Predicting Recessions

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1 Introduction

There are probably few things that businessmen, journalists and politicians agree on. However, an exception might well be that economists can't forecast recessions. Many reasons have been given for this. One is that the economy is complex. Many systems such as weather are also complex, but the record of meteorologists seems quite good. Another is that there are too few recessions, so one is faced with a small number of events. Now there are more bear equity markets than recessions, yet the extra observations available on these does not seem to have produced a more succesful prediction record of stock market crashes. Clearly, the answer must lie somewhere else.

It is also the case that a large number of papers appear every year that claim to be able to forecast recessions with either some new model or some new estimation method e.g. recent ones are Kaufman (?) and Galvao (?). Evidence is presented which seems to substantiate this. How then does one reconcile the existence of such a literature with the sceptism noted above?. It seems to us that the resolution to this discrepency stems from the very nature of the definition of a recession and this paper aims to flesh out that observation. Understanding why it is so difficult to predict a recession leads to an appreciation of the barriers to be faced in this task and also suggests that many of the claims made about how the forecasting record can be improved should be treated with scepticism. This is not to deny that some of these suggestions might improve our understanding of business cycle issues, even if they do not improve the forecasting capacity. Moreover, being aware of the limits to forecasting suggests that we should focus our research on questions that we might have a better chance of addressing e.g. whether it is possible to predict how long a recession will last once it is initiated.

In the next section we give a definition of a recession that revolves around isolating peaks and troughs in a series that represents economic activity. Although our presentation will concentrate upon quarterly data it can be extended to monthly series, although there is little to be gained for an understanding of the prediction issues from doing so. Section 3 then uses that definition to explain why it is unlikely that recessions can be predicted in either Turkey or the US utilizing most currently available data sets. Any improvement will have to come from access to information that captures the future shocks that are likely to affect the economy. At this stage we slightly modify our focus away from predicting recessions *per se* and ask what is the probability of predicting negative growth one period ahead? Because a recession gets inititiated with a period of negative growth, if such an event cannot be predicted it will be hard to predict a recession which involves studying a sequence of signs of future growth rates. We find it useful to concentrate on this simpler event.

Section 4 then asks whether the problem lies in the perspective taken in section 3. Essentially this views growth in economic activity as being determined linearly by the state of the economy and past growth rates. We allow for various types of non-linearity. In particular a Markov Switching model is fitted to Turkish growth. Using this model to produce forecasts it is found that there is even less ability to predict future negative growth, unless a negative growth rate has already been observed.

Section 5 investigates what other information might be useful in predicting future growth rates. Here we focus upon the US as there is a substantial literature on that question, some of which claims success for variables such as the yield spread. For Turkey only one series seems to have been suggested as useful in predicting future GDP growth - capacity utilization - but that did not provide any substantial extra predictive power. Some of the US literature also uses a range of variables that might be taken as indicators of future developments, and then incorporates them into multivariate models that might be utilized to predict future events. One that has received some attention is the Qual-VAR of Dueker (2005). We discuss some econometric problems with this paper, and its use in forecasting the 2001 recession, which makes us feel that the claims for its effectiveness are perhaps exaggerated.

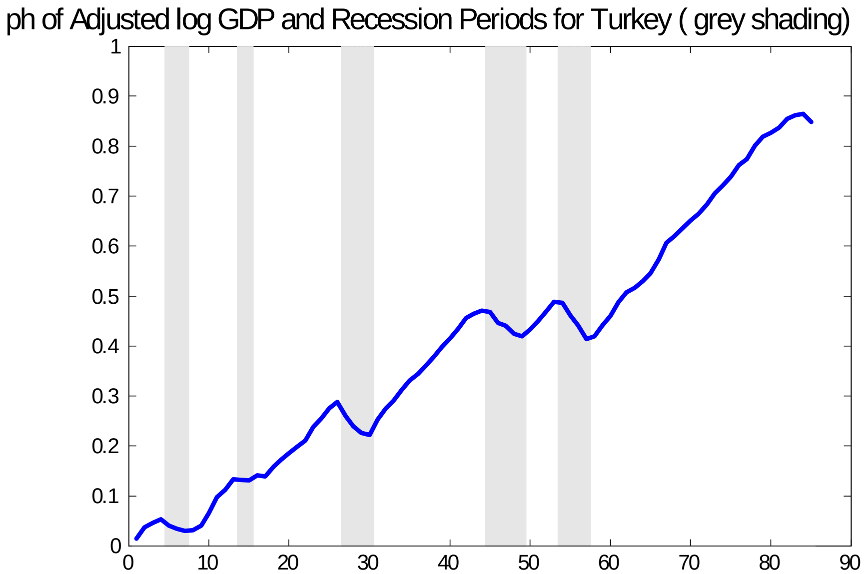
Finally, section 6 looks at how one defines a "recession event" in contrast to a recession. Some papers in the literature claim to have an impressive record of predicting recessions whereas they are actually predicting what we will term a "recession-derivative indicator" (RDI). We show that there are two effects of switching to an RDI perspective. Firstly, the unconditional probability of encountering an RDI event is much higher than that of a recession - often twice as high- and what may look like predictive success is simply an artifact of the definition. Secondly, the timing of the origin of an RDI event is very different to that of a recession. Mostly the event happens before a recession and so it may look as if one has managed to predict recessions in advance. Again, this is an artifact of the definition.

2 Recognizing a Recession

Figure 1 shows the log of Turkey's real GDP, y_t, after we have performed a seasonal adjustment on it[1].

For graphical purposes the data has been mean corrected and .4 added

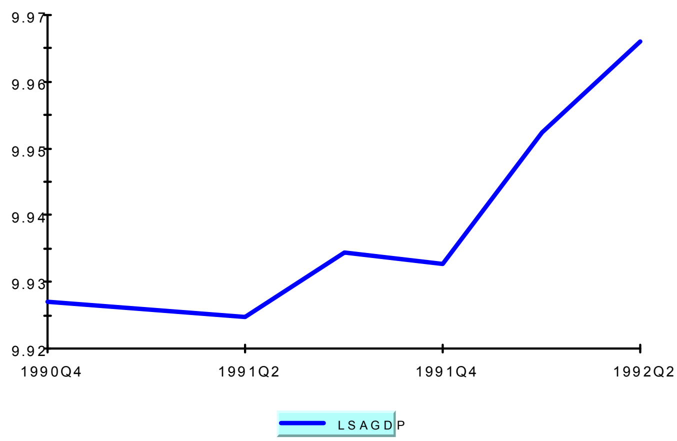
[1] With y(t) being the log of GDP the data used in our analysis is the average (y(t)+y(t-1)+y(t-2)+y(t-3))/4, which is known to eliminate an evolving seasonal pattern. Other methods of seasonal adjustment such as X11-ARIMA might be employed but this method is simple and isolates the business cycle quite well.



on so as to keep the series between zero and unity. There are four obvious recessions. The one between 1990:4 and 1991:2 is the least striking. Fig 2 shows it was a shallow recession and one in which there was not a smooth rise from the trough in 1991:2.

Now rather than look at the pictures to decide where the turning points are we can automate their determination. A *peak* marks the end of an expansion and a trough the end of a recession. A program that we use to date quarterly series like this is the BBQ program which derives from the philosophy set out in Bry and Boschan (1983) and underlies much of the NBER business cycle dating philosophy. BBQ is a WYSIWYG program, as evidenced by putting the turning points identified by BBQ on the graph of the log of Turkish GDP - see figure 1 (the grey areas being BBQ define recessions). Note that the shallow recession we have just talked about was identifed by BBQ. Indeed, following our normal strategy, we ran BBQ on this data first and then just visually verified the outcomes identified by the program. So the first point to make is that BBQ isolates turning points in a series representing economic activity. It is worth noting that we could have just used the original GDP data rather than the log of it to locate the turning points as they are the same in both series due to log being a monotonic transformation, but it is more convenient to work with logs, as the changes

Fig 2 The Early 90s Recession



in that series are approximately growth rates. Given the fact that BBQ reliably finds turning points we can think more formally about how one detects a recession by looking at the rules that are written into BBQ.

The rules that BBQ uses to locate a set of turning points are as follows.

1. A *peak* occurs at time t if y_t is greater than $\{y_{t-1}, y_{t-2}, y_{t+1}, y_{t+2}\}$. Thus 2000:4 is a peak since the values in $\{ y_{2000:2}, y_{2000:3}, y_{2000:4}, y_{2001:1}, y_{2001:2}\}$ are {10.2432, 10.2621, 10.2826, 10.2801, 10.2543}. Why choose two quarters on either side of the potential peak? The reason is the feeling that a recession (time between peak and a trough) should last for some minimal time, otherwise recessions will be called too often. By convention this has become 2 quarters (or five months if one uses monthly data)[2]. This could be changed if one wished. For Turkey it would matter only a little if one moved to one quarter as the minimum length of a recession, since there is just one period of negative growth in what one would most likely think of as an expansion (1991:4, see Figure 2). In countries such as the US and Australia it would matter a lot, as these countries often have a single quarter of negative growth in expansions (for the US examples are 1959:3 and 1977:4) while for Australia single quarters of negative growth happened in 1964:1, 1968:1, 1986:2, 1993:3, 2000:4 and 2003:1. The point is that a recession is an extreme event and so some convention needs to be established about how we recognize that

[2]The NBER Dating Committee uses the five monthly rule when finding the turning points in the US economy.

the behaviour of GDP is extreme enough. One might also apply some quantitative rules e.g. the decline in GDP has to be larger than some specified value. This might be used to eliminate the recession of 1990:4-1991:2 but, although this is sometimes done informally, it is not common. It should be noted that the BBQ rule does not coincide with that often used in the press that a recession is two consecutive periods of negative growth, nor rules that sometimes appear in the academic literature e.g. Fair (1993) has a recession occuring in time t if there are two consecutive negative growth rates in GDP in the five quarters that begin in t. We return to these alternative definitions in section 6.

2. There are other constraints that BBQ uses such as a minimal length for a complete cycle i.e. the period from a peak to peak, but these are of smaller importance and won't detain us here.

3. Once the turning points have been isolated it is possible to determine where recessions and expansions occurred. It is convenient to summarize this information by designating a series S_t that takes the value 1 when we are in an expansion and zero when we are in recession. Thus when we are concerned with predicting a recession at time t we will be asking what the chance is that $S_{t+1} = 0$.

4. The condition for a peak can be expressed in terms of growth rates. When that is done a peak at t occurs when $\{\Delta y_t > 0, \Delta_2 y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\}$, where $\Delta_2 y_t = y_t - y_{t-2} = \Delta y_t + \Delta y_{t-1}$ is six-monthly growth. Another way of expressing this is to adopt the conventional definition that a recession starts the period after a peak while an expansion begins the period after a trough - see Estrella and Trubin (2006). Using that perspective we can alternatively express a turning point as a change in state viz. $S_t = 1 \rightarrow S_{t+1} = 0$ if there is a peak at t. Thus, if $\{\Delta y_t > 0, \Delta_2 y_t > 0, \Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\}$, then we have a change from expansion to recession. If these conditions are not satisfied then we remain in the current state i.e. $S_t = 1 \rightarrow S_{t+1} = 1$. Thus to know if there has been a change in state we will need to know future information in the form of $\{\Delta y_{t+1} < 0, \Delta_2 y_{t+2} < 0\}$.

To look at this more formally we observe that the S_t generated by BBQ can be written in the recursive form

$$\begin{aligned} S_{t+1} &= S_t S_{t-1} \left[1 - \mathbf{1}\left(\Delta y_{t+1} \leq 0\right) \mathbf{1}\left(\Delta y_{t+1} + \Delta y_{t+2} \leq 0\right)\right] \\ &\quad + S_t \left(1 - S_{t-1}\right) \\ &\quad + \left(1 - S_t\right)\left(1 - S_{t-1}\right) \mathbf{1}\left(\Delta y_{t+1} > 0\right) \mathbf{1}\left(\Delta y_{t+1} + \Delta y_{t+2} > 0\right), \end{aligned} \tag{1}$$

where $\mathbf{1}(\mathcal{A}) = 1$ if $\mathcal{A}$ is true and zero otherwise.[3] We wish to predict S_{t+1}. To make this more concrete we will look at whether one could have predicted a recession happening in 2001:1. So $t+1$ is 2001:1, when we know the Turkish economy slipped into recession ($S_{t+1} = 0$) and t is 2000:4, when the economy was still in an expansion ($S_t = 1$). More generally, (1) points to the fact that to predict S_{t+1} we need to have some idea of S_t, S_{t-1} and the future signs of Δy_{t+1} and $\{\Delta y_{t+1} + \Delta y_{t+2}\}$. Thus we need to indicate what information is available when predicting S_{t+1}, and also how one is to predict S_t, S_{t-1}, and the future signs of Δy_{t+1},$\{\Delta y_{t+1} + \Delta y_{t+2}\}$ with that information.

It is worth looking at (1) when we are interested in predicting a recession given that, at time t, we are in an expansion i.e. $S_t = 1, S_{t-1} = 1$. Then it becomes

$$S_{t+1} = [1 - \mathbf{1}(\Delta y_{t+1} \leq 0)\, \mathbf{1}(\Delta y_{t+1} + \Delta y_{t+2} \leq 0)]$$

and

$$\begin{aligned}
\Pr(S_{t+1} &= 1|F_t) = [1 - E\{\mathbf{1}(\Delta y_{t+1} \leq 0)\, \mathbf{1}(\Delta y_{t+1} + \Delta y_{t+2} \leq 0)\,|F_t\}] \\
&\geq [1 - E\{\mathbf{1}(\Delta y_{t+1} \leq 0)\,|F_t\}] \\
&\therefore \Pr(S_{t+1} = 0) = 1 - \Pr(S_{t+1} = 1) \leq E\{\mathbf{1}(\Delta y_{t+1} \leq 0|F_t) = \Pr(\Delta y_{t+1} \leq 0|F_t),
\end{aligned}$$

where F_t is the information available at t (including $S_{t-1} = 1, S_t = 1$). So an upper bound to the probability of a recession at $t = 1$ is found by looking at $\Pr(\Delta y_{t+1} \leq 0|F_t)$. In Turkey the upper bound is also close to being attained and so it is simplest to study the event $\mathbf{1}(\Delta y_{t+1} \leq 0)$, rather than the joint event $\mathbf{1}(\Delta y_{t+1} \leq 0)\, \mathbf{1}(\Delta y_{t+1} + \Delta y_{t+2} \leq 0)$, and we will do that later. But it should be borne in mind that the joint event will be harder to predict than the single period of negative growth and for other countries this can be significant.

3 Predicting a Recession with State and GDP Growth Data

We are then ultimately interested in whether a recession can be predicted at time $t+1$ when we are at t i.e. in predicting whether $S_{t+1} = 0$. It is necessary to specify what information would have been available at time t that would be

[3]There is a small complication caused by completed cycles having a minimum duration of five quarters. Only occasionally does this constraint bite.

useful to predict the value of S_{t+1}.[4] To make this concrete position ourselves at 2000:4 and ask whether there will be a recession in 2001:1. To perfectly predict $S_{2001:1}$ we need to know the sign of the GDP growth rates in 2001:1 and 2001:2. If the growth rates were independent then knowing these past values will be of no use in predicting the future growth rates *per se*. Now in many countries there is very little persistence in growth rates of GDP e.g. the UK and Australia. But in Turkey there is quite strong first order serial correlation in growth rates of the order of .7. *Prima facie* this might look advantageous but we will see later that it is not.

Suppose we know that $S_t = 1$ and $S_{t-1} = 1$. Then

$$\begin{aligned} \Pr(S_{t+1} &= 0|F_t) = E\{\mathbf{1}\left(\Delta y_{t+1} \leq 0\right)\mathbf{1}\left(\Delta y_{t+1} + \Delta y_{t+2} \leq 0\right)|F_t\} \\ &= g(F_t) \end{aligned}$$

The functional relation $g(\cdot)$ will generally be non-linear for two reasons. One is that the conditional expectations will be non-linear in F_t as they must lie between zero and unity, but it also may be that Δy_{t+j} $(j = 1, 2)$ depends in a non-linear way upon F_t. In most instances $g(\cdot)$ will not be analytically derivable. If the number of elements in F_t is limited then one can use non-parametric methods to estimate $g(\cdot)$ as in Harding and Pagan (2010). Unlike that paper it is important to make the $g(\cdot)$ function monotonic, given that it is a probability, and Harding (2010) shows how one can adjust the non-parametric estimates to impose monotonicity in a reasonably simple way. Sometimes, for example if the mapping between Δy_{t+1} and Δy_{t+2} and F_t is linear, then one can evaluate $E\{\mathbf{1}\left(\Delta y_{t+1} \leq 0\right)\mathbf{1}\left(\Delta y_{t+1} + \Delta y_{t+2} \leq 0\right)|F_t\}$ by simulation methods. An example would be if Δy_t followed an $AR(1)$ process of the form

$$\Delta y_{t+1} = \mu + \rho\Delta y_t + \sigma\varepsilon_t, \qquad \varepsilon_t\tilde{\ } iid\, N\left(0, 1\right). \tag{2}$$

and $F_t = \Delta y_t$.

As foreshadowed earlier we will focus upon the ability to predict a negative growth rate i.e. $\Pr(\Delta y_{t+1} < 0|F_t)$. Figure 3 looks at the ability to predict $1(\Delta y_{t+1} < 0)$ when information Δy_t is available. It is necessary to estimate $E\{1(\Delta y_{t+1} < 0)|\Delta y_t)$ and this could be done non-parametrically but again for simplicity we will assume that $E\{1(\Delta y_{t+1} < 0)|\Delta y_t) = \Phi(\Delta y_t)$, where $\Phi(\cdot)$ is the c.d.f for the standard normal. Such a functional form appeals as it can be fitted with a Probit model.

[4] Although we will write Δy_t, S_{t-1}etc. as the available information we will mean all past values of these quantities.

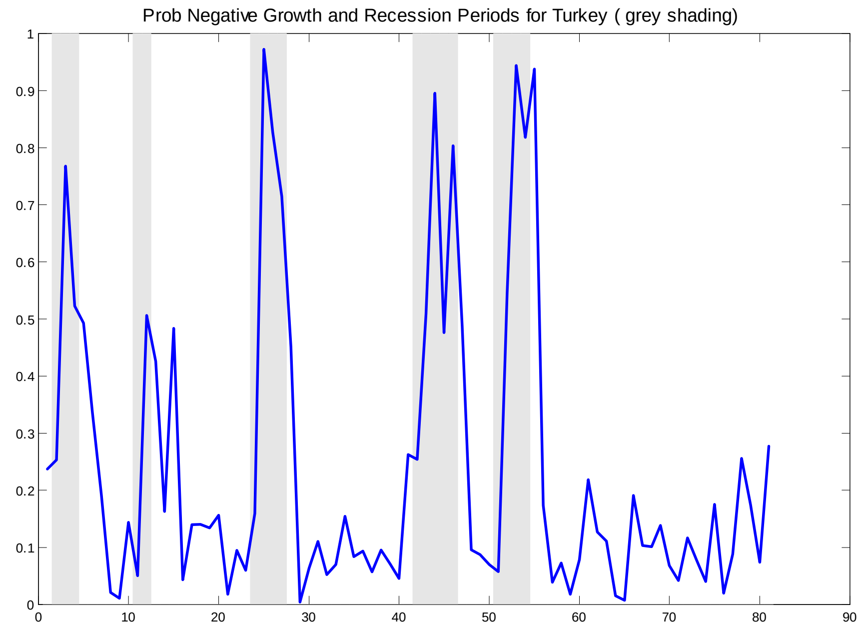


Table 1 shows the probabilities of predicting $1(\Delta y_{t+1} < 0|\Delta y_t)$ for the Turkish 2001 recession.

Table 1 : Probabilities of Predicting the Turkish 2001 Recession			
Prediction At/For	A		S_t
2000:4/2001:1	.06		0
2001:1/2001:2	.55		0
2001:2/2001:3	.94		0
2001:3/2001:3	.82		0
2001:4/2002:1	.94		1

.

This is a typical pattern - the first period of the recession is predicted with very low probability but it then rises as the recession gets underway. Thus at the time the recession emerges i.e. $S_{t+1} = 0$, we would have prediction probabilities for the various Turkish recessions of .25 (1988:3), .05 (1990:4), .16 (1994:1), .25 (1998:3) and .06 (2000:4). If we think that a critical value here is .5 (a fairly common choice) then none of the five recessions would have been predicted using the the most favourable information.[5] To put these numbers into context, since 21% of the time was spent in recession, if you just allocated a value of .21 every period you would generally be doing better than trying to exploit the information available.

Because a common element in all of these is $\Pr(\Delta y_{t+1} \leq 0)$ an examination of this is needed. Now the fact that there is strong positive serial correlation in GDP growth in Turkey militates against successfully predicting $\Delta y_{t+1} < 0$, since a positive growth in the previous period points towards it being positive again. Indeed, the correlation of $\psi_t = 1(\Delta y_{t+1} < 0)$ with ψ_{t-1} is .59.[6] Hence it is very difficult to predict negative growth coming out of an expansion and it is only after the recession has arrived that the strong dependence will make the probability of Δy_{t+1} <0 substantial. It is useful to see this visually and it will become a useful device for later analysis. Thus figure 4 shows a non-parametric estimate of the $\Pr(\Delta y_{t+1} < 0|\Delta y_t)$[7] . It is clear from this that, if there is positive growth in t , even if it is small, we would not generate a prediction probability that reaches .5. For small positive growth rates it is around .4.

However, it seems unlikely that one would know Δy_t. In practice we rarely know what the growth rate in the current quarter Δy_{t-1} is e.g. in Australia the best we would get would be GDP growth for $t-1$ in t. Even then this quantity can be subject to substantial revision and even a possible sign change. This has two consequences. One is that it will no longer be the case that S_t can be known to be unity. If it was the case that S_{t-1} was known to be unity, then a positive Δy_t would mean that $S_t = 1$, since the peak in y_t would not be at $t-1$. But if we don't know Δy_t then it might be negative. Since a negative growth can occur in an expansion, S_t could be either 0 or 1, will not be known and so we will need to predict this as well as $\Delta y_{t+j} (j = 1, 2)$.

To see the effect of only knowing Δy_{t-1} we consider $E\{1(\Delta y_{t+1}|\Delta y_t)\}$ and

[5] The issue of deciding on a threshold is a difficult one and something we will deal with in a later version of this paper. The choice raises similar issues to balancing Type 1 and Type 2 errors in hypothesis testing.

[6] Under a normality assumption for Δy_t Kedem(1980) gave an expression for the serial correlation coefficients of $1(\Delta y_{t+1} > 0)$ in terms of the serial correlation coefficients of Δy_t.

[7] The probability is identical to $E(1(\Delta y_{t+1} < 0)|\Delta y_t)$ given the binary nature of the event $1(\Delta y_{t+1} < 0)$ so we can estimate the probability with a non-parametric estimate of the conditional mean of $1(\Delta y_{t+1} < 0)$.

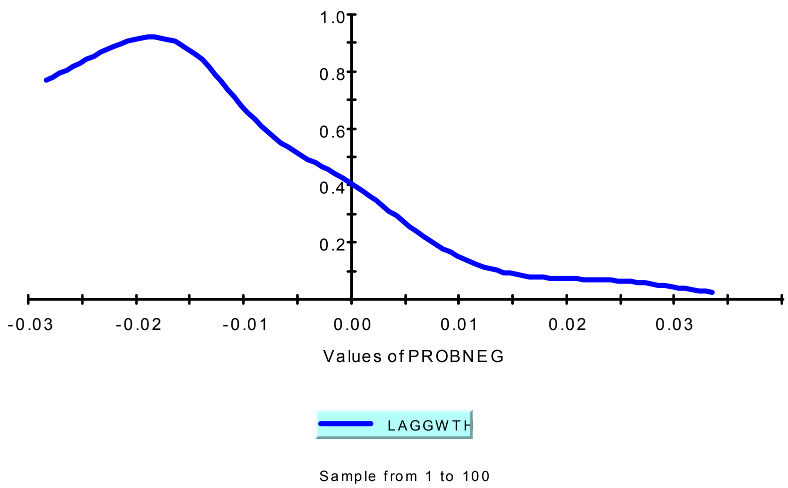


Figure 1: Plot of Non-Parametric Estimate of $\Pr(\Delta y_{t+1} < 0|\Delta y_t)$ against Δy_t

$E\{1(\Delta y_{t+1}|\Delta y_{t-1})\}$ one quarter into a recession. The probabilities for Turkish recessions are then $\{.77,.26\},\{.51,.12\},\{.97,.20\},\{.51,.25\},\{.55,.13\}$ respectively. The poor prediction record of the second set of information (Δy_{t-1}) comes from the fact that, even after the recession has started, we have not yet gained the information that there has been a negative growth rate. Thus the extra information has a significant impact on the ability to predict a recession. Of course we also need to predict what S_t is rather than assuming that $S_t = 1$. Thus we might think that at best the information available for predicting S_{t+1} would be S_t and Δy_t and at worst it would be $S_{t-2}, \Delta y_{t-1}$. This problem of trying to come up with the latest GDP growth outcome is often referred to as "now-casting".

When S_t is not known we need to make an estimate of it. To see how this is done lag (1) by one period to get

$$
\begin{aligned}
S_t &= S_{t-1}S_{t-2}\left[1 - \mathbf{1}\left(\Delta y_t \leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+1} \leq 0\right)\right] && (3)\\
&\quad + S_{t-1}\left(1 - S_{t-2}\right) && (4)\\
&\quad + \left(1 - S_{t-1}\right)\left(1 - S_{t-2}\right)\mathbf{1}\left(\Delta y_t > 0\right)\mathbf{1}\left(\Delta_2 y_{t+1} > 0\right)
\end{aligned}
$$

Substituting (3) into (1), after some re-arrangment we would obtain

$$\begin{aligned} S_{t+1} &= S_{t-1}S_{t-2}\left[1-\mathbf{1}\left(\Delta y_{t+1}\leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+2}\leq 0\right)\right]\left[1-\mathbf{1}\left(\Delta y_t\leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+1}\leq 0\right)\right] \\ &\quad +S_{t-1}\left(1-S_{t-2}\right)\left[1-\mathbf{1}\left(\Delta y_{t+1}\leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+2}<0\right)\right] \\ &\quad +\left(1-S_{t-1}\right)\left(1-S_{t-2}\right)\mathbf{1}\left(\Delta y_t>0\right)\mathbf{1}\left(\Delta_2 y_{t+1}<0\right) \\ &\quad +\left(1-S_{t-1}\right)\mathbf{1}\left(\Delta y_{t+1}>0\right)\mathbf{1}\left(\Delta_2 y_{t+2}>0\right) \\ &\quad -\left(1-S_{t-1}\right)\left(1-S_{t-2}\right)\mathbf{1}\left(\Delta y_{t+1}>0\right)\mathbf{1}\left(\Delta_2 y_{t+2}>0\right)\mathbf{1}\left(\Delta y_t>0\right)\mathbf{1}\left(\Delta_2 y_{t+1}>0\right) \end{aligned}$$

Hence if $S_{t-1}=1, S_{t-2}=1$ is in F_t we have

$$S_{t+1}=\left[1-\mathbf{1}\left(\Delta y_{t+1}\leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+2}\leq 0\right)\right]\left[1-\mathbf{1}\left(\Delta y_t\leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+1}\leq 0\right)\right]$$

and

$$\begin{aligned} \Pr(S_t &= 0|F_t)=E\{\mathbf{1}\left(\Delta y_{t+1}\leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+2}\leq 0\right)+\mathbf{1}\left(\Delta y_t\leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+1}\leq 0\right) \\ &\quad -\mathbf{1}\left(\Delta y_{t+1}\leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+2}\leq 0\right)\mathbf{1}\left(\Delta y_t\leq 0\right)\mathbf{1}\left(\Delta_2 y_{t+1}\leq 0\right)|F_t) \\ &= h(F_t). \end{aligned}$$

Clearly $h(\cdot)$ will be different to $g(\cdot)$. Again this can be evaluated numerically. A similar expression can be found if only S_{t-2} is known.

4 Predicting Recessions: Can Non-linear Models of GDP Growth Help?

The previous section drew attention to studying $\Pr(\Delta y_{t+1}<0|F_t)$ as a first test of the ability to predict a recession. So far we have assumed that there is effectively a linear model connecting Δy_t and past growth in GDP. One might allow Δy_t to also depend upon the state of the economy at $t-j$,S_{t-j}, as this is often mentioned as a possibility. Of course, since S_{t-j} depends on growth rates in GDP, one could assert that all that is needed is observable growth rates. But this ignores the fact that S_t is a parsimonious summary of these and that it also introduces some non-linear structure through the fact that S_t depends on the sign of the growth rate and not the magnitude. Fitting a Probit model to $1(\Delta y_{t+1}<0)$, with explanatory variables Δy_{t-1} and S_{t-1} suggests that there is little separate influence of S_{t-1}.

An alternative modification is to allow for a non-linear model to determine growth in economic activity. Specifically, we will consider whether GDP growth depends in a non-linear way on its past history. Many non-linear models for Δy_t have been proposed, and often one sees comments that these produce better forecasts of GDP growth than linear models. A popular one that is used in a lot in the business cycle literature is that of a Hidden Layer Markov Chain, introduced into econometrics by Hamilton (1979). This is often given the shortened descriptor of a Markov Switching (MS) model, with the simplest variant having the form

$$\Delta y_t = \mu_t + \beta \Delta y_{t-1} + \sigma \varepsilon_t \quad (5)$$

$$\mu_t = \mu_1 \xi_t + (1 - \xi_t)\mu_0 \quad (6)$$

$$p_{ij} = \Pr(\xi_t = i | \xi_{t-1} = j), \quad (7)$$

where ξ_t is a binary random variable that follows a first order Markov process with transition probabilities p_{ij} and ε_t is $n.i.d(0,1)$. More complicated models are available but we doubt that these improve the recession predictions - see for example the discussion in Engel et al (2005). The MS model in (5) – (7) was estimated for Turkey, producing the results in Table 2[8]

Table 2 Estimated Parameters of MS Model for Turkey

	est	t
β	.275	2.5
μ_1	1.215	4.2
μ_0	-1.013	-2.4
p_{11}	.92	5.1
p_{01}	.26	2.0
p_{10}	.08	2.0
p_{00}	.74	1.8
σ^2	.625	5.4

The probability of getting $\Delta y_{t+1} < 0$ given Δy_t from this model was found by simulation to be .36 for small positive values of Δy_t. Comparing this to figure 4 it is clear that there is actually a smaller probability of getting a negative growth rate at $t+1$ than what would have been found from a model

[8]The package used for estimation was Perlin (2009).

in which growth just depended linearly on past growth. It may be that the MS model gives a better fit to the data but it produces a worse record at predicting recessions.

5 Predicting Recessions: Using Multivariate Information to Model GDP Growth

So far we have looked at whether one can predict Δy_{t+1} with past growth and state information and found that this is not likely. The fact that we are looking for the shocks that cause movements in future growth suggests that more success might be had by looking at variables that contain some forward-looking information. For Turkey Aysoy and Kipici (2005) give a GDP equation for real GDP of the form[9]

$$\Delta y_t = a_1 \Delta y_{t-1} + a_2 r_t + a_3 \Delta_4 cps_{t-1} + a_4 \Delta_4 pcu_t$$

where r_t is the real Treasury Bill rate, cps_t is total credit in real terms extended to the private sector, and pcu_t is the private sector's capacity utilization rate. Using capacity utilization and Δy_{t-1} one can get a prediction pattern that improves on just using Δy_{t-1}, but it is not as good as having Δy_t available, although it does go some way to filling in for any lack of current GDP information.

Another variable often used in an attempt to improve predictive power is the spread between long and short interest rates and there is now a large literature suggesting that this spread variable is good at doing so e.g. Estrella and Mishkin (1978). There are also models that use multi-variate information to make predictions. Thus Canova and M. Ciccarelli (2004) use a VAR which is estimated across a number of economies. One that has attracted substantial attention is the Qual-VAR model of Deuker (2005) in which a vector of variables including $S_t, \Delta y_t$ and interest rates are modelled as a latent variable VAR. This structure is then used to generate forecasts of S_{t+1}. In the following sub-sections we look at the utility of spreads and the Qual-VAR model in predicting recessions.

[9] They actually use $y_t^u - y_{t-4}^u$ where y_t^u is seasonally unadjusted data but since we have $y_t = y_t^u + y_{t-1}^u + y_{t-2}^u + y_{t-3}^u$, $\Delta y_t = \Delta_4 y_t^u$.

5.1 The Predictive Utility of Spreads

It has often been suggested that spreads are informative. Indeed there are hundreds of articles looking at this for a variety of countries. In what follows we look at the US case as the predictive efficiacy of spreads for that country's recessions has been a constant theme in the literature.[10] As before, focussing on the first period the recession begins in, we get probabilities of negative growth for the ten recessions between 1953:4 and 2008:4 of .30, .4, .32, .38, .59, .55, .40, .22, .25, .25. The information used in this prediction was current growth and the spread lagged one period (the best for predicting S_{t+1} according to Estrella and Mishkin). The estimated probit model shows a probability of negative growth of .307 when there is zero spread, rising to .48 when the spread is -100 basis points. The latter is is a very rare occurrence in US history. In fact, with the exception of 1973:3, the only other time the term structure inverted to such an extent was during the "Volcker experiment" over 1979-1982, when the Fed targeted money supply. In only two recessions would the spread have managed to indicate a negative growth rate - 1974/5 and 1980. If one looks at the pseudo-R^2 from the Probit model one finds it is .076 (with GDP growth), .163 (when spread is added to growth as a regressor), .178 (with growth, spread and S_{t-2}), and .28 (with growth, spread and S_t). So spreads do contribute but not a great deal. We introduced S_t as suggested by Deuker (1997), and much applied research since then has used it e.g. Kauppi and Saikkonen (2008). Of course, as explained earlier, S_t is not available for prediction purposes, and to construct it one needs to know future growth outcomes. Because it is a function of future outcomes, it should not therefore be surprising that it will produce a much better fit to future data. In order to do a proper comparision it is not $S(t)$ that we should use but the expected value of $S(t)$ conditional upon whatever information is available at t, and we have discussed how to compute this earlier. If one does that there is only a marginal improvement. One recommendation therefore is that Probit models which include S_t as regressors can easily produce misleading accounts of the explanatory power of spreads, and so the inclusion of the variable in these relations needs to be treated with caution. Because

[10] Moneta (2003) reports even stronger results for predicting Euro-area recessions. He defines a recession as two periods of negative growth in GDP. We used data on the long and short rates and GDP in the Area Wide Model Data base, but could not get anything like what he reports. Mostly the predicted probabilities were quite low, certainly for the first quarter of the recession.

S_t would be known to be S_{t-2} if both Δy_t and Δy_{t-1} were positive (as the peak in y_t could not be at $t-2$ and $t-1$),using S_t as a regressor utilizes information that Δy_t and Δy_{t-1} have different signs.

5.2 Qual VAR

Dueker(2005) described a Qual-VAR which has the (simplified) form

$$\begin{aligned} z_t &= \alpha_{yy} z_{t-1} + \alpha_{yz} \psi_{t-1} + \varepsilon_t & (8) \\ \psi_t &= \alpha_{zy} z_{t-1} + \alpha_{zz} \psi_{t-1} + \upsilon_t & (9) \\ \zeta_t &= 1(\psi_t > 0), \zeta_t = S_t & (10) \end{aligned}$$

where z_t is a vector of observable variables, ψ_t is a latent variable describing economic activity and the shocks $\varepsilon_t, \upsilon_t$ are normally and independently distributed with a zero expectation. z_t, ψ_t follow a VAR(1) in (8) and (9). Basically one can think of this as a model in which the "level of activity" is captured by a latent variable ψ_t. The NBER Dating Committee are assumed to express opinions about the variable ψ_t through their published decisions about whether the economy is in an expansion or a contraction (S_t), and that is described in (10). This model is then estimated using data on z_t *and* S_t. The values of S_t determine what distribution one should draw ψ_t from. Specifically, at time t, if we observe that $S_t = 1$, a draw is made from the truncated distribution $\psi_t|(\psi_t > 0)$. Because S_t have been formed by the NBER after recessions and expansions have occurred, S_t should be related either directly to z_t or variables that are correlated with it. Thus S_t is a forward-looking endogenous variable, but Deuker effectively treats it as if it was exogenous. To see this note that the implications of the model above are

$$\begin{aligned} E(\zeta_t|z_{t-1}, \psi_{t-1}) &= \Pr(\psi_t > 0|z_{t-1}, \psi_{t-1}) \\ &= \Phi(\alpha_{zy} z_{t-1} + \alpha_{zz} \psi_{t-1}) \end{aligned}$$

so that, adopting a linear approximation for exposition,

$$E(\zeta_t|z_{t-1}, \psi_{t-1}) = a z_{t-1} + b \psi_{t-1},$$

produces an explanation of ζ_t of the form

$$\zeta_t = S_t = a z_{t-1} + b \psi_{t-1} + \eta_t,$$

where $\eta_t = \zeta_t - E(\zeta_t | z_{t-1}, \psi_{t-1})$. Inverting this equation gives $\psi_{t-1} = \frac{1}{b}(S_t - az_{t-1} - \eta_t)$ so that the model for z_t becomes

$$z_t = cz_{t-1} + dS_t + \xi_t. \tag{11}$$

Consequently, even though S_t did not appear explicitly in the original VAR, it was implicitly there because of the presence of the latent variable ψ_{t-1}.

Now if $\alpha_{zy} = 0$ we know from Kedem (1980) that ζ_t will be an infinite dimensional Markov Chain whose parameters depend only upon α_{zz}, and so the combination of (11) and the stationary process for S_t will effectively be the system that is being estimated. Conditioning upon S_t raises the issue of whether S_t actually equals ζ_t. We know that there will be an estimate of ζ_t that can be generated from $\{z_{t-j}\}_{j=0}^{\infty}$, but whether that corresponds to the way that the NBER construct S_t from z_t is problematic. There is probably some specification for the ψ_t process that will lead to a dating rule using z_t that will agree with the NBER states but whether it is the one in the Qual-VAR is another matter. Consequently, this difficulty points to the need to check for specification errors in the latent variable part of the Qual-VAR. In any case it is clear that the fact that S_t is being treated as if it is exogenous, when it actually depends on future values of z_t, will lead to inconsistent estimation of d unless one recognizes that dependence.

A more direct solution to this issue which uses the nature of the S_t in (1), is to define y_t in that equation as the latent variable ψ_t. We then have the VAR system

$$\begin{aligned}
z_t &= \alpha_{yy} z_{t-1} + \alpha_{yz} \psi_{t-1} + \varepsilon_t && (12)\\
\psi_t &= \alpha_{zy} z_{t-1} + \alpha_{zz} \psi_{t-1} + \upsilon_t && (13)\\
S_{t+1} &= S_t S_{t-1} \left[1 - \mathbf{1}\left(\Delta\psi_{t+1} \le 0\right) \mathbf{1}\left(\Delta_2 \psi_{t+1} + \Delta\psi_{t+2} \le 0\right)\right] + S_t \left(1 - S_{t-1}\right) \\
&\quad + \left(1 - S_t\right)\left(1 - S_{t-1}\right) \mathbf{1}\left(\Delta\psi_{t+1} > 0\right) \mathbf{1}\left(\Delta_2 \psi_{t+1} > 0\right). && (14)
\end{aligned}$$

The VAR in (12)-(14) incorporates both a latent variable and some non-linear structure and has some similarities to DSGE models with forward looking expectations. Estimation can be done by simulation methods. The simplest estimation approach would be to use indirect estimation and various auxiliary models could be used.

Deuker (2005) reports an application of his Qual-VAR to predicting the recession of 2001. The task was to forecast 2000:4-2003:3. Dueker assumes that $S_t = 1$ in 2000:3. It is necessary to have a way of generating $E_t z_{t+j}$

over the forecast period. One possibility is to produce forecasts of z_t using an AR(5) in z_t. Another is to use the broader set of variables in Deuker's paper and take the forecasts of the spread to be from a VAR(5) in GDP growth, inflation, the spread and an interest rate. We use both below.

Table 3 shows the probability of a recession over the period 2000:4-2003:3 given that the information used is just the knowledge of S_t in 2000:1 and 2000:2 along with the yield spread up to and including 2000:3 (as well as the other variables in the case of the VAR).

Table 3: Probability of a Recession, 2000:4-2003:3

	AR(5)	VAR(5)
2000:4	.24	.24
2001:1	.36	.36
2001:2	.42	.45
2001:3	.42	.49
2001:4	.40	.49
2002:1	.37	.47
2002:2	.34	.44
2002:3	.30	.40
2002:4	.27	.36
2003:1	.24	.32
2003:2	.22	.29
2003:3	.20	.26

Comparing these to the results in Deuker (Table 2, p 100) we see that the pattern is the same but the latter reports a maximum probability of .57, which is a very high probability given such a weak recession (indeed Dueker notes that the Qual-VAR forecast of GDP growth never becomes negative). We were not able to replicate his Table 2 probabilities with the program Dueker supplied to us, getting instead a maximum probability of .55 and a probability in 2003:3 of .32, but these seem reasonably consistent.

To understand why our estimates are smaller consider first the comparison between the AR and VAR. The VAR produces higher probabilities because it features three forecasts of the spread that are negative, with values of -38,-34 and -2 basis points, whereas the AR never has any negative forecast, although there is one quarter of a small positive value. Since Dueker utilizes a VAR in his work one would therefore expect a probability of at least .5. It should be said that the AR produces a much better forecast of the actual path of the spread than the VAR, as there are no negative spreads ex-post in the forecast period. So it is not clear whether one gets a large probability of a recession simply due to an incorrect forecast of the spread over the recession. It would seem important that one would provide information on exactly what causes the probability of a recession to rise, and should be able to do this with the multivariate model that is used, rather than just treating it as a black box.

The other source of difference is that in 2003:3 the probability of a recession from the Qual-VAR is .29 which is high compared to the unconditional probability of .17 in the data. This suggests that the model has a tendency to assign a high probability to a recession. If we extend the forecast period for the AR and VAR models to 25 periods, the probability of a recession would be given as .174 i.e. both models return to the unconditional mean of S_t as the forecast. In contrast, simulating out the Qual-VAR produces an unconditional forecast of the probability of a recession of .39.[11] One problem may be that in the simulations used to do the Bayesian forecasts unstable VARs were retained provided the maximum root was less than 1.02. In a sense this is a specification test since the forecast should return to the unconditional mean in a stationary context, and S_t will be a stationary random variable. Thus the Qual-VAR does not seem to have this property and directs attention to the possibility of specification errors in it based on treating S_t as exogenous that were mentioned earlier. The methodology of the Qual-VAR seems not seem to have been well documented so predictions from it are more like those from a judgement rather than something that can be re-produced.

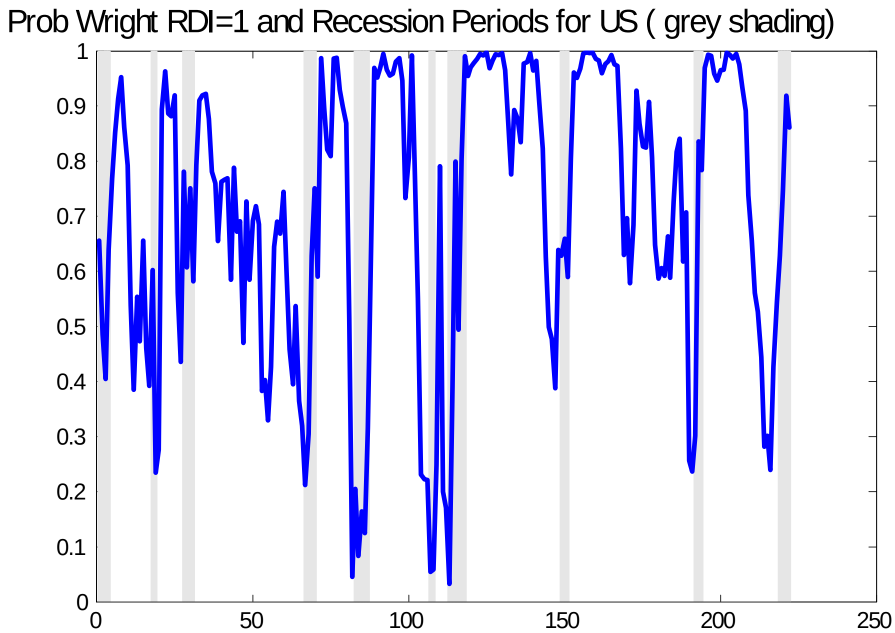
6 Changing The Event Defining Recessions and Turning Points

There are two ways that this can happen. In one approach a variety of series might be combined together to measure the level of economic activity. Many suggestions have been made e.g. the level of unemployment as well

[11] In an econbrowser blog http://www.econbrowser.com/archives/2008/02/how_to_balance.html Deuker reports what seem to be probabilties of S_{t+1} being zero in the 2001 recession of around .42 which seems more in line with our finding. In his 12 period horizon forecast the probability of .17 is close to the unconditional forecast. There is no explanation of how the Qual VAR has been adjusted to produce such different results from that given in Deuker (2005). We also note that in the internet piece Dueker takes .35 to be the critical value that one compares the predicted probability to when deciding if a recession is being forecast, which again raises the issue of what is an appropriate threshold.

as industrial production. Indeed, the NBER S_t involve an analysis of the turning points of a number of series and this information is then combined in an unknown way by the NBER Dating Committee to produce the final recession and expansion dates. Coincident indices generally combine together a number of series with fixed weights, while many factor models aiming to extract a common factor from a variety of series use a set of weights that may be varying. Obviously, it cannot be any easier to predict an indicator based on a variety of series than a single one, since one now has to forecast the sign of future growth in many series to find their turning points.

A different approach is to re-define the recession event. We will refer to these as "recession-derived indicators", RDI. A number of papers that suggest high probabilities of predicting a recession have used RDI's. One example that is often cited is Wright (1996). Wright has an RDI taking the value one if an NBER defined recession happens in the next four quarters and zero otherwise. To see how this affects outcomes take the following series for S_t,$\{1, 1,1, 1, 1, 0, 0, 0, 1, 1, 1, 1\}$. The binary numbers here correspond to when the NBER define expansions and recessions. The RDI is then $R_t = 1(\{S_{t+j} = 0\}_{j=1}^{4})$ and, for the S_t above, it will be $\{0, 1, 1, 1, 1, 1, 1, 0, ?, ?, ?, ?\}$, where ? indicates that a decision can't be made as not enough future information is available. It is clear from this that the mean of R_t is different from that for $(1 - S_t)$ i.e. the unconditional probability that $S_t = 0$ is much lower than the probability that $R_t = 1$ (for the US it is .17 versus .31). If one then describes a high $\Pr(R_t = 1|F_t)$ as "predicting a recession" it can look as if there is much greater predictive success but it is an artifact of the re-definition of the event being predicted. Note that the rise in probability comes because expansions are not treated symmetrically with recessions. Thus, for the seventh observation in the S_t sequence, the following four values for S_t are $\{0, 1, 1, 1\}$, and R_4 will be taken to be unity because of the fact that $S_8 = 0$. But given that the quadruple $\{0, 1, 1, 1\}$ largely consists of expansions it would seem more appropriate that $R_7 = 0$. Except for the instances in which ties occur e.g. the quadruple {0,0,1,1} could presumably either eimply an R of zero or one, treating expansions and contractions symmetrically would just mean that $R_j = \{S_{j-4}\}$, and then the probabilties of $S_t = 0$ and the symmetric RDI $R_t = 1$ would be the same. Another important effect of moving to R_t is that there is a timing change. In the example above consider predicting whether $R_7 = 1$. Because thisincludes two $S_t = 0$ and so is effectively the second period into a recession, comparing it with the S_t outcomes will make it look as if one has managed to predict the recession well



in advance, but again it is an artifact of changing the event being predicted. One can see these effects in Figure 5.

Another RDI used by Fair (1993), Anderson and Vahid (2001), and Galvao (2006) is similar except that it defines a recession starting at t if the five quarters starting at t have two successive periods of negative growth. One can see that the R_t formed this way will be $R_t = \{1, 1, 0, 0, 0, 0, 0, 1, ?.?, ?, ?\}$. Again the timing has been changed and the unconditional probability of the new recession defined event will likely be higher than for S_t. If fact, for the US over 1953/1-2009/4, it is actually lower since the recession definition does not record the 2001 recession, because there were not two successive periods of negative growth over 2001.

Canova and Ciccarelli (2004) look at turning points but they define a peak and trough in the *growth rates* and so are looking at a *growth rate cycle*. Of course this means that one is intereested in whether $\Delta^2 y_t$ etc are negative, not Δy_t. Thus predictibility of a turning point in the series on growth rates would involve a "first period test" of $\Pr(\Delta y_{t+1} - \Delta y_t < 0|F_t)$. Suppose $F_t = \Delta y_t$. Now, if Δy_t had a unit root, then Δy_t would have no predictive power for the event $1(\Delta y_{t+1} - \Delta y_t < 0)$ but, if it was white noise, then there is quite a bit. For the US regressing $1(\Delta y_{t+1} - \Delta y_t < 0)$ against Δy_t gives an R^2 of .24.

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