EDUCATE OR ADJUDICATE?
SOCIO-ECONOMIC HETEROGENEITY AND WELFARE

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EDUCATE OR ADJUDICATE?
Socio-Economic Heterogeneity and Welfare

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ABSTRACT

This paper presents a model to explore the welfare effects of the government’s choice over two types of public goods provision: domestic regulatory and security spending (adjudication) versus education. Output is a function of physical and social capital, both of which can be heterogeneous across the regions. Local social capital is exposed to spillover effects of other regions. Education spending increases social capital, whereas adjudication spending increases total factor productivity. The solution in an OLG framework indicates that the welfare maximizing ratio of education spending is negatively related with the past levels of social capital stock and the degree of social cohesion, but positively related with the current levels of aggregate income and the tax rate. Simulations of the model’s temporal solution reveal the short-run and long-run difference, reversing the positive effects of the tax rate and the income level, which is a crucial point. Income and cultural homogeneity are associated positively with the level of aggregate income and social cohesion whereas the relationship between income distribution and social cohesion is non-linear in the short-run.

JEL Codes: E02, E6, H11, H52, I24, I25, I31, Z18

Key Terms: Economic Development; Income Equality; Public Spending; Social Capital; Social Cohesion

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EDUCATE OR ADJUDICATE?
Coping with Socio-Economic Heterogeneity

ABSTRACT

This paper presents a model to explore the welfare impact of the government’s choice over two types of public goods provision: domestic security and education. Output is a function of physical and social capital, both of which can be heterogeneous across the regions. Social capital has direct spillover effects whose sign and magnitude for a specific region depend on the exposure to the rest of the regions. Education spending increases social capital, whereas domestic security spending increases total factor productivity. Simulations of the model’s solution indicate that the welfare maximizing ratio of education spending is negatively related with the levels of social capital and income, the degree of social cohesion, tax rate and fiscal decentralization. Income and cultural homogeneity are associated positively with the level of aggregate income and social cohesion whereas the relationship between income distribution and social cohesion is not linear.

JEL Codes: E02, E6, H11, H52, I24, I25, I31. Z18

Key Terms: Culture, Economic Development, Income Equality
“...we have money to give military equipment to police forces, when we don’t have money for training and money for public education,” (Rev. Al Sharpton at the funeral of James Brown in Ferguson, U.S.A., Aug 2014).

1. Introduction

There is hardly any country in the world that is homogenous in every dimension. Each country, regardless of its size, portrays heterogeneity of some sort.\(^1\) Economic development literature is laden with studies that argue that heterogeneity, be it in cultural and economic dimensions, is a major source of instability; it may reduce the allocative efficiency as well as and the quality (technical efficiency) of public spending (see, for example, Alesina and Spolaore, 1995, Alesina et al, 1997 and Kujis, 2000). Easterly and Levine (1997) and Sachs and Warner (1997) point out that socio-political conflicts and poor policy decisions have negative effects on economic development, which are mostly observed in heterogeneous countries. Barro (1991) and Montalvo and Reynal-Querol (2005), among others, have argued that ethnic heterogeneity or polarization leads to political conflict that in turn hinders economic development due to reduced investment and increased rent-seeking. Bluedorn (2001), however, demonstrates that institutions of governance and democracy ameliorate the negative effects of ethnic diversity. Horowitz (1985) argue that ethnic diversity and political conflict are related non-monotonically; in a similar vein, Montalvo and Reynal-Querol show that it is polarization, rather than fractionalization that matters for the potential of conflict and therefore is more relevant for developmental outcomes.\(^2\)

Looking deeper into the relationship between heterogeneity and economic development, it can be stated that heterogeneous societies, which are characterized by substantial differences in social preferences, have greater difficulty than others in deciding on the level and the type of public good provision. The potential of inequality and conflict increases when ethnic fractionalization coincides with the differential degree of groups’

\(^1\)Appendix 2, Figure 1, shows, for example, that ethnic fractionalization is very weakly related with the land area, although size is often considered positively related with various degrees of heterogeneity. The Republic of Korea seems to be the only country which has no ethnic fractionalization

\(^2\) Figures 1 and 2 in Appendix 1 show that neither land size nor income level show a direct association with ethnic heterogeneity.
access to economic opportunities. The case in point is the continued conflict in former colonies where duality has been especially prevalent (see, for example, Goudie and Neyapti, 2000). Rodrik (1999) argues that the effect of external shocks is wider in case of social divisions and distributional conflicts.

Olson (1965 and 1982)’s collective action theory links economic inefficiencies to interest group dynamics. In order to explain the differential development processes, Acemoglu and Robinson (2012) coin the term **extractive institutions** that are run by a small number of elite that redistributes resources towards themselves. These institutions may be sustained until the technological frontier is pushed through creative destruction; till then, economic success relies mainly on the central delivery of essential public good, which establishes inclusivity to some degree. Heterogeneity may lead to rent-seeking and inefficient institutions in democratic market economies also. Neyapti (2013) shows that economic and cultural homogeneity is associated with higher levels of development. Even in politically stable countries, the empowerment of certain special interests may lead to **institutional sclerosis** (Olson, 1982) that causes increasing inequality overtime; Acemoglu and Robinson (2012) describe this phenomenon as the devolution of **inclusive institutions** into extractive ones. The emergence of financial interest groups that reap large degree of rents at the expense of the messes in the period that led to the great recession is a recent manifestation of this. Increase in economic inequality in turn generates the potential of political conflict.

This paper presents a framework to analyze the impact of the government’s choice between the two types of public good provision: domestic security versus education, in order to maximize the welfare given the degree of fragmentation in the society. We argue that spending on education and security can be viewed as policy alternatives that help

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3 While several studies (see, for example, Collier and Hoeffler, 2004) find no significant correlation between conflict and inequality, Huber and Mayoral (2014) point out that within-group inequality has significant positive effects on conflict.

4 In that regard, Lange (2004) notes that dispersed power structure under colonialism reduces governance quality and leads to increased potential of political instability. Moreover, minority direct ruling under colonialism is observed to increase income inequality, as compared to the case where settlers constituted the majority; high inequality remained in those countries even after independence (Angelas, 2007).

5 Neyapti (2013) models the change in the formal institutional structure in interaction with economic progress.
achieve social order and harmony: Increasing education enhances employment, productivity and growth, all of which are associated with improved organization of the social life. Increasing the domestic security spending, on the other hand, aims to attain the social order mainly through coercion or enforcement, which may be fundamentally unproductive. While education affects social harmony as a side-product of improved production, security targets it directly. Nonetheless, the literature is inconclusive in regard to the relationship between defense spending and growth; Shieh (2002), argues, for example, that the relationship is non-linear.

It is possible to predict that increasing complexity in economic transactions requires equally complex regulatory and supervisory mechanisms, and hence increasing “policing” of those functions, which is in the form of increased spending on regulatory, supervisory and judicial activities. On the other hand, greater levels of development associated with high education may be thought of reducing the potential of the violent forms of conflict and increasing social capital, thus reducing the need for the internal security spending. Increased regulatory, supervisory and judicial spending that result from increasing economic complexity may thus lead to the institutionalized forms of social capital accumulation.

Given the above arguments, I hypothesize that economic development is associated with a decreasing need for domestic security spending. As for the education spending, even though it is beneficial for the entire society over the long run, some groups benefit from it more than the others due to diminishing returns. Hence, the provision of education as the public good may effectively imply some redistribution from the rich towards the poor. I assume a heterogeneous society where differences in social capital across different regions affect regional spillovers, which may be positive or negative and need not be reciprocal between the two regions either in their signs and magnitudes. Heterogeneity is productive in case regional spillovers have net positive effects onto each other; it reduces

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6 We abstain from political economy aspects of decentralization and public good provision.
8 The relationship between the level of education of the citizens and the preference for the level of security spending seems ambiguous in the US Based on the study of a US survey (Wong, 2010). It is unfortunate that data on domestic security spending is too scant to make observations on the relationship between domestic security spending and the level of development.
total productivity, however, if the sum of all the effects, coming from the other regions, is negative for a given region. I consider that the government maximizes welfare by taking into account those spillovers, and hence the level of socio-economic diversity, in order to determine the relative amounts of two alternative public goods: education and security.

The solution of the model indicates that the welfare-optimizing share of education spending decreases in the level of heterogeneity, which is defined by the sum of regional spillovers arising from social capital differentials. That is, the more positive net spillovers across the different regions of a country, the lower the welfare-optimizing level of education spending. An increase in both social capital stock and income make the allocation of public goods into education spending less necessary. Furthermore, the optimal level of education spending by the central authority is negatively associated with both fiscal decentralization and the tax rate.

This paper is similar to Shieh (2002) in its effort to assess the growth and welfare impacts of defense versus non-defense spending. As different from the former, however, the current paper treats social heterogeneity as endogenous to government policy. In addition, domestic security is modeled explicitly.

The rest of the paper is organized as follows. Section 2 provides the description of the model and outlines its implications. Section 3 concludes by presenting the policy implications of the model.

2. The Model

The country consists of \( n \) distinct regions. The sequence of events is as follows. Facing the income tax and regional incomes, representative consumers in each region choose their saving and thus consumption and investment. Investment turns to productive capital next period. The government allocates its tax revenue optimally between education, which augments the social capital of each region by the same amount, and domestic order, which increases total factor productivity; both types of spending affect the next period.

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9 The authors suggest that the optimal defense spending that maximizes economic growth is smaller than the level that maximizes the welfare. While that model introduces the military spending into both the utility and the production functions, the current model only focuses on the productivity effects of security spending.
period’s production. The government lives for two periods only; maximizing period $t+1$ welfare by taking decisions in period $t$. Hence, total spending in any region differs from the size of local production, the difference being due to the provision to each region of pure public good that is financed by the overall tax revenue, to which each region contributes only partially. Heterogeneity across the regions takes the form of initial levels of social and physical capital, in addition to the extent that regional differences in social capital affect the local social capital.

The formal presentation of the model is as follows. Given the income of region $i$ at time $t$ (denoted by $Y_{i,t}$) the consumption of the representative consumer is given by:

$$C_{i,t} = (1-\tau)(1-s_i)Y_{it}$$  (1)

where the tax rate, $\tau$, is given exogeneously and $s_i$ is the rate of saving of a representative consumer of region $i$. The rest of the disposable income is used to augment the physical capital ($I_{i,t}$):

$$I_{i,t} = s_i(1-\tau)Y_{it}$$  (2)

where the rate of savings in each region determines the level of investment in each period. Hence, from the spending side, $Y_{i,t}$ consists of local private consumption and investment, and the government spending on the pure public good that is financed by taxing all localities’ incomes: $G_t = \tau \sum_i Y_{i,t}$. It is assumed that the government budget always balances.

Since $G_t$ is public good, part of the financing from the rest of the regions constitutes a transfer to any given locality, which can be expressed as follows:

$$\tilde{Y}_{i,t} - Y_{i,t} = \tau \sum_i Y_{i,t}$$  (3)

where $\tilde{Y}_{i,t}$ stands for the local income after transfers. That is, all the regions receive transfers by the amount of $G_t$ minus its own tax payments. Thus, the goods market equilibrium condition for any region $i$ is given by:

$$\tilde{Y}_{i,t} = C_{i,t} + I_{i,t} + G_t.$$  (4)

There are two factors of production in each region: social and physical capital, denoted by $A_i$ and $K_i$, respectively for regions $i=1,...,n$. The rules of accumulation for each are given by:

$$A_{i,t} = A_{i,t-1}(1+f_{i,t-1}) + G_t^{E}$$  (5)

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10 This means that pure public good provision generates positive transfers for all the regions.
\[ K_{it} = K_{it-1}(1-\delta) + I_{it-1} \]  

where \( \delta \) stands for the rate of depreciation of the physical capital stock and \( t-1 \) denotes the initial period. \( G_t \) is spent for the provision of two types of pure public goods: education investment \( (G^E_t) \), and security spending \( (G^S_t) \):

\[ G^E_t = m_t G_{t-1} ; \quad \text{and} \quad G^S_t = (1-m_t) G_{t-1}, \]  

where \( m \in [0,1] \) is the proportion of public spending allocated to education and \( (1-m) \) goes to domestic security and enforcement. Government makes this allocation in period \( t-1 \) but the effects of this allocation are observed in period \( t \), similar to the effect of investment spending by the households.

The accumulation rule in equation (5) indicates that the prevailing level of social capital carries over by the extent of \( f_{it-1} \), which is a measure of the cultural spillovers that region \( i \) receives from the rest of the regions that it has varying degrees of socio-economic interaction with.\(^{11}\) The cultural spillover parameter for region \( i \) and for the aggregate economy \((f_i \text{ and } F, \text{ respectively})\) are thus given by:

\[ f_{i,t} = \sum_j h_i (A_{j,t} - A_{i,t}) \quad \text{and} \quad F_t = \sum_i f_{i,t} \]  

where \( h_i \in [0,1] \) is the region specific exposure factor. It is possible that either individual \( f_i \)'s or \( F_t \) take a negative value, the latter of which would indicate lack of social cohesion or conflict at the aggregate level.

Note that the education spending of the central government \((G^E_t \geq 0)\) is made in the previous period which augments each \( A_{i,t} \). Since education spending is the public good, the proportional increase the human capital is greater in the regions with lower initial \( A_{i,b} \).

\( K \) follows the usual accumulation process given Equation (6), with the depreciation term \( \delta \). Unlike \( G^E_t \), the investment spending on physical capital: \( I_t \), is financed by local private savings from the previous period, as given in Equation 2.

The production function is assumed to have the same form in each region:

\[ 11 \text{ Empirically, the relevant variable for production is labor (}L\text{); }A \text{ here encompasses the quantity of labor, as its quality, which is also referred to as the human capital (}H\text{), as well as the interactive value it produces in a society. Hence, we define } A \text{ as the social capital, which can be measured as the effective labor, which further augments } H \text{ by incorporating the cultural aspects such as trust and cooperation.} \]
\[ Y_{t,i}^* = G_i^S K_i^{\alpha} A_i^{1-\alpha} \]  

(9)

where the term \( G_i^S \) stands for total factor productivity that is generated by government spending on security and rule enforcement; the production function exhibits constant returns to scale, where \( 0 \leq \alpha \leq 1 \). For simplicity, the income shares of physical and social capital are taken the same across the regions.

The government chooses \( m_t \) in period \( t \) in order to maximize the social welfare in period \( t \), which is the sum of utilities obtained from regional consumption. Using Equations (1) and (8), and assuming a logarithmic utility function, the government’s problem can hence be written for period \( t \):

\[
\text{Max}_{m_t} \sum_i \log(G_i^S K_i^{\alpha} A_i^\beta (1-s_i)(1-\tau))
\]

(10)

Substituting Equations (5) and (6) into Equation (10), and utilizing Equations (7) and (8), the first order condition is:

\[
m^* - (1-n)/(1-\alpha)\tau \sum_j Y_{t-1} \sum_i (1/(A_i^{t-1}(1+f_{i,t-1}) + m^* \tau Y_{t-1})) = 0
\]

(11)

In the remainder of the paper we assume that \( n=2 \) for simplicity. The solution of Equation (11) for \( m^* \) yields distinct two roots. Based on the numerical simulations detailed below, one of these roots is identified as the global maximum, and is used in the analysis reported below.

Optimal \( m \) in period \( t \) is determined based on the model parameters \( \{\alpha, h_i, s_i, \tau\} \) and the initial values \( \{A_{i,t-1}, K_{i,t-1}, G_i^{S_0}\} \). Table 1 reports the feasible ranges of these parameters and the set of initial values based on which the numerical simulations are carried out.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( h_i )</th>
<th>( s_i )</th>
<th>( \tau )</th>
<th>( A_{i,0} )</th>
<th>( K_{i,0} )</th>
<th>( G_i^{S_0} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[0.1-0.4]</td>
<td>[0.1-1.0]</td>
<td>[0.1-0.7]</td>
<td>[0.1-0.7]</td>
<td>[10-100]</td>
<td>[10-100]</td>
<td>[1-100]</td>
</tr>
</tbody>
</table>

12 It turns out that the government cannot decide on both \( \tau \) and \( m \) simultaneously.
13 The roots are obtained using the Matlab program.
The problem is solved temporally (for a single period, t) due to the highly non-linear nature of the model indicated by Equations (5) to (8).\textsuperscript{14} We consider that the government lives for two periods, faces initial values at period t and the world ends in period t+1.\textsuperscript{15}

Hence, the solution procedure of the model is thus characterized as follows.

\begin{enumerate}
  \item Given a set of \{\textit{A}_{i,t-1}, \textit{K}_{i,t-1}, \textit{G}_{i,t-1}\} and \textit{\alpha}, \textit{Y}_{i,t-1} is found.
  \item Given \textit{A}_{i,t-1} and \textit{h}_{i,t-1}, \textit{f}_{i,t} is found.
  \item The government chooses the optimal \textit{m}_{t}(\textit{m}_{t}^{*}) that allocates \textit{G}_{t-1} between \textit{dA}_{t} and \textit{G}_{t} so as to maximize social welfare in period t.
  \item Given a set of parameters, \textit{A}_{i}, \textit{K}_{i}, \textit{Y}_{i} and \textit{C}_{i}, are calculated for period t.
  \item The economy disappears.
  \item The numerical simulations are carried out repeatedly for different combinations of the parameters and initial values, with the additional feasibility constraints of: 0≤\textit{m}^{*}≤1; \textit{C}_{i}>0; \textit{Y}_{i}>0; \textit{A}_{i}>0 leading to a data set of size 1,131,618.
\end{enumerate}

3. Simulation Results

Numerical analysis of the comparative statics of \textit{m}^{*} is carried out in Matlab, revealing the following unambiguous results that it is related negatively with the levels of \textit{A}, \textit{Y}, \textit{\alpha} and \textit{F}.\textsuperscript{16} These results are expected due to the diminishing returns nature of the production function.

\textbf{Remark 1:} Given the budget, the optimal share of education spending decreases with the level of income, social capital and social cohesion.

A major finding of the paper is that social-capital investment is positively associated with aggregate of the regional spillovers (see Figure 1, Appendix 2).\textsuperscript{17}

\textsuperscript{14} Appendix 1 presents the OLG modeling and the solution, which can be considered as the long-run decision making alternative.

\textsuperscript{15} This is a reasonable assumption given the commonly short-sighted nature of democratically-elected governments.

\textsuperscript{16} The partial derivatives are too long to report here and to sign, which are therefore obtained thorough numerical simulations.

\textsuperscript{17} Increasing the range of values for the initial values lead to the further observation that the upper range of \textit{m}^{*} first increases in \textit{F} and then decreases in it.
Remark 2: The lower bound of $m$ increases in the aggregate level of cultural spillovers: $F$.

Remark 3: The greater is cultural (social-capital) heterogeneity, the worse is the income distribution. (see Figure 2, Appendix 2)

It is also observed that both types of inequality are associated positively with higher $m^*$; considering that the model is solved periodically, this indicates the need for higher education to correct for the socio-economic inequality.

Remark 4: Income distribution first worsens and then improves (follows a bell-shaped pattern) as $F$ increases. (see Figure 3, Appendix 2).

A similar pattern is observed between the regional distribution of social capital and $F$: an increase in $F$ is associated with first worsening and then improving social capital distribution (not shown).

Remark 5: The greater is the homogeneity in social capital (the closer are $A_i$’s to each other), the higher is the range of aggregate incomes that can be attained (see Figure 4, Appendix 2).

The last remark, along with with Remark 2, implies that the higher is the level of aggregate income, the more equitable is income distribution (not shown). Hence, it is also observed that welfare is associated positively with more equitable distribution of both income and social capital. This is consistent with Neyapti (2014), who confirms the empirical observations of Easterly et al. (2006), Boettke et al. (2008) and Williamson (2009) in a theoretical model, by showing that formal institutional quality and welfare improves with cultural homogeneity.

3.1 Extension: Decentralization of the Public Good Provision

The model is modified to allow for the case that public good provision can be made both locally and centrally. In that case, the fiscal decentralization parameter $fd$ (where $0 \leq fd \leq 1$) is added to the model in the following way: $G_t = (1 - fd) \tau \sum_i Y_{it}$, so as to make the central government spending only a fraction of the fiscal revenues. Hence, $dA$ and $G^s$ defined earlier is now multiplied by the factor of $(1-fd)$; if $fd>0$, this means that the pure public
food provision is less than the case above. In addition, Equation (4) is modified by the addition of the local public good that is financed by the amount of local tax revenues: $G_{i,t}=fd(tY)$.

$$ A_{i,t} = A_{i,t-1}(1+f_{i,t-1})+dA_{t-1} + G_{i,t} \quad (4') $$

The optimum value of $m$ that is obtained as the solution of this modified version yields findings that leave the formerly reported remarks intact. This extension, however, leads to the additional observation that there is a negative association between the degree of fiscal decentralization and the optimum rate of education spending by the central government. This can be expected since local public good spending is a substitute of the education spending of the central government. Hence, ceteris paribus, increasing $fd$ is associated with increased share of regulatory or security spending of the central, though not necessarily total, government spending.

**Remark 6:** An increase in fiscal decentralization is associated with a decrease in the optimum rate of central education spending.

4. Conclusions

This paper presents an original model to analyze the optimum allocation of fiscal resources between education and security spending by the central government that faces heterogeneous regions with respect to their initial social and physical capital. A two-period lived government decides on the allocation of its budget across these two types of pure public goods in order to maximize the aggregate utility. The numerical simulations of the model’s solution indicate that the optimal share of education spending decreases in income and fiscal decentralization, while it increases in social cohesion, measured by the aggregate spillover effects of social-capital across the regions. The findings of this paper support several earlier empirical studies that point at the importance of social cohesion for economic development.

As an alternative to the temporal model, an OLG framework is provided (Appendix 1) to account for the long-run view. The interesting finding that results from the comparison between OLG and the temporal model is that in the long-run, an increase in the tax revenue leads to increased education spending. This finding points at the important
distinction between the preference of a short-sighted government and the one that faces institutions to induce a long-term view, such as fiscal rules that helps balance generational concerns against short-term political gains.
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Appendix 1: Solution in an OLG Framework

Because the solution of the model is not possible in a Ramsey framework due to the non-linearities in the model, the OLG version of the model is presented as a proxy to the long-run equilibrium. To set the framework, the above set up is easily modified into one where, in period \( t \), the young saves and the old consumes the proceeds of his or her saving in period \( t-1 \): 
\[
(1+r)s_t Y_{t-1}(1-\tau).
\]
The capital accumulation formula (Equation 6) is changed into: 
\[
K_{i,t} = I_{i,t-1},
\]
since the cumulative depreciation rate over a generation is assumed to be 100 percent. The young chooses \( s_t \) optimally, by solving the problem below:
\[
\max_{s_t} \ln(G_i^S K_{i,t}^{\alpha} A_{i,t}^{\beta} (1-s_t)(1-\tau)) + \beta \ln[(G_i^S K_{i,t}^{\alpha} A_{i,t}^{\beta} (1-s_t)(1-\tau))(1+r)]
\]
which yields (assuming identical decision making for all the households in a region \( i \)): 
\[
s^* = \frac{\beta}{1-\beta}.
\]

Keeping the rest of the model specification above, the government’s problem becomes one of maximizing the joint utility of the old and young of time \( t \):
\[
\max_{m_t} \sum_{i} \left[ \ln(G_i^S K_{i,t}^{\alpha} A_{i,t}^{\beta} (1-s_{i,t})(1-\tau)) + \ln(G_i^S K_{i,t}^{\alpha} A_{i,t}^{\beta} (1-s_{i,t})(1-\tau))(1+r) \right]
\]
which yields: 
\[
m^*_{OLG} = \left(\frac{1-\alpha}{2-\alpha}\right) - \left[ \frac{\sum_i A_{i,t-1}(1+f_{i,t-1})}{n(2-\alpha)\tau \sum_i Y_{i,t}} \right]
\]
Note that the second term on the right hand side can be read as the ratio of the average social capital stock value to current tax revenue. Given that \( 0 < m^*_{OLG} < 1 \), the optimality requires that the ratio of the average level of past social capital to tax revenue lies within the interval: 
\[
-1 < \left[ \frac{\sum_i A_{i,t-1}(1+f_{i,t-1})}{n\tau \sum_i Y_{i,t}} \right] < (1-\alpha).
\]
The negative lower bound indicates that the value of (negative) average social capital may not exceed the tax revenue.

It is clear from the solution that the optimal share of education over the long run decreases with past social capital and social cohesion (as in the temporal model), although it increases in the tax rate and the aggregate income level which contradicts with the short-run results reported in the paper (unless the aggregate social capital is negative). This finding is important as it reveals the difference between the short-run and
the long-run decision making by a government: in the long-run, ceteris paribus, an increase in the tax revenue increases the education spending (decreases adjudication) if aggregate social capital is positive, whereas it increases optimal adjudication under temporal decision making.

**Proposition 1:** The optimal share of education spending increases in the tax revenue if the government has a long-run perspective (and social capital is positive\(^\text{18}\)), but decreases in it if the government is short-sighted. Spending on adjudication follows the reverse pattern.

Proof: \[
\frac{\partial m^*_{\text{OLG}}}{\partial [\tau_i \sum Y_i]} = \left[ \sum_i A_{i,t-1} \left(1 + f_{i,t-1}\right) \right] > 0, \text{ if } \sum_i A_{i,t-1} \left(1 + f_{i,t-1}\right) > 0 ; < 0 \text{ otherwise.}
\]

Comparative static analysis further indicates that \(m^*\) is related with the income share of capital (\(\alpha\)) negatively, given the feasibility condition for \(m^*_{\text{OLG}}\) stated above. For the set feasible \(m^*_{\text{OLG}}\) values, an increase in the income share of capital, ceteris paribus, leads to a reduction in the education spending.

**Proposition 2:** Welfare maximizing share of education spending decreases with the share of capital income.

Proof: \[
\frac{\partial m^*_{\text{OLG}}}{\partial \alpha} = -\frac{1}{(2 - \alpha)^2} \left[ \sum_i A_{i,t-1} \left(1 + f_{i,t-1}\right) \right] < 0, \text{ if } \sum_i A_{i,t-1} \left(1 + f_{i,t-1}\right) > 0 ; < 0 \text{ otherwise.}
\]

\(^{18}\) This can be assumed to be always the case for the aggregate index.
Appendix 2. Ethnic heterogeneity, land area and income (Source: The World Bank)

Figure 1: Country size and ethnic fractionalization

Figure 2: Income and ethnic polarization
Appendix 2: Simulation Results

Figure 1. Social cohesion ($F$) and optimal $m$ (vertical scale:[0-1] is cropped)

Figure 2: Heterogeneity in social capital (vertical) and income distribution.
Figure 3: Social cohesion and income distribution (vertical scale)

Figure 4: Aggregate income and the distribution of social capital (horizontal scale)