to $x^{'}$ under σ_{ϵ_x}.

4 Calibration and the Steady State

This section first determines parameter values for the above model, except for those concerning idiosyncratic productivity. Their values are standard and are chosen so that the model economy replicates several features of the U.S. economy. Next, I determine parameter values for idiosyncratic productivity, matching moments of individual wages in the model with moments of the PSID wages. The end of this section briefly presents the steady state.

Parameter	Description	Value
β	Discount factor	0.9829
B	Disutility of labor	1.0203
$\overline{h}$	Working hours	1/3
$\overline{k}$	Borrowing limit	-2.0
α	Labor share	0.64
δ	Capital depreciation rate	0.025
ρ_z	Persistence in aggregate TFP	0.95
σ_{ϵ_z}	Volatility of aggregate TFP shocks	0.007

Table 2: Parameters other than idiosyncratic productivity.

4.1 Parameters Other Than Idiosyncratic Productivity

Table 2 lists the parameter values other than those related to idiosyncratic productivity. The values are comparable to those used in existing incomplete asset markets models (e.g., Krusell and Smith (1998) and Chang and Kim (2007)). Each period is equal to one quarter. The discount factor β is 0.9829, which generates a one percent quarterly rental rate of capital at the steady state. The momentary utility when individuals work is $u(c, h) = \ln c - B$. When they do not work, $u(c, h) = \ln c$. The disutility parameter is $B = 1.0203$, producing a steady-state employment rate of 60%. The employment rate is close to the average U.S. employment-population ratio during the period of 1948Q1–2009Q3. Individuals use one third of their time when working ($\overline{h} = 1/3$). The borrowing limit is $\overline{k} = -2.0$. Under the constraint, individuals can borrow up to 44% of the average annual income at the steady state, which is similar to the constraint set by Krusell and Smith (1998) for their model with borrowing.[11]

As for the firm side, the production function is $Y = zK^{1-\alpha}L^{\alpha}$, and labor's share α is 0.64. The capital depreciation rate δ is 0.025. Aggregate TFP z follows an AR(1) process, $\ln z' = \rho_z \ln z + \epsilon_z'$, where $\epsilon_z' \sim N(0, \sigma_{\epsilon_z}^2)$. As in Cooley and Prescott (1995), $\rho_z = 0.95$, and $\sigma_{\epsilon_z} = 0.007$.

[11] I conducted a sensitivity analysis with respect to the borrowing limit $\overline{k}$. The business cycle results with $\overline{k} = -4.0$ and 0.0 did not substantially differ from those with $\overline{k} = -2.0$.

	PSID	Varying risk	Constant risk
A. *Moments (Annual)*			
$\widehat{\rho}_x$	0.854	0.855	0.855
$\widehat{\overline{\sigma}}_{\epsilon_x}$	0.282	0.283	0.279
$std(\hat{\sigma}_{\varepsilon_x,t})$	0.032	0.032	0.008
$corr(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1})$	0.185	0.158	-0.240
B. *Parameters (Quarterly)*			
ρ_x	-	0.930	0.930
$\overline{\sigma}_{\epsilon_x}$	-	0.223	0.223
$\rho_{\sigma_{\epsilon_x}}$	-	0.900	-
λ	-	0.090	-

Table 3: Calibration moments and parameter values for idiosyncratic productivity. Panel A lists the moments of individual wages used for calibration. Panel B shows the parameter values.

4.2 Parameters on Idiosyncratic Productivity

Four parameters concern idiosyncratic productivity x. Since x follows an AR(1) process in (2), the first parameter is the persistence ρ_x. The other three parameters concern fluctuations in idiosyncratic wage risk σ_{ε_x}. I assume a three-state Markov chain: high (H), middle (M), and low (L). The analysis in Section 2 finds no strong cyclicality in σ_{ε_x}. Thus, σ_{ε_x} evolves independently of aggregate TFP z. Furthermore, the symmetry of risk variation shown in Section 2 suggests a symmetric transition matrix: A risk state remains unchanged with a quarterly probability of $\rho_{\sigma_{\varepsilon_x}}$ and transitions to each of the other two states with a probability of $(1-\rho_{\sigma_{\varepsilon_x}})/2$. The risk levels also should be symmetric: $\sigma_{\varepsilon_x,H} = (1+\lambda)\bar{\sigma}_{\varepsilon_x}$, $\sigma_{\varepsilon_x,M} = \bar{\sigma}_{\varepsilon_x}$, and $\sigma_{\varepsilon_x,L} = (1-\lambda)\bar{\sigma}_{\varepsilon_x}$, where $\bar{\sigma}_{\varepsilon_x}$ is the steady-state risk and λ is the size of risk variation.

I determine the values of these four parameters $(\rho_x, \bar{\sigma}_{\varepsilon_x}, \rho_{\sigma_{\varepsilon_x}}, \lambda)$ such that model wages reproduce the four moments of the PSID wages listed in Table 3A. The two moments are the persistence in individual wages $\hat{\rho}_x$ and the long-run idiosyncratic wage risk $\widehat{\bar{\sigma}}_{\varepsilon_x}$. For both the model and the PSID data, I compute these moments by estimating (3) with year dummies using the pooled OLS.[12] The other two moments are the annual standard deviation

[12] I simulate the model with 10,000 individuals for 1,500 periods and generate annual panel

$std(\hat{\sigma}_{\varepsilon_x,t})$ and the first-order autocorrelation coefficient $corr(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1})$ of idiosyncratic wage risk. For the model moments, I estimate (3) each year with OLS, compute idiosyncratic wage risk as $\hat{\sigma}_{\varepsilon_x,t} = std(\hat{\varepsilon}_{x,i,t})$, and remove trend using the Hodrick-Prescott filter with a smoothing parameter of 10. To keep the comparability, I use the PSID moments estimated with simple OLS, which were originally shown in the second column of Table 2. Note that targeting the risk variation estimated with simple OLS gives a conservative estimate for the impact of uncertainty shocks because as shown in Table 2, the estimated risk variation is smaller than that estimated with the two other methods.

The second column of Table 3A shows the PSID moments. The persistence in individual wages is 0.854. The long-run value of idiosyncratic wage risk is 0.282. Idiosyncratic wage risk varies by a standard deviation of 3.2% (0.032) and shows a first-order autocorrelation coefficient of 0.185.

These PSID moments pin down the model parameters as shown in the third column of Table 3B. The persistence of productivity is $\rho_x = 0.930$, while the steady-state risk is $\bar{\sigma}_{\varepsilon_x} = 0.223$. These values are comparable to those used in models assuming constant idiosyncratic wage risk. For example, Chang and Kim (2007) use $\rho_x = 0.929$ and $\bar{\sigma}_{\varepsilon_x} = 0.227$. As for risk variation, the persistence $\rho_{\sigma_{\varepsilon_x}}$ is 0.90 and the size λ is 0.09, implying that idiosyncratic wage risk varies between $\sigma_{\varepsilon_x,L} = (1-\lambda)\bar{\sigma}_{\varepsilon_x} = 0.201$ and $\sigma_{\varepsilon_x,H} = (1+\lambda)\bar{\sigma}_{\varepsilon_x} = 0.245$.[13] As shown in the same column of Table 3A, the calibrated varying risk model successfully reproduces the PSID moments.

In contrast, as shown in the last column, the constant risk model fails to replicate the risk variation in the PSID. Because of endogenous employment choice, idiosyncratic wage risk exhibits some variation, even when estimated using individual wages in the constant risk model. However, the volatility and persistence are much smaller than those estimated using the PSID data. This finding provides further evidence for cyclical fluctuations in idiosyncratic wage

data on hourly wages (discarding the first 500 periods). The same sequence of aggregate TFP is used for varying and constant risk models.

[13]Footnote 19 explains how the labor market dynamics depends on $\rho_{\sigma_{\varepsilon_x}}$ and λ in the present model.

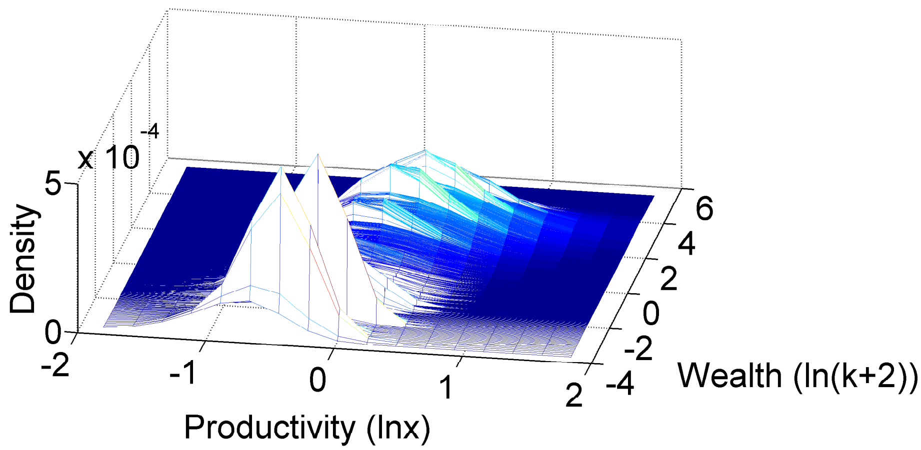

Figure 3: Steady-state distribution of wealth and productivity.

risk.

4.3 Steady State

The steady state of the present model shows the inequality of wealth and labor income that is comparable to that found in the U.S. economy. The Gini coefficient of annual labor income is 0.59 in the model and 0.65 in the 1991 PSID.[14] As for the wealth inequality, the Gini coefficient is 0.64 in the model. Since it is difficult to define individual wealth in the actual economy, I compare this individual-level wealth inequality with the household-level inequality in the U.S. According to Díaz-Giménez, Quadrini, and Ríos-Rull (1997), the Gini coefficient is 0.78 in the 1992 Survey of Consumer Finances.

Further, the present model generates a weakly positive correlation between wealth and labor income (0.30), which is close to that found by Díaz-Giménez, Quadrini, and Ríos-Rull (1997) for the U.S. economy (0.23).[15] The positive

[14] Appendix B1 explains the solution method for the steady state. I generate the distribution of annual labor income through simulation with 10,000 individuals. Appendix A2 explains the PSID data.

[15] The steady state of the present model is quite similar to that of Chang and Kim (2007)'s model. Table 2 of Chang and Kim (2007) provides additional evidence that their model's

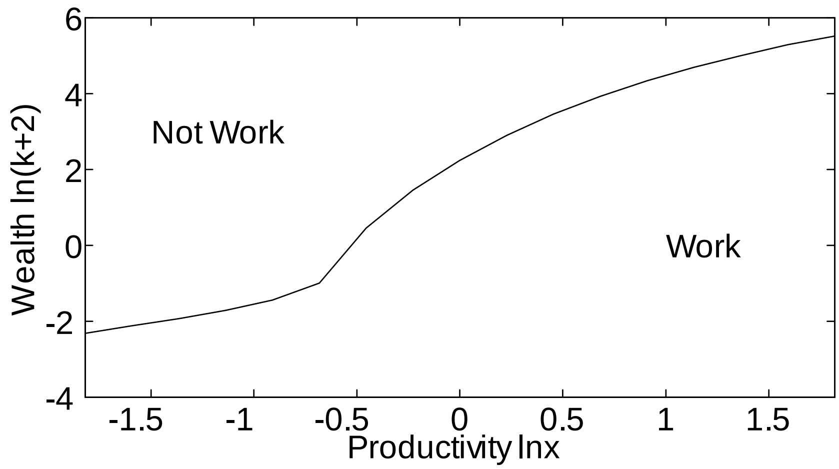

Figure 4: Individual employment decisions at the steady state.

correlation in the model arises from two factors. First, since idiosyncratic productivity is persistent, individuals with higher current productivity tend to hold larger wealth, as shown in Figure 3.[16] Second, as shown in Figure 4, individuals are more likely to work when they have higher current productivity and smaller wealth. These two factors interact in generating the weakly positive correlation between wealth and labor earnings.

5 Business Cycle Results

This section compares business cycles of the varying and constant risk models calibrated in the last section. Next, in order to understand the result, I analyze how the varying risk model responds to exogenous variation in idiosyncratic wage risk and aggregate TFP.

joint distribution of wealth and income is comparable to that in the U.S.

[16] There are a large number of individuals near the borrowing limit ($k = -2.0$). The figure excludes those individuals to highlight the rest of the distribution.

	U.S. economy	Varying risk	Constant risk	Psych risk
σ_Y	1.69	1.43	1.37	1.37
σ_C/σ_Y	0.54	0.33	0.32	0.33
σ_I/σ_Y	2.85	3.15	3.10	3.10
σ_H/σ_Y	1.00	0.81	0.57	0.61
$\sigma_{Y/H}/\sigma_Y$	0.63	1.00	0.48	0.52
σ_{wedge}/σ_Y	1.40	1.26	0.23	0.38
$Corr(Y,C)$	0.78	0.86	0.90	0.89
$Corr(Y,I)$	0.80	0.99	0.99	0.99
$Corr(Y,H)$	0.80	0.41	0.96	0.91
$Corr(Y,Y/H)$	0.31	0.67	0.95	0.87
$Corr(H,Y/H)$	−0.32	−0.40	0.83	0.58
$Corr(H,wedge)$	–0.94	–0.84	–0.96	–0.83

Table 4: Varying idiosyncratic wage risk and business cycle moments. I take logs of all of the series and use the Hodrick-Prescott filter with a smoothing parameter of 1,600. The volatility of output σ_Y is the standard deviation of output (multiplied by 100). Other volatilities are their ratios with respect to σ_Y. $Corr$ denotes a contemporaneous correlation.

5.1 Time-Varying Idiosyncratic Wage Risk and Business Cycles

Table 4 lists business cycle statistics of the U.S. economy, the varying risk model, and the constant risk model.[17] I generate the two model results through simulations using the same sequence of aggregate TFP. Hence, the difference between the two reveals the impact of variation in idiosyncratic wage risk on aggregate fluctuations.

Introducing fluctuations in idiosyncratic wage risk improves the model's performance on two key labor market statistics. One is the volatility of the labor wedge. The labor wedge is the ratio of average labor productivity to the marginal rate of substitution of leisure for consumption, computed assuming a representative individual. It is computed here by $\ln wedge = \ln Y/H -$

[17] I take the U.S. macroeconomic data from the source listed in Appendix A3. I use the Krusell and Smith (1997, 1998) algorithm for the model simulations. Appendix B2 explains the numerical method.

$\ln H^{1/\gamma}C$, setting $\gamma = 1.5$ at the aggregate level, as in Chang and Kim (2007).[18] The labor wedge is quite volatile in the U.S. economy. The varying risk model successfully reproduces the feature: The volatility of the labor wedge (relative to the output volatility) is 95% of that in the U.S. data. In contrast, the constant risk model can account for only 17% of the volatility of the labor wedge seen in the U.S.

The second improvement appears in the correlation between total hours worked and average labor productivity. The varying risk model generates a weakly negative correlation of -0.40 that is similar to the empirical value of -0.32 from the U.S. data, whereas the constant risk model produces a counterfactually strong positive correlation of 0.83. Thus, the hours-productivity puzzle is also resolved by shifting idiosyncratic wage risk in a manner consistent with the micro-level wage data.[19]

Furthermore, introducing the risk variation increases the volatility of hours worked and reduces the correlation between output and labor productivity, moving their values closer to the U.S. data. The volatility of productivity and the correlation between output and hours also move towards the U.S. data, although their values in the varying risk model overshoot their data counterparts.

In contrast, changes in idiosyncratic wage risk have a relatively mild impact on the fluctuations in other aggregate variables. The varying and constant risk models exhibit similar volatilities and comovements of output, consumption, and investment. Thus, introducing variation in idiosyncratic wage risk strengthens the model's ability to explain labor market fluctuations, without weakening the model's ability to account for other business cycle moments.

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[18] Total hours worked is $H \equiv \int h(k, x; z, \mu, \sigma_{\epsilon_x})\mu([dk \times dx])$, which is different from efficiency-weighted total labor $L \equiv \int xh(k, x; z, \mu, \sigma_{\epsilon_x})\mu([dk \times dx])$.

[19] These improvements are largely insensitive to the risk persistence parameter $\rho_{\sigma_{\epsilon_x}}$. For example, even under almost no persistence ($\rho_{\sigma_{\epsilon_x}} = 0.4$), the volatility of the labor wedge is 77% of that in the U.S. data, while the correlation between hours and productivity is –0.21. As for the size of risk variation λ, even when it is decreased by almost 50% ($\lambda = 0.05$), the volatility of the labor wedge is still 57% of that in the U.S. data and the hours-productivity correlation is almost zero (0.09).

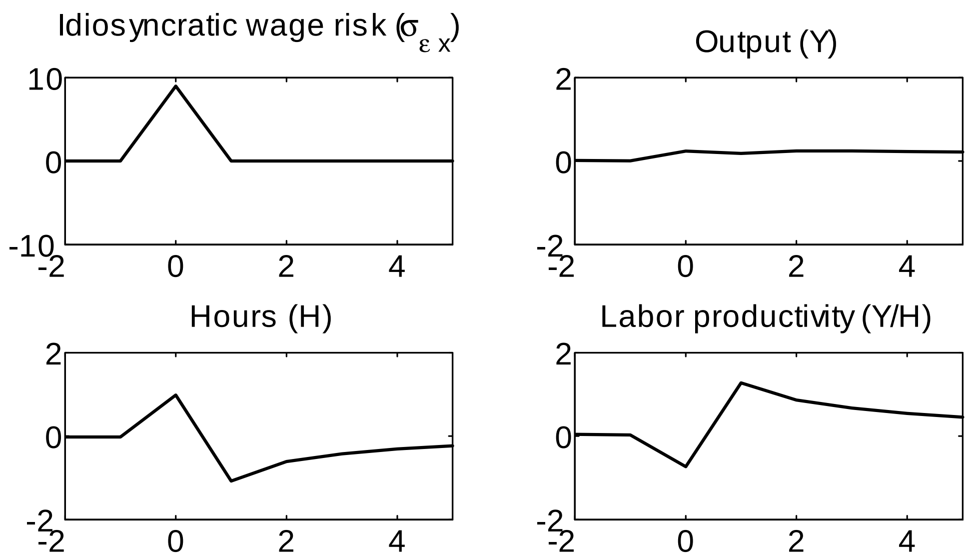

Figure 5: Increase in idiosyncratic wage risk (varying risk model). Vertical axis - period. Horizontal axis - percent deviation from the pre-shock average.

5.2 Responses to Changes in Idiosyncratic Wage Risk

Next, in order to identify the mechanism through which variation in idiosyncratic wage risk σ_{ε_x} generates labor market fluctuations, I analyze the response of the varying risk model to an increase in σ_{ε_x}. The simulation starts from the steady state (period –29), and as shown in the upper-left panel of Figure 5, σ_{ε_x} increases by 9% (i.e., changes from the middle to high states) for one period in period 0. The timing assumption made earlier implies that while individuals learn this rise in σ_{ε_x} in period 0, the elevated σ_{ε_x} increases the dispersion of idiosyncratic productivity in period 1. In contrast, aggregate TFP is constant at its steady-state level throughout.

As the remaining panels of Figure 5 show, output, total hours worked, and average labor productivity move in different ways. Output increases slightly in period 0 and then slowly returns to the pre-shock level. Hours increases in period 0, drops below the pre-shock level in period 1, and then gradually recovers. Productivity moves in a direction exactly opposite to that of hours.

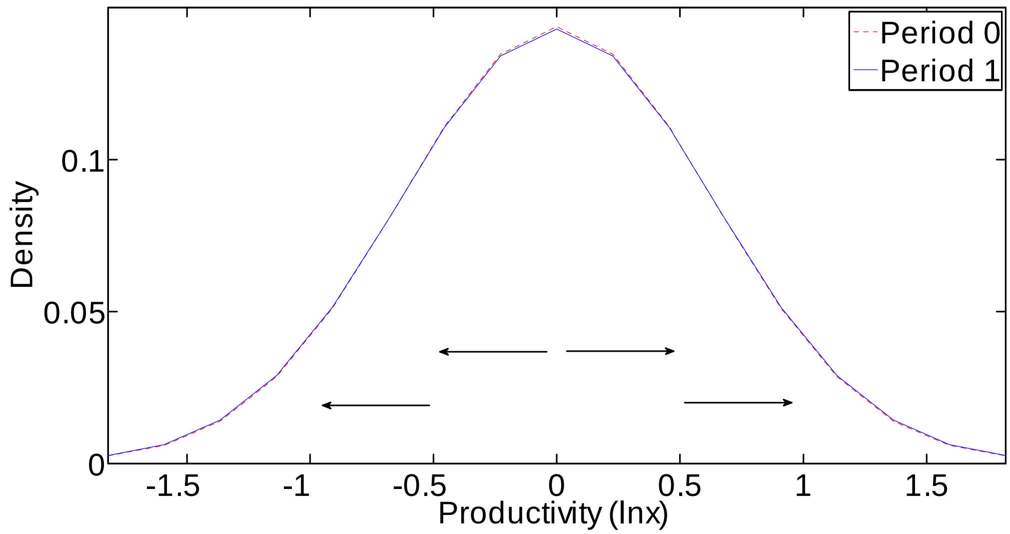

Figure 6: Shift in the productivity distribution.

Hence, a one-period increase in idiosyncratic wage risk generates a long-lasting negative comovement of hours with productivity, without moving output and hence consumption substantially. The result explains why introducing variation in idiosyncratic wage risk increases the volatility of the labor wedge and breaks the strong comovement of hours with productivity. It also explains why varying idiosyncratic wage risk increases the volatilities of hours and productivity relative to the output volatility.

The timing assumption of the present model implies two effects underlying these responses of aggregate variables to the rise in idiosyncratic wage risk. The first is an *uncertainty effect.* In period 0, individuals become more uncertain about their future productivity. Because of the greater uncertainty on their future wages, individuals increase labor supply in period 0. The second effect is a *distribution effect.* When the increased volatility of idiosyncratic productivity shocks materializes in period 1, there is an increase in the cross-sectional dispersion of idiosyncratic productivity. As shown in Figure 6, the increase in the productivity dispersion is small because the increase in risk is small and temporary. However, it shifts the joint distribution of wealth and productivity, changing employment choices of a relatively large number of individuals in period 1. In contrast, the uncertainty effect disappears in period 1 because uncertainty on future productivity returns to the pre-shock level.

First, in period 0, the uncertainty effect increases the employment of individuals with relatively low productivity and to a lesser extent decreases the employment of individuals with relatively high productivity. As shown in Figures 3 and 4, many individuals with low productivity hold small wealth close to their employment boundary. When uncertainty on their future wages increases, these individuals increase savings in order to self-insure and some of them switch from not working to working. In contrast, individuals with high productivity and near the employment boundary are wealthy and are thus well-insured. The employment of these individuals decreases slightly because the increase in the employment of low-productivity individuals lowers the equilibrium wage rate. At the aggregate level, hours worked increases in period 0. Output increases less than hours, not only because aggregate TFP and capital remain unchanged, but also because employment disproportionately increases among low-productivity individuals. Hence, average labor productivity decreases in period 0.

Second, in period 1, the distribution effect decreases the employment of individuals with lower-than-average productivity and to a lesser degree increases the employment of individuals with higher-than-average productivity. As the arrows in Figure 6 indicate, there is a net flow of individuals from the middle to lower and higher levels of productivity at the beginning of period 1. Since there was a positive correlation between productivity and wealth in the pre-shock distribution (Figure 3), the flow of individuals generates a population shift from the "Work" region to the "Not work" region for lower-than-average productivity in Figure 4. The opposite mechanism works for higher-than-average productivity, and some of those individuals shift from the "Not work" region to the "Work" region. The pre-shock distribution and the decline in the equilibrium wage rate imply that even a small increase in the productivity dispersion decreases the employment of low-productivity individuals substantially, while it increases high-productivity employment much less. Hence, total hours worked decreases. Output remains slightly above the pre-shock level because this time the composition of workers shifts towards individuals with higher productivity. Average labor productivity increases more than output.

Crucially, even though idiosyncratic wage risk increases only for one period, it takes quite a few periods for the wealth-productivity distribution to settle down. Hence, employment gradually recovers among individuals with lower-than-average productivity, whereas the employment of higher-than-average productivity individuals slowly decreases. As a result, output and average labor productivity gradually decrease to their pre-shock levels, whereas total hours worked recovers sluggishly.

In order to quantify the roles of the uncertainty and distribution effects in generating the above results, I solve the model including only the uncertainty effect and shutting down the distribution effect. Specifically, I assume that individuals perceive and respond to changes in idiosyncratic wage risk, but those changes in risk do not materialize and the distribution of idiosyncratic productivity remains unchanged. Following Bachmann and Bayer (2013), I call it the psych risk model.

The last column of Table 3 shows the result of the psych risk model. As shown, the uncertainty effect alone increases the volatility of the labor wedge and lowers the correlation between total hours worked and average labor productivity relative to the constant risk model. However, the impact is relatively small. The psych risk model can only account for 27% of the empirical volatility of the labor wedge and generates a positive correlation between hours and productivity of 0.58. Figure 7 shows the response of the psych risk model to the one-period increase in σ_{ε_x} of 9% considered above. The response is qualitatively similar to that of the varying risk model.[20] However, when compared to the varying risk model, the fluctuations in hours and productivity are much smaller and the negative comovement of the two disappears much more quickly in the psych risk model. These results indicate that the major impact

[20] Total hours worked in period 1 is slightly lower than the pre-shock level even in the psych risk model. This is because of ex-post wealth effect. Individuals accumulate unusually large savings in period 0 due to the elevated uncertainty. See Marcet, Obiols-Homs, and Weil (2007) on the ex-post wealth effect under constant idiosyncratic income risk.

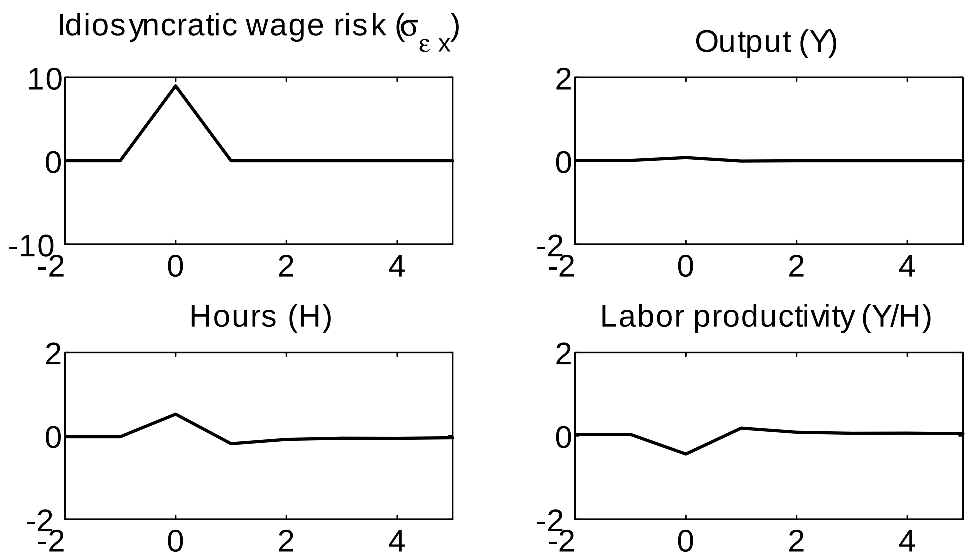

Figure 7: Increase in idiosyncratic wage risk (psych risk model). Vertical axis - period. Horizontal axis - percent deviation from the pre-shock average.

of changes in idiosyncratic wage risk works not through the uncertainty effect, but through the distribution effect.[21]

5.3 Responses to Aggregate TFP Shocks

This subsection examines the response of the varying risk model to the other aggregate shock, namely, an exogenous change in aggregate TFP z. The simulation starts from the steady state (period –29), and as shown in the upper-left panel of Figure 8, z declines by 1.67% in period 0. Idiosyncratic wage risk remains constant.

As the other panels indicate, output, total hours worked, and average labor productivity all decrease following the decrease in aggregate TFP. In this model

[21] Including unemployment benefits strengthens the distribution effect for low-productivity individuals and weakens the distribution effect for high-productivity individuals and the uncertainty effect. Since the distribution effect of low-productivity groups is dominant in the present model, the inclusion of unemployment benefits is unlikely to change the main result substantially.

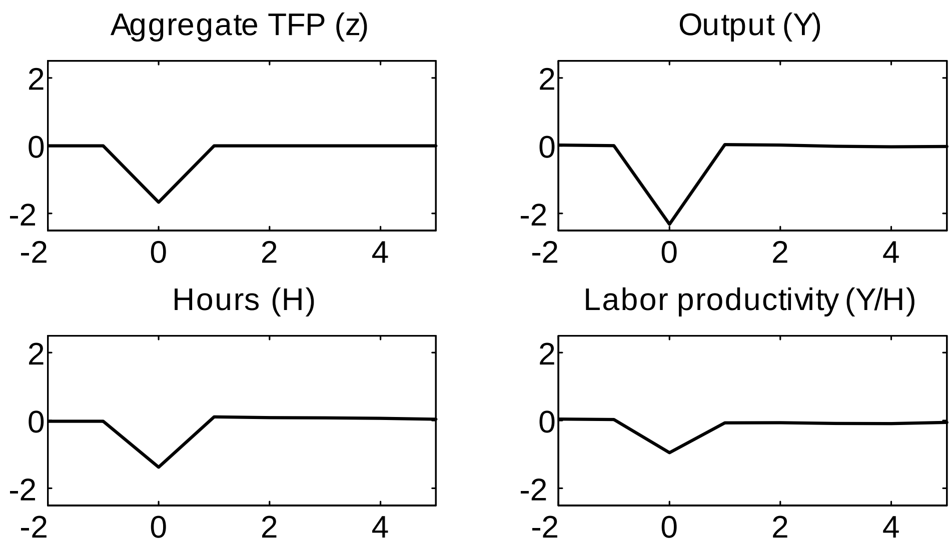

Figure 8: Decrease in aggregate TFP (varying risk model). Vertical axis - period. Horizontal axis - percent deviation from the pre-shock average.

economy, as in the prototype equilibrium business cycle model, a decline in aggregate TFP reduces labor demand, without significantly affecting labor supply. In equilibrium, the wage rate falls, and employment decreases across all productivity groups. Since aggregate TFP decreases, output decreases more substantially than hours, lowering average labor productivity. Furthermore, since the wealth-productivity distribution hardly shifts, output, hours, and productivity recover quickly. Hence, a temporary decrease in aggregate TFP generates a short-lived positive correlation between hours and productivity.

Including only aggregate TFP shocks, the constant risk model generates relatively small fluctuations in the labor wedge and a strong positive correlation between total hours worked and average labor productivity.[22] In contrast, since fluctuations in idiosyncratic wage risk generate a persistent negative cor-

[22] Chang and Kim (2007) report large variation in the labor wedge and a nearly zero correlation between hours and productivity in the constant risk model with slightly different parameter values from those used herein. However, as Takahashi (2014) shows, their findings are a result of computational errors. See also Chang and Kim (2014).

relation between hours and productivity, the varying risk model exhibits large cyclical fluctuations in the labor wedge and a weakly negative correlation between hours and productivity.

6 Countercyclical Idiosyncratic Wage Risk

Up to this point, I have assumed no correlation between idiosyncratic wage risk and aggregate TFP. Although the finding in Section 2 and that of Heathcote, Perri, and Violante (2010) rationalize the assumption, countercyclical risk is worth examining. First, my analysis of PSID data indicates that idiosyncratic wage risk increased in the 1973–1975 and the early 1990s recessions, and hence it would be interesting to examine the impact of elevated risk during a recession. Second, the seminal paper by Storesletten, Telmer, and Yaron (2004) documents countercyclicality of *income* risk and existing macro models typically assume countercyclical income risk. Although the finding by Heathcote, Perri, and Violante (2010) suggests that labor income and wage risk move differently, I analyze how cyclicality of wage risk affects aggregate fluctuations by introducing countercyclical idiosyncratic wage risk into the present model.

6.1 Increase in Idiosyncratic Wage Risk During Recessions

In order to analyze a smooth change in idiosyncratic wage risk as seen in the actual economy, I conduct 500 model simulations and compute the average response. Each simulation starts from the steady state (in period –250) and idiosyncratic wage risk σ_{ε_x} increases by one state in period 0. While individuals learn of this rise in σ_{ε_x} in period 0, the elevated σ_{ε_x} actually increases the idiosyncratic productivity dispersion in period 1. Aggregate TFP z decreases by one grid point in period 0.[23] At other times, σ_{ε_x} and z evolve according to their independent stochastic processes. As shown in the upper two panels

[23] If σ_{ε_x} was in the high state in period –1, then σ_{ε_x} remains there in period 0. Similarly, if z was the lowest in period –1, then z remains unchanged in period 0.

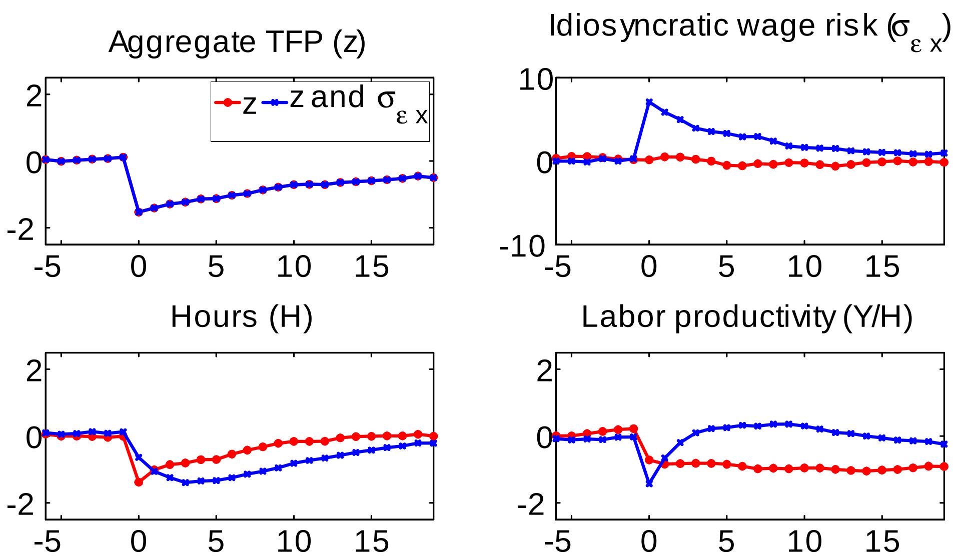

Figure 9: Increase in idiosyncratic wage risk during a recession. Vertical axis - period. Horizontal axis - percent deviation from the pre-shock average.

of Figure 9, σ_{ε_x} increases by 7.09% on average relative to the pre-shock level in period 0 and slowly decreases, whereas z decreases by 1.53% initially and gradually recovers. I compare the results of this experiment with those when only z decreases.

The lower panels show the movements of total hours worked and average labor productivity in these two cases. Compared with the case in which only aggregate TFP decreases, labor productivity recovers faster when idiosyncratic wage risk increases simultaneously. However, total hours worked recovers more slowly.

This result is similar to the U.S. experience. Figure 10 shows the cyclical components of total hours worked and average labor productivity between 1947Q3 and 2009Q3. The recovery of hours relative to labor productivity is slower after the 1973–1975 and the 1990–1991 recessions, during which idiosyncratic wage risk rose, than after the 1981–1982 recession, during which risk remained low. This finding provides additional evidence that the movement of idiosyncratic wage risk plays an important role in labor market dynamics

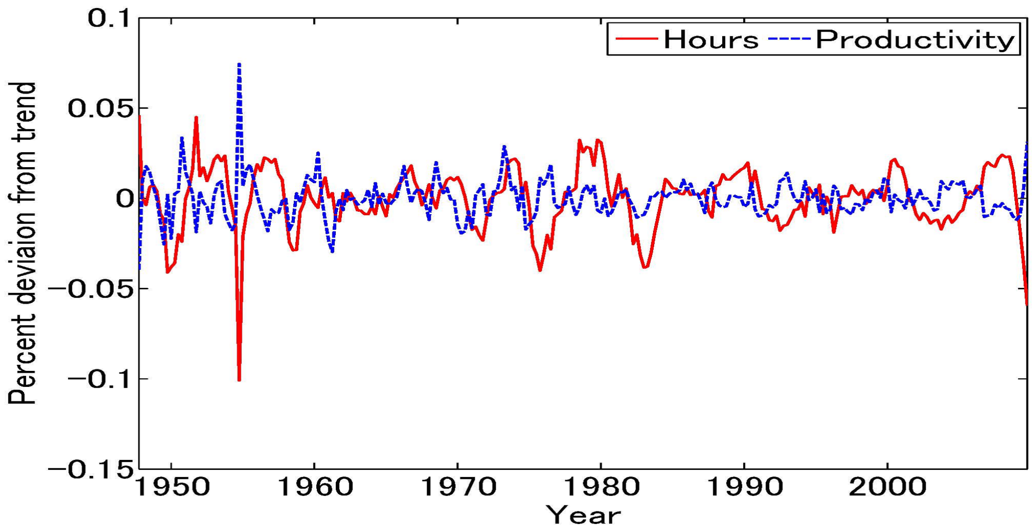

Figure 10: Total hours worked and average labor productivity in the U.S. I take logs of the variables and use the Hodrick-Prescott filter with a smoothing parameter of 1,600.

over the business cycle.

6.2 Cyclicality of Risk and Business Cycles

Next, in order to evaluate the impact of countercyclical idiosyncratic wage risk on overall business cycles, I introduce negative comovement of idiosyncratic wage risk σ_{ε_x} with aggregate TFP z as follows. When z is approximately equal to the mean (approximately $\pm 1.7\%$ relative to the steady-state level), $\sigma_{\varepsilon_x} = \sigma_{\varepsilon_x,M} = \bar{\sigma}_{\varepsilon_x}$. When z is lower than this range, $\sigma_{\varepsilon_x} = \sigma_{\varepsilon_x,H} = (1+\lambda)\bar{\sigma}_{\varepsilon_x}$. When z exceeds the range, $\sigma_{\varepsilon_x} = \sigma_{\varepsilon_x,L} = (1-\lambda)\bar{\sigma}_{\varepsilon_x}$. All of the other parameters maintain their values determined in Section 4, including $\bar{\sigma}_{\varepsilon_x} = 0.223$ and $\lambda = 0.09$, except that here $\rho_{\sigma_{\varepsilon_x}}$ is determined by the law of motion for z.

Importantly, the results for the countercyclical risk model are not directly comparable with those for the varying risk model in Section 5. This is because the fluctuations of idiosyncratic wage risk σ_{ε_x} are different in these two models. Specifically, σ_{ε_x} in the countercyclical risk model is substantially less volatile and slightly more persistent than σ_{ε_x} in the varying risk model calibrated to

	Countercyclical risk	Recalibrated varying risk	Constant risk
σ_Y	1.32	1.38	1.37
σ_C/σ_Y	0.37	0.33	0.32
σ_I/σ_Y	2.98	3.11	3.10
σ_H/σ_Y	0.56	0.68	0.57
$\sigma_{Y/H}/\sigma_Y$	0.61	0.74	0.48
σ_{wedge}/σ_Y	0.60	0.83	0.23
$Corr(Y,C)$	0.91	0.88	0.90
$Corr(Y,I)$	0.99	0.99	0.99
$Corr(Y,H)$	0.84	0.68	0.96
$Corr(Y,Y/H)$	0.87	0.73	0.95
$Corr(H,Y/H)$	0.46	0.00	0.83
$Corr(H,wedge)$	–0.67	–0.77	–0.96

Table 5: Cyclicality of idiosyncratic wage risk and business cycle moments. I take logs of all of the series and use the Hodrick-Prescott filter with a smoothing parameter of 1,600. The volatility of output σ_Y is the standard deviation of output (multiplied by 100). Other volatilities are their ratios with respect to σ_Y. $Corr$ denotes a contemporaneous correlation.

reproduce the risk fluctuations in the PSID data. Therefore, in order to isolate the impact of the cyclicality of wage risk from the impact of the volatility and persistence of risk, I reset λ and $\rho_{\sigma_{\varepsilon_x}}$ of the varying risk model, targeting the standard deviation and the first-order autocorrelation of σ_{ε_x} in the countercyclical risk model. The results are $\lambda = 0.058$ and $\rho_{\sigma_{\varepsilon_x}} = 0.925$. All of the other parameters inherit their original values.

Table 5 lists the results for the countercyclical risk and recalibrated varying risk models along with the results for the constant risk model for comparison. Note that these results should not be compared with the U.S. data moments because these models are not calibrated to reproduce the risk fluctuations seen in the PSID data. As shown, acyclical and countercyclical idiosyncratic wage risk move the model's labor market statistics in the same direction, although countercyclical risk has a smaller impact. In particular, the increase in the volatility of the labor wedge and the reduction in the hours-productivity correlation under acyclical risk are about twice as large as those under countercyclical risk. Hence, variation in idiosyncratic wage risk and its independence from aggregate TFP play roughly equal roles in the shaping the labor market dynamics of the calibrated varying risk model in Section 5.

7 Conclusion

This paper examined how cyclical variation in idiosyncratic wage risk affects business cycles, in particular, labor market dynamics. The model analyzed here is based on the framework commonly used in labor market analyses and includes indivisible labor and realistic heterogeneity in wealth and wages across individuals. I found that changes in idiosyncratic wage uncertainty lead to heterogeneous employment responses among different wage groups. As a result, at the aggregate level, total hours worked and average labor productivity move persistently in opposite directions. When moved by shocks to idiosyncratic wage uncertainty and aggregate TFP, the calibrated model successfully reproduces the large volatility in the labor wedge and the weakly negative hours-productivity correlation seen in the U.S. data. Without uncertainty shocks, the success is lost. These findings indicate that variation in idiosyncratic wage uncertainty is an important driver for the U.S. labor market fluctuations.

References

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109(3), 659–684.

Alonso-Ortiz, J., and R. Rogerson (2010): "Taxes, Transfers and Employment in an Incomplete Markets Model," *Journal of Monetary Economics*, 57(8), 949–958.

Arellano, C., Y. Bai, and P. J. Kehoe (2012): "Financial Frictions and Fluctuations in Volatility," *Federal Reserve Bank of Minneapolis Research Department Staff Report*.

Bachmann, R., and C. Bayer (2013): "'Wait-and-See' Business Cycles?," *Journal of Monetary Economics*, 60(6), 704–719.

Benhabib, J., R. Rogerson, and R. Wright (1991): "Homework in Macroeconomics: Household Production and Aggregate Fluctuations," *Journal of Political Economy*, 99(6), 1166–1187.

Bloom, N. (2009): "The Impact of Uncertainty Shocks," *Econometrica*, 77(3), 623–685.

Bloom, N., M. Floetotto, N. Jaimovich, I. Saporta-Eksten, and S. J. Terry (2012): "Really Uncertain Business Cycles," *NBER Working Paper*, 18245.

Card, D. (1999): "The Causal Effect of Education on Earnings," *Handbook of Labor Economics*.

Chang, Y., and S.-B. Kim (2006): "From Individual to Aggregate Labor Supply: A Quantitative Analysis Based On A Heterogeneous Agent Macroeconomy," *International Economic Review*, 47(1), 1–27.

——— (2007): "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations," *American Economic Review*, 97(5), 1939–1956.

——— (2014): "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations: Reply," *American Economic Review*, 104(4), 1461–1466.

Chari, V., P. J. Kehoe, and E. R. McGrattan (2007): "Business Cycle Accounting," *Econometrica*, 75(3), 781–836.

Christiano, L. J., and M. Eichenbaum (1992): "Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations," *American Economic Review*, 82(3), 430–450.

Cociuba, S. E., E. C. Prescott, and A. Ueberfeldt (2009): "U.S. Hours and Productivity Behavior Using CPS Hours Worked Data: 1947-III to 2009-III," mimeo.

Cooley, T. F., and E. C. Prescott (1995): "Economic Growth and Business Cycles," in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley. Princeton University Press.

Díaz-Giménez, J., V. Quadrini, and J.-V. Ríos-Rull (1997): "Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income, and Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, 21(2), 3–21.

Flodén, M. (2006): "Labor Supply and Saving under Uncertainty," *Economic Journal*, 116(513), 721–737.

Gali, J., and L. Gambetti (2009): "On the Sources of the Great Moderation," *American Economic Journal: Macroeconomics*, 1(1), 26–57.

Hansen, G. D. (1985): "Indivisible Labor and the Business Cycle," *Journal of Monetary Economics*, 16(3), 309–327.

Heathcote, J., F. Perri, and G. L. Violante (2010): "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967–2006," *Review of Economic Dynamics*, 13, 15–51.

Karabarbounis, L. (2014): "The Labor Wedge: MRS vs. MPN," *Review of Economic Dynamics*, 17(2), 206–223.

Khan, A., and J. K. Thomas (2003): "Nonconvex Factor Adjustments in Equilibrium Business Cycle Models: Do Nonlinearities Matter?," *Journal of Monetary Economics*, 50(2), 331–360.

——— (2007): "Inventories and the Business Cycle: An Equilibrium Analysis of (S,s) Policies," *American Economic Review*, 97(4), 1165–1188.

——— (2008): "Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics," *Econometrica*, 76(2), 395–436.

Krusell, P., T. Mukoyama, A. Şahin, and A. A. Smith, Jr. (2009): "Revisiting the Welfare Effects of Eliminating Business Cycles," *Review of Economic Dynamics*, 12, 393–404.

Krusell, P., T. Mukoyama, R. Rogerson, and A. Şahin (2010): "Aggregate Labor Market Outcomes: The Roles of Choice and Chance," *Quantitative Economics*, 1(1), 97–127.

——— (2011): "A Three State Model of Worker Flows in General Equilibrium," *Journal of Economic Theory*, 146(3), 1107–1133.

Krusell, P., and A. A. Smith, Jr. (1997): "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns," *Macroeconomic Dynamics*, 1(2), 387–422.

——— (1998): "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, 106(5), 867–896.

——— (1999): "On the Welfare Effects of Eliminating Business Cycles," *Review of Economic Dynamics*, 2(1), 245–272.

Kydland, F. E., and E. C. Prescott (1982): "Time to Build and Aggregate Fluctuations," *Econometrica*, 50(6), 1345–1370.

Lopez, J. I. (2010): "Consumption and Labor Income Risk, Aggregation and Business Cycles," mimeo.

Marcet, A., F. Obiols-Homs, and P. Weil (2007): "Incomplete Markets, Labor Supply and Capital Accumulation," *Journal of Monetary Economics*, 54(8), 2621–2635.

Mukoyama, T., and A. Şahin (2006): "Costs of Business Cycles for Unskilled Workers," *Journal of Monetary Economics*, 53(8), 2179–2193.

Parker, S. C., Y. Belghitar, and T. Barmby (2005): "Wage Uncertainty and the Labour Supply of Self-Employed Workers," *Economic Journal*, 115(502), C190–C207.

Pijoan-Mas, J. (2007): "Pricing Risk in Economies with Heterogeneous Agents and Incomplete Markets," *Journal of the European Economic Association*, 5(5), 987–1015.

Rogerson, R. (1988): "Indivisible Labor, Lotteries and Equilibrium," *Journal of Monetary Economics*, 21(1), 3–16.

Schaal, E. (2012): "Uncertainty, Productivity, and Unemployment in the Great Recession," mimeo.

Shimer, R. (2010): *Labor Markets and Business Cycles*. Princeton University Press, New Jersey.

Storesletten, K., C. I. Telmer, and A. Yaron (2001): "The Welfare Cost of Business Cycles Revisited: Finite Lives and Cyclical Variation in Idiosyncratic Risk," *European Economic Review*, 45(7), 1311–1339.

——— (2004): "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *Journal of Political Economy*, 112(3), 695–717.

——— (2007): "Asset Pricing With Idiosyncratic Risk and Overlapping Generations," *Review of Economic Dynamics*, 10(4), 519–548.

Swanson, E. T. (2007): "Real Wage Cyclicality in the Panel Study of Income Dynamics," *Scottish Journal of Political Economy*, 54(5), 617–647.

Takahashi, S. (2014): "Heterogeneity and Aggregation: Implications for Labor-Market Fluctuations: Comment," *American Economic Review*, 104(4), 1446–1460.

Tauchen, G. (1986): "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 20(2), 177–181.

8 Appendix A: Data

8.1 A1: Individual Wage Data

I take data for heads of households from the family-level file of the PSID. Individual wages are the ratios of annual labor income (1975: V3863–1992: V21484) to annual hours worked (1975: V3823–1992: V20344), converted to real wages in terms of 1983 dollars using the CPI data.[24,25] I only use interview years of 1975–1992 because data on years of education are discontinuous in 1974.[26]

I also exclude the following observations.

- Observations whose heads change in the year (1975: V4114–1992: V21388).
- Observations with major assignments assigned to the labor income and/or hours.[27]
- Observations with wages of less than one dollar (in 1983 dollars) or higher than 500 dollars (in 1983 dollars).
- The most recent Latino sample and the Survey of Economic Opportunity sample.
- Observations with fewer than 100 annual hours.
- Self-employment observations (1975: V3970–1992: V20696).

[24] Numbers in parentheses are variable labels of the PSID.

[25] I take Monthly "Consumer Price Index for All Urban Consumers: All Items" from the FRED database at the Federal Reserve Bank of St. Louis. The annual CPI in a year is the average monthly CPI in the year.

[26] I exclude the interview year of 1993 because data on major assignment for labor income are not available.

[27] See Swanson (2007) for details. For labor hours, I use the total hour accuracy (1975: V3824–1984: V10038) until 1984, and after 1984, the main job hour accuracy (1985: V11141–1992: V20339), the overtime hour accuracy (1985: V11143–1992: V20341), and the extra job accuracy (1985: V11145–1992: V20343). For labor income, I use the accuracy code for wages and salaries (1975: V3859–1992: V20430) and the accuracy code for labor income except wages and salaries (1975: V3864–1992: V20435).

- Observations in the agricultural sectors (1975: V3968–1992: V20701).
- Top-coded observations for income.

Other variables are age (1975: V3921–1992: V20651), sex (1975: V3922–1992: V20652), marital status (1975: V4053–1992: V21522), and the number of children (1975: V3924–1992: V20654). For years of education, I select "Grade Completed" (1975: ER30169–1992: ER30748) from the individual-level file and define experience as age minus education minus six.

8.2 A2: Individual Labor Income Data

The 1992 PSID individual-level file provides annual labor income data in 1991 for individuals, including those other than heads of households. I take total labor income (ER30750), excluding individuals younger than 16 (ER30736) and individuals with major assignments on their income and/or hours worked (ER30751, ER30755).

8.3 A3: U.S. Macroeconomic Data

The data period is from 1947Q3 to 2009Q3. Output is "Real Gross Domestic Product (billions of chained 2005 dollars)" taken from Table 1.1.6 of the Bureau of Economic Analysis (BEA). Consumption is "Personal Consumption Expenditures (PCE)" less durable goods obtained from Table 2.3.5 of the BEA. Investment is the sum of durable goods consumption in Table 2.3.5 and private fixed investment (including residential investment) in Table 5.3.5. I compute the real values of consumption and investment using the price index for Gross Domestic Product in Table 1.1.4. The data on total labor hours are the data constructed by Cociuba, Prescott, and Ueberfeldt (2009).[28]

[28] I am grateful to the authors for making the data available.

9 Appendix B: Solution Methods

9.1 B1: Steady State

The solution method for the steady state is similar to that of Chang and Kim (2007).

1. Discretize the idiosyncratic state (k, x). Set 100 log-spaced points over [–2, 250] for k. For x, set 17 evenly spaced points over $[-3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}$, $3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}]$ and compute the transition matrix using the method of Tauchen (1986).

2. Set a guess for the discount factor β.

3. Solve the individual optimization problem and obtain the beginning-of-period value function $V(k, x)$. I omit the aggregate state $(z, \mu, \sigma_{\varepsilon_x})$, which is constant at the steady state.

   (a) Compute the steady-state wage rate $\bar{w} = \alpha\bar{z}((1-\alpha)\bar{z}/(\bar{r}+\delta))^{(1-\alpha)/\alpha}$ with the target steady-state rental rate of capital $\bar{r} = 0.01$ and the steady-state aggregate TFP $\bar{z} = 1.0$.

   (b) Set a guess for the beginning-of-period value function $V_0(k, x)$.

   (c) Solve the consumption-saving problem for each employment choice:

$$V_1^E(k,x) = \max_{k' \geq \bar{k}}\{u(w\bar{h}x + (1+r)k - k', \bar{h}) + \beta\sum_{x'}\pi_x(x'|x)V_0(k', x')\}$$

   and

$$V_1^N(k,x) = \max_{k' \geq \bar{k}}\{u((1+r)k - k', 0) + \beta\sum_{x'}\pi_x(x'|x)V_0(k', x')\},$$

   where $\pi_x(x'|x)$ is the transition probability from x to x'. Use cubic spline interpolation to approximate the conditional expectation at

k' off the grid points. If $V_1^E(k,x) \geq V_1^N(k,x)$, then individuals with k and x choose to work. Otherwise, they do not work. Set $V_1(k,x) = \max\left\{V_1^E(k,x), V_1^N(k,x)\right\}$.

(d) If $V_1(k,x)$ becomes sufficiently close to $V_0(k,x)$, then set $V(k,x) = V_1(k,x)$ and proceed to the next step. Otherwise, update the value function as $V_0(k,x) = V_1(k,x)$ and return to (c).

4. Compute the steady-state distribution $\bar{\mu}(k,x)$.

   (a) Choose points used for approximating the distribution. Use 2,000 log-spaced points over $[-2, 250]$ for k and the points chosen in Step 1 for x.

   (b) Replace $V_0(k,x)$ of the problems in Step 3 (c) with $V(k,x)$ obtained in Step 3 (d). Solve the problems this time for $2,000 \times 17$ pairs of (k,x) and find their optimal asset holding $k'(k,x)$ and employment $h(k,x)$.

   (c) Suppose $k_m \leq k'(k,x) < k_{m+1}$, where k_m and k_{m+1} are two sequential asset points. Starting from an initial guess, keep updating the distribution until the distribution converges as follows: Individuals with (k,x) move to (k_m, x') with probability $\omega\pi_x(x'|x)$ and to (k_{m+1}, x') with probability $(1-\omega)\pi_x(x'|x)$, where $\omega = (k_{m+1} - k')/(k_{m+1} - k_m)$. The result is the steady-state distribution $\bar{\mu}(k,x)$.

5. Compute the steady-state aggregate capital $\bar{K} = \displaystyle\int k\bar{\mu}([dk \times dx])$ and aggregate efficiency-weighted labor $\bar{L} = \displaystyle\int xh(k,x)\bar{\mu}([dk \times dx])$. Calculate the implied steady-state rental rate of capital $\bar{r} = (1-\alpha)\bar{z}\bar{K}^{-\alpha}\bar{L}^{\alpha} - \delta$. If $\bar{r}$ becomes sufficiently close to the target rate (1 percent), then stop. Otherwise, set a different value for β and repeat Steps 3–5.

9.2 B2: Business Cycles

I analyze the model's business cycle generalizing the Krusell and Smith (1997, 1998) algorithm. The method is similar to that used in Takahashi (2014).

1. Discretize the aggregate state $(z, \mu, \sigma_{\epsilon_x})$. For aggregate TFP z, set nine evenly spaced points over $[-3\bar{\sigma}_{\varepsilon_z}/\sqrt{1-\rho_z^2}, 3\bar{\sigma}_{\varepsilon_z}/\sqrt{1-\rho_z^2}]$, and compute the transition matrix using the method of Tauchen (1986). Replace the individual distribution μ with aggregate capital K. Use seven evenly spaced points over $[0.80\bar{K}, 1.20\bar{K}]$, where $\bar{K}(= 11.57)$ is the steady-state aggregate capital. For σ_{ϵ_x}, use the three risk states.

2. Discretize the individual state (k, x). For k, use the 100 points chosen in the steady-state solution. For x, use 17 evenly spaced points over $[-3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}, 3\bar{\sigma}_{\varepsilon_x}/\sqrt{1-\rho_x^2}]$ for all of the risk states. The transition probabilities vary with the risk states. Compute these probabilities using the method of Tauchen (1986).

3. Individuals forecast K' and w using the following rules:

$$\ln \hat{K}' = a_{0,i} + a_{1,i} \ln K + a_{2,i} \ln z \tag{12}$$

   and

$$\ln \hat{w} = b_{0,i} + b_{1,i} \ln K + b_{2,i} \ln z, \tag{13}$$

   for each risk state $(i = H, M, L)$. Individuals compute $\hat{r} = z(1-\alpha)(\hat{w}/(\alpha z))^{-\alpha/(1-\alpha)}$.

4. Solve the individual optimization problem and obtain the beginning-of-period value function $V(k, x; z, K, \sigma_{\epsilon_x})$.

   (a) Set a guess for the beginning-of-period value function $V_0(k, x; z, K, \sigma_{\epsilon_x})$.

   (b) Solve the consumption-saving problem for each employment choice:

$$V_1^E(k,x;z,K,\sigma_{\epsilon_x}) = \max_{k^{'}\geq\bar{k}}\{u(\hat{w}\bar{h}x + (1+\hat{r})k - k^{'}, \bar{h})$$
$$+\beta\sum_{x^{'}}\sum_{z^{'}}\sum_{\sigma^{'}_{\epsilon_x}}\pi_x(x^{'}|x,\sigma_{\epsilon_x})\pi_z(z^{'}|z)\pi_{\sigma_{\epsilon_x}}(\sigma^{'}_{\epsilon_x}|\sigma_{\epsilon_x})V_0(k^{'},x^{'};z^{'},\hat{K}^{'},\sigma^{'}_{\epsilon_x})$$

and

$$V_1^N(k,x;z,K,\sigma_{\epsilon_x}) = \max_{k^{'}\geq\bar{k}}\{u((1+\hat{r})k - k^{'}, 0)$$
$$+\beta\sum_{x^{'}}\sum_{z^{'}}\sum_{\sigma^{'}_{\epsilon_x}}\pi_x(x^{'}|x,\sigma_{\epsilon_x})\pi_z(z^{'}|z)\pi_{\sigma_{\epsilon_x}}(\sigma^{'}_{\epsilon_x}|\sigma_{\epsilon_x})V_0(k^{'},x^{'};z^{'},\hat{K}^{'},\sigma^{'}_{\epsilon_x}),$$

where $\pi_x(x^{'}|x,\sigma_{\epsilon_x})$ is the transition probability from x to $x^{'}$ under σ_{ϵ_x}, $\pi_z(z^{'}|z)$ is the transition probability from z to $z^{'}$, and $\pi_{\sigma_{\epsilon_x}}(\sigma^{'}_{\epsilon_x}|\sigma_{\epsilon_x})$ is the transition probability from σ_{ϵ_x} to $\sigma^{'}_{\epsilon_x}$. Use bivariate cubic spline interpolation in (K,k) to approximate the conditional expectation at $(\hat{K}^{'},k^{'})$ off their grid points. If $V_1^E(k,x;z,K,\sigma_{\epsilon_x}) \geq V_1^N(k,x;z,K,\sigma_{\epsilon_x})$, individuals with k and x work. Otherwise, they do not. Set $V_1(k,x;z,K,\sigma_{\epsilon_x}) = \max\{V_1^E(k,x;z,K,\sigma_{\epsilon_x}), V_1^N(k,x;z,K,\sigma_{\epsilon_x})\}$.

(c) If $V_1(k,x;z,K,\sigma_{\epsilon_x})$ becomes sufficiently close to $V_0(k,x;z,K,\sigma_{\epsilon_x})$, then proceed to the next step, setting $V(k,x;z,K,\sigma_{\epsilon_x}) = V_1(k,x;z,K,\sigma_{\epsilon_x})$. Otherwise, update the value function as $V_0(k,x;z,K,\sigma_{\epsilon_x}) = V_1(k,x;z,K,\sigma_{\epsilon_x})$, and return to (b).

5. Generate 3,500-period data using the beginning-of-period value function $V(k,x;z,K,\sigma_{\epsilon_x})$.

   (a) Set conditions for the initial period: $z_1 = \bar{z}, \sigma_{\epsilon_x 1} = \sigma_{\epsilon_x,M}, \mu_1(k,x) = \bar{\mu}(k,x)$, and $K_1 = \int k\mu_1([dk \times dx])$.

   (b) Set $\tilde{w}_1$, as a guess for w_1. Then, $\tilde{r}_1 = (1-\alpha)z_1(\tilde{w}_1/\alpha z_1)^{-\alpha/(1-\alpha)} - \delta$. The forecasting rule gives the individuals' forecast of the next period approximate aggregate state $\hat{K}_2$. Replacing $V_0(k, x; z, K, \sigma_{\epsilon_x})$ with $V(k, x; z, K, \sigma_{\epsilon_x})$, solve the individual problems shown in Step 4 (b) under $w = \tilde{w}_1$, $r = \tilde{r}_1$, and $K^{'} = \hat{K}_2$, this time for $2,000 \times 17$ pairs of (k, x). Record the optimal asset holding $k_2(k, x)$ and employment $h_1(k, x)$.

(c) Check labor market clearing: $\tilde{L}_1 \equiv (\alpha z_1/\tilde{w}_1)^{1/(1-\alpha)} K_1 = \int x h_1(k, x)\mu_1([dk \times dx])$. If the labor market clears, proceed to the next step. Otherwise, reset $\tilde{w}_1$ and return to (b).[29]

(d) Compute aggregate variables: $L_1 = \int x h_1(k, x)\mu_1([dk \times dx]), K_2 = \int k_2(k, x)\mu_1([dk \times dx]), H_1 = \int h_1(k, x)\mu_1([dk \times dx]), Y_1 = z_1 K_1^{1-\alpha} L_1^{\alpha}$, $I_1 = K_2 - (1-\delta)K_1, C_1 = Y_1 - I_1$, and $r_1 = (1-\alpha) z_1 K_1^{-\alpha} L_1^{\alpha} - \delta$.

(e) Obtain the next period distribution $\mu_2(k, x)$ as described in Step 4 (c) of the steady-state solution.

(f) Repeat (b)–(e) for 3,500 periods.

6. Using the simulated data (disregarding the first 500 periods), update the coefficients of the forecasting rules by ordinary least squares. If these coefficients converge, then proceed to the next step. Otherwise, repeat Steps 4 and 5 using the new forecasting rules.

7. Check whether the converged forecasting rules are sufficiently accurate. If not, assume different functional forms and repeat Steps 3–6. The forecasting rules of (11) and (12) are quite accurate, as reported in Appendix C.

10 Appendix C: Forecasting Rules

The tables below list the coefficients of the forecasting rules ($\ln \hat{K}^{'} = a_0 + a_1 \ln K + a_2 \ln z$ and $\ln \hat{w} = b_0 + b_1 \ln K + b_2 \ln z$) and the accuracy of the

[29] Ensuring market clearing is an essential step of the Krusell and Smith (1998) algorithm and included for the bond market by Krusell and Smith (1997) and Pijoan-Mas (2007), for the goods market by Khan and Thomas (2003, 2007, 2008), and for the labor market by Takahashi (2014).

rules. Two accuracy measures are the coefficient of determination R^2 and the standard deviation of the forecasting error $\hat{\sigma}$. I use separate rules for each of the risk states.

		Constant risk	Varying risk (H / M / L)	Psych risk (H / M / L)
$\hat{K}'$	a_0	0.115	0.073 / 0.081 / 0.080	0.116 / 0.114 / 0.115
	a_1	0.953	0.971 / 0.967 / 0.967	0.953 / 0.954 / 0.953
	a_2	0.101	0.082 / 0.086 / 0.093	0.101 / 0.100 / 0.102
	R^2	1.000	1.000 / 1.000 / 1.000	1.000 / 1.000 / 1.000
	$\hat{\sigma}$	0.0079%	0.1122% / 0.0939% / 0.0969%	0.0101% / 0.0106% / 0.0107%
$\hat{w}$	b_0	–0.209	0.031 / –0.027 / –0.024	–0.209 / –0.209 / –0.209
	b_1	0.438	0.338 / 0.364 / 0.365	0.438 / 0.438 / 0.439
	b_2	0.818	0.928 / 0.900 / 0.859	0.818 / 0.818 / 0.815
	R^2	1.000	0.972 / 0.982 / 0.972	1.000 / 1.000 / 1.000
	$\hat{\sigma}$	0.0407%	0.6154% / 0.5032% / 0.5415%	0.0456% / 0.0503% / 0.0452%

		Countercyclical risk (H / M / L)	Recalibrated varying risk (H / M / L)
$\hat{K}'$	a_0	0.245 / 0.204 / 0.229	0.096 / 0.099 / 0.091
	a_1	0.900 / 0.917 / 0.906	0.961 / 0.960 / 0.963
	a_2	0.093 / 0.100 / 0.106	0.091 / 0.096 / 0.093
	R^2	0.999 / 0.999 / 0.999	1.000 / 1.000 / 1.000
	$\hat{\sigma}$	0.0401% / 0.0418% / 0.0423%	0.0728% / 0.0592% / 0.0773%
$\hat{w}$	b_0	–0.988 / –0.722 / –0.855	–0.102 / –0.125 / –0.078
	b_1	0.756 / 0.648 / 0.703	0.393 / 0.404 / 0.387
	b_2	0.848 / 0.817 / 0.793	0.873 / 0.847 / 0.862
	R^2	0.974 / 0.983 / 0.981	0.989 / 0.990 / 0.986
	$\hat{\sigma}$	0.2518% / 0.2366% / 0.2291%	0.3958% / 0.3123% / 0.4054%