UNEMPLOYMENT FLOWS, PARTICIPATION AND THE NATURAL RATE FOR TURKEY

Gonul Sengul
Murat Tasci

Working Paper 1404
February 2014

This Working Paper is issued under the supervision of the ERF Directorate. Any opinions expressed here are those of the author(s) and not those of the Koç University-TÜSİAD Economic Research Forum. It is circulated for discussion and comment purposes and has not been subject to review by referees.
Unemployment Flows, Participation and the Natural Rate for Turkey*

Gonul Sengul†
Murat Tasci‡
Central Bank of Turkey Federal Reserve Bank of Cleveland

February 7, 2014

Abstract

This paper measures flow rates into and out of unemployment for Turkey and uses these rates to estimate the unemployment rate trend, that is the level of the unemployment rate the economy converges to in the long-run. In doing so, the paper explores the role of the labor force participation in determining the trend unemployment. We find an inverse V-shaped pattern for the unemployment rate trend over time in Turkey, currently standing between 8.5 and 9 percent, with an increasing labor market turnover. We also find that allowing for an explicit role for participation changes the results substantially, reducing the “natural” rate at first, but then getting closer to the baseline over time. Finally, we show that this parsimonious model can be used for forecasting unemployment in Turkey with relative ease and accuracy.

*The views expressed herein are those of the authors and not necessarily those of the Central Bank of the Republic of Turkey, Federal Reserve Bank of Cleveland or the Federal Reserve System. We would like to thank the participants of the seminar at the Istanbul School of Central Banking, the Turkish Labor Market Research Network Conference, and the Internal Conference of the Central Bank of the Republic of Turkey.
†Istanbul School of Central Banking, Central Bank of the Republic of Turkey. Email: Gonul.Sengul@tcmb.gov.tr.
‡Research Department, Federal Reserve Bank of Cleveland. Email: Murat.Tasci@clev.frb.org
1 Introduction

The rate of unemployment in the long-run, or the underlying trend, has attracted a lot of attention since the Great Recession. In an environment where a lot of developed countries as well as developing ones face exceptionally high levels of unemployment, policy makers and economists focused on identifying the level of the unemployment rate that is feasible in the long-run, i.e. the “natural” rate, to gauge the extent of the labor market slack. In an effort to face this challenge, recent studies approached the problem by estimating the unemployment rate trend using the underlying flow rates. For instance, Tasci (2012) uses data on flows between employment and unemployment and, in the context of the U.S. labor markets, argues that this method provides an estimate of the natural rate that has several desirable statistical features while being theoretically very close to the language of the modern theory of unemployment. In this paper, we adopt his methodology to estimate the natural rate of unemployment for Turkey.

We believe that this exercise not only is valuable in its own right, but also allows us to highlight usefulness of the approach taken by Tasci (2012) in the face of interesting challenges posed by various structural issues experienced by many economies. For instance, many developing countries, Turkey included, have a very limited data span that covers substantial changes in the aggregate economy. Turkey has gone through significant changes in the monetary policy environment followed by a sharp decline in inflation in the early 2000s. The traditional approach of estimating a natural rate by focusing on the relationship between the labor market variables and the price level, that is NAIRU, will not necessarily inform us about the underlying dynamics of the Turkish labor market. Section 4.1 shows that natural rate estimates extracted using the NAIRU method imply an almost invariant level of unemployment, which is the average of the sample period, while our method reveals variation over time. Moreover, our method implies recent values of the natural rate of unemployment that are below the sample period average.

Moreover, the method developed by Tasci (2012) is flexible enough to be modified to incorporate different labor market structures of economies. As such, when we implement the same approach for Turkey, we need to take into account the

---

1The Central Bank of Turkey implemented implicit inflation targeting from 2002 to 2006, and has been officially targeting inflation since then. Please see Kara (2006) and Kara and Orak (2008), among others, for more information regarding the monetary policy in Turkey.
tive role of the participation margin in the labor market. The role of participation rate in estimating the long-run trend for unemployment becomes very evident in the Turkish data, a country whose participation rate is three times more volatile then the U.S.’s (see Sengul (2014)). Using flow rates to identify a trend rate for unemployment provides us with a way to carefully address the problem in a country where the persistence in unemployment is quite different from a developed country, where labor markets are relatively more dynamic.

Building on Tasci (2012), we estimate the unemployment rate trend for Turkey from 2001 to 2012, extending the methodology to include labor force participation. In doing so, we also exploit the work by Sengul (2014), which estimates monthly flow rates from 2005 to 2012 for Turkey, including the flows from nonparticipation to unemployment. We first estimate quarterly flow rates from 2001 to 2012, following Sengul (2014). Then, using a parsimonious unobserved components method as in Tasci (2012), we decompose the flow rates into their trend and cyclical components. Once we infer the trend components, we provide an estimate of the unemployment rate trend, that is the natural rate, implied by the steady state description of the unemployment rate in a standard labor market search model that relies on these flow rates.

Our results show a distinct pattern for the trend unemployment. As such, the trend unemployment increases during the first two thirds of the sample period, and then starts declining, which occurs after the 2008-2009 recession. This pattern holds regardless of allowing for a time varying labor force participation explicitly. However, with an explicit role for labor force participation, the estimated unemployment trend stays significantly below the level implied by the baseline, where we assume a constant participation rate over time. Moreover, we find that this pattern is led by a similar pattern by the inflow rate into unemployment - first increasing and then declining by 2008-09 - and a secular rise in the outflow rate from unemployment over the whole sample. Taken together, these findings imply that Turkish labor markets look a lot more dynamic at the end of 2012 relative to 2001. We also highlight another potentially useful feature of our framework; improving unemployment forecast accuracy in the short term, even though it is not designed for this purpose. In a country where unemployment data releases lag by more than two months, this is an important additional benefit of the framework discussed in the paper.

The rest of the paper proceeds as follows: In the next section, we lay out
the baseline model with the assumption that labor force participation does not move over time. After describing the methodology for measurement of the flow rates and the estimation of the trends, we extend the baseline model to incorporate variations in the participation rate in Section 3. Section 4 presents a more detailed discussion of the natural rate concept we develop here in conjunction with the more conventional measures of the natural rate used in the literature, including a NAIRU. We also address the robustness of the estimation in this section. Section 5 presents the forecasting performance of the model. The last section concludes.

2 Baseline Model

We first present our approach for identifying an unemployment trend for Turkey under the simplifying assumption that workers can only move between two labor market states, employment and unemployment, and the labor force participation does not move between two consecutive periods. These simplifications not only allow us to implement the approach proposed in Tasci (2012) for Turkey with relative ease, but also illustrates the main ideas behind our methodology in a simpler way. Later in Section 3 we extend the model to include movements in and out of the labor force, though the basic premise of using underlying flow rates and a measure of the business cycle to distinguish the cyclical movements from the trend fluctuations in unemployment is common in both cases.

Following Tasci (2012), we write down a simple reduced form unobserved components model that incorporates the comovement of flows into and out of unemployment into previous attempts at estimating the natural rate, such as Clark (1987, 1989) and Kim and Nelson (1999). The reduced form model assumes that real GDP - or any other measure of the aggregate business cycle - has both a stochastic trend and a stationary cyclical component, where only real GDP is observed by the econometrician. We also assume that both unemployment outflow and inflow rates (\( F_t \) and \( S_t \), respectively) have a stochastic trend as well as a stationary cyclical component. Furthermore, the stochastic trend follows a random walk, but the cyclical component in the flow rates depends on the cyclical component of real GDP. More specifically, let \( Y_t \) be log real GDP, \( \bar{y}_t \) be a stochastic trend component, and \( y_t \) be the stationary cyclical component. Similarly, let \( F_t \) (\( S_t \)) be the quarterly outflow (inflow) rate, \( \bar{f}_t \) (\( \bar{s}_t \)) be its stochastic trend component, and \( f_t \) (\( s_t \)) be its stationary cyclical component. Then we consider the following
unobserved components model:

\[
Y_t = \bar{y}_t + y_t, \\
y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon^{y_c}_t, \\
\bar{y}_t = r_{t-1} + \bar{y}_{t-1} + \varepsilon^{yn}_t, \\
r_t = r_{t-1} + \varepsilon^r_t, \\
\]

(1)

where \(r_t\) is a drift term in stochastic trend component of output, which is also a random walk, and cyclical component of output follows an AR(2) process, as in Ozbek and Ozlale (2005). The time series behavior of flow rates similarly take the following form:

\[
F_t = \bar{f}_t + f_t, \quad \bar{f}_t = \bar{f}_{t-1} + \varepsilon^{f_n}_t, \\
f_t = \tau_1 y_t + \tau_2 y_{t-1} + \tau_3 y_{t-2} + \varepsilon^{fc}_t, \\
\]

(2)

and

\[
S_t = \bar{s}_t + s_t, \quad \bar{s}_t = \bar{s}_{t-1} + \varepsilon^{sn}_t, \\
s_t = \theta_1 y_t + \theta_2 y_{t-1} + \theta_3 y_{t-2} + \varepsilon^{sc}_t. \\
\]

(3)

We assume that all the error terms are independent white noise processes.

As equations (2) and (3) show, we also assume that the cyclical component of the inflow and outflow rates move with the aggregate cycle. This idea captures the empirical pattern that recessions are times when a substantial number of matches dissolve because they cease to be productive enough and significantly fewer new matches are formed because firms do not demand as much labor anymore. Hence, a priori, we expect a negative co-movement between the cyclical components of the flow rates, \(s_t\) and \(f_t\). This basic description of the comovement between flow rates and the aggregate cycle can be easily reconciled with the extensions of the basic labor market search model with endogenous separations, as in Mortensen and Pissarides (1994).

We are agnostic about the existence of any co-movement between the trends of the flow rates, if any, as long as they are not correlated with the aggregate output. Even though such interaction is possible, we abstract away from it as, given the short sample we are working with, any more complication in the form of another
latent variable will substantially reduce the precision of the estimates we get in this unobserved components model. Tasci (2012) argues that the low-frequency movements in the trends, $\bar{f}_t$ and $\bar{s}_t$, will capture the effects of institutions, demographics, tax structure, labor market rigidities, and the long-run matching efficiency of the labor markets, which will be more important in determining the steady state of unemployment.

One can express the empirical model laid out in equations (1) through (3), in a convenient state-space representation as

\[
\begin{bmatrix}
Y_t \\
F_t \\
S_t
\end{bmatrix}
= 
\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & \tau_1 & \tau_2 & \tau_3 & 0 & 1 & 0 \\
0 & \theta_1 & \theta_2 & \theta_3 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\bar{y}_t \\
y_t \\
y_{t-1} \\
r_t \\
\bar{f}_t \\
\bar{s}_t
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
\varepsilon_{fc}^t \\
\varepsilon_{sc}^t
\end{bmatrix},
\]

where all error terms come from an i.i.d. normal distribution with zero mean and variance $\sigma_i$, such that $i = \{y_n, y_c, r, f_n, f_c, s_n, s_c\}$.

We use the Kalman filter to filter the unobserved components and write the log-likelihood function to estimate the model via maximum likelihood. Since we are interested in the unobserved stochastic trend and cyclical components, once we estimate the model, we use the Kalman smoother to infer them over time. These time-varying trend estimates for the flow rates, $\bar{f}_t$ and $\bar{s}_t$, determine the unobserved unemployment rate trend over time. More specifically, our definition of the long-run trend for the unemployment rate is given by

\[
\bar{u}_t = \frac{\bar{s}_t}{\bar{s}_t + \bar{f}_t},
\]
which is consistent with the search theory of the labor market. Tasci (2012) interprets the unemployment rate trend expressed in (6) as the steady state unemployment rate that is implied by the current trend estimates of the flow rates. Note that, since trend flow rates are random walks, current trend estimates are also the best estimates for future trend values. Hence, we interpret this rate as the rate of unemployment in the long run, to which the actual unemployment rate would converge. The intuition behind this equation as well as how we measure the observed flow rates, $F_t$ and $S_t$, are described in the following subsection.

Before proceeding to computation of the flow rates, we would like to discuss an issue that needs to be tackled in estimating the model. The model, as spelled out in equations (4)-(5), has three observable series and seven shock parameters that needs estimating, and hence is subject to a potential identification problem. The solution involves normalizing the standard deviation of the cyclical component of a variable relative to its trend component, thereby reducing the number of parameters to estimate. We address this issue in more detail and describe the process in Section 4.

2.1 Computing the Flow Rates

First step in estimating our measure for the trend unemployment requires us to obtain quarterly flow rates, $F_t$ and $S_t$. There is now an extensive literature on the importance of the flow rates in accounting for unemployment fluctuations. Most of this literature uses a simple unemployment duration based measurement to infer these rates (i.e., Shimer (2012), Elsby et al. (2009), Fujita and Ramey (2009), Elsby et al. (2013), Petrongolo and Pissarides (2008)). In particular, we follow the methodology presented in Elsby et al. (2013), which focuses on computing flow rates for a sample of the OECD countries. They extend the earlier work, as in Shimer (2012) and Elsby et al. (2009), to explicitly account for low flow hazard rates as not doing so will bias the estimates of the flow rates in some of the countries in their sample.

In what follows, we assume that time is continuous, and the data is available at discrete months $t$. Hence, “period $t$” refers to the interval $[t, t + 1)$. Let $L_{t+\tau}$, $U_{t+\tau}$, and $U_{t+\tau}^{\leq 5}(\tau)$ be the number of labor force, the number of unemployed, and the number of unemployed for less than 5 weeks at time $t + \tau$, respectively.

In this section we assume that all worker transitions are from unemployment
into and out of employment. People become unemployed because they separate from their employment and leave unemployment because they find a job. Let $S_t$ and $F_t$ be the job-separation (inflow) and job-finding (outflow) rates during period $t$. We can write the law of motion for unemployment as follows:

$$\dot{U}_{t+\tau} = (L_{t+\tau} - U_{t+\tau})S_t - U_{t+\tau}F_t. \quad (7)$$

Solving equation (7) and iterating it three months, we get the evolution of unemployment rate in the data, observed in discrete intervals, as:

$$u_t = u_{t-3}(1 - \lambda_t) + \lambda_t \frac{S_t}{S_t + F_t}. \quad (8)$$

where $\lambda_t = (1 - e^{-3(S_t+F_t)})$ is the quarterly convergence rate. Note that this is the original equation of [Elsby et al. (2013)](https://doi.org/10.1086/671680). Solving this equation for the steady state leads to the definition of the flow steady state unemployment as follows

$$u_t^{ss} = \frac{S_t}{S_t + F_t}. \quad (9)$$

If there is a trend in the underlying flow rates, then we get the expression in equation (6) as the time-varying trend estimate of the unemployment rate. This simple accounting framework forms the foundation of the measurement exercise, which relies heavily on exploiting the changes in the stock of unemployed at different durations across time to infer the high-frequency flow rates.

To compute the flow rates, we also need the law of motion for short-term unemployed, unemployed for less than five weeks, which is:

$$\dot{U}_{t<1}(\tau) = (L_{t+\tau} - U_{t+\tau})S_t - U_{t<1}(\tau)F_t. \quad (10)$$

The change in the number of short-term unemployed consists of workers separating from their jobs and workers who became unemployed after the last time data were available and did not leave unemployment, respectively. Subtracting equation (10) from equation (7) yields:

$$\dot{U}_{t+\tau} = \dot{U}_{t<1}(\tau) - (U_{t+\tau} - U_{t<1}(\tau))F_t. \quad (11)$$

Solving the differential equation above provides us with a simple measurement
equation for the outflow hazard:

\[ u_t = e^{-F_t}u_{t-1} + u_t^{<1}, \]  
(12)

where \( u_t \) denotes the unemployment rate in period \( t \).

If unemployment exit occurs with a Poisson process with parameter \( F_t \), then the probability of exiting unemployment within a month is \( \hat{F}_t = 1 - e^{-F_t} \). Therefore, equation (12) can be rewritten as

\[ \hat{F}_t = 1 - \frac{u_t - u_t^{<1}}{u_{t-1}}. \]
(13)

The intuition behind (13) is that we infer the average outflow probability, job-finding probability, by measuring the size of the decline in the unemployment pool who is a not short-term unemployed. The monthly outflow probability relates to associated monthly outflow hazard rate, \( F_t^{<1} \), through the following equation:

\[ F_t^{<1} = -\ln(1 - \hat{F}_t). \]
(14)

Equation (13) works well to estimate the outflow probability in labor markets for which the flow rate out of unemployment is high (duration of unemployment is low). For countries with longer durations, like Turkey, there are relatively few people in \( u_t^{<1} \) at any time since exit rates are low. Hence, the variance of the estimate will be higher (\( \hat{F} \) will be noisy). We follow Elsby et al. (2013) and use additional duration data to increase the precision of the estimate of \( \hat{F}_t \). Based on the unemployment data by duration, we can calculate the probability that an unemployed worker exits unemployment within \( d \) months as

\[ \hat{F}_t^d = 1 - \frac{u_t - u_t^{<d}}{u_{t-d}}. \]
(15)

As before, we can calculate the outflow rates as

\[ F_t^{<d} = -\ln(1 - \hat{F}_t^d)/d, \]
(16)

for different durations, \( d = 1, 3, 6, 9, 12 \). This rate is interpreted as the rate at which an unemployed worker exits unemployment within the subsequent \( d \) months.

If the exit rate from unemployment is independent of the duration of unem-
ployment, then $F_{t}^{<d}$ for different values of $d$ would not be much different from each other, and we have the monthly outflow hazard rate as $F_{t}^{<1}$. However, if the exit rate from unemployment depends on the duration of unemployment, then the $F_{t}^{<1}$ rate would not be a consistent estimate of the average outflow rate. We formally test the duration dependence by testing the hypothesis that $F_{t}^{<1} = F_{t}^{<3} = F_{t}^{<6} = F_{t}^{<9} = F_{t}^{<12}$. The approach in general is to derive the asymptotic distribution of unemployment rates and unemployment rates for different durations, and then to apply the Delta method to compute the joint asymptotic distribution of the outflow rate estimates. For Turkey, the hypothesis that there is no duration dependence (i.e., the hypothesis that $F_{t}^{<d}$ is the same for all $d$) can be rejected at 95 percent confidence level. We use the asymptotic distribution to compute an optimally weighted estimate of outflow rate that minimizes the mean squared error of the estimate. Once, we compute $F_{t}$, we use equation (8) and data on $u_{t}$ to back out the inflow rate $S_{t}$.

2.2 Data and Estimation Results

Before discussing the results, we describe our data sources and the treatments we have to implement to address some concerns before getting the desired flow rates at a quarterly frequency. We then present our results for the baseline model. The data used in estimating the flow rates is from the Turkish Statistical Agency (TurkStat). We have quarterly data from 2000:Q1 to 2012:Q4 on the number of workers in the labor force, and unemployed persons for less than $d$ months, where $d \in \{1, 3, 6, 9, 12\}$.

Unfortunately, the raw data requires some adjustments due to breaks prior to construction of the flow hazard rates, $F_{t}$ and $S_{t}$. First, there is a break in the 2005:Q1 data, due to a change in population projection methods. TurkStat updated quarterly data until 2005:Q1 and yearly data until 2004. To correct the data prior to 2005, we make use of the availability of unadjusted quarterly and

---

2Formal details of the test can be found in Elsby et al. (2013) with the only difference being that this paper has an extra term, the duration $d < 9$.

3For more information go to http://www.tuik.gov.tr.

4$d = 1$ corresponds to the number of workers unemployed for less than five weeks and this data is provided by TurkStat upon request.

5In 2007, Turkey implemented an address-based population registration system (ADNKYS), which allows yearly data for population. Turkstat was using population numbers based on projections from census data prior to this change, and it realized a discrepancy between the projections and the actual numbers delivered by ADNKYS.
adjusted annual values for 2004. As such, we update the unadjusted quarterly
values for 2004 such that quarterly growth rates within 2004 are the same for
adjusted and unadjusted series and the average of the new quarterly data for
2004 is the same as the adjusted annual value reported by TurkStat. Once we
adjust the quarterly series of 2004, we also update the data prior to 2004 such
that the quarterly growth rates are the same as in the unadjusted series.

In addition, there is a break in 2004 in the data for unemployed with different
durations. To correct for this, we assume that the growth rate of the share of
unemployed with a duration of $d$ months among all unemployed from 2003:Q4 to
2004:Q1 is the average of the growth rate of the same quarter of the two previous
and the following years’ shares. Then, we back up the new shares for periods prior
to 2003:Q4 from the new growth rates, and readjust all duration data so that the
shares add up to 1. We adjust the number of unemployed for less than one month
such that their share among unemployed for less than three months (in unadjusted
series) remains the same. All these treatments are unfortunately dictated by the
concerns we have due to data breaks, survey redesign, and methodological changes.
However, the fact that there was no major aggregate economic shock hitting the
economy around this time reassures us that the impact of our treatments on the
estimation results will be nonsubstantial. Finally, we also use the aggregate real
GDP data from the TurkStat.

\begin{table}[ht]
\centering
\begin{tabular}{ccc}
\hline
 & $u$ & F & S \\
\hline
 & 0.105 & 0.089 & 0.011 \\
 & (0.014) & (0.022) & (0.003) \\
\hline
\end{tabular}
\caption{Flow Rates}
\end{table}

Note: Standard deviations are in parentheses.

Once we make necessary adjustments to the data, we compute the aggregate
flow rates following our discussion in the preceding section. Table 1 presents the
basic moments of the data. Average unemployment in Turkey has been about

\footnote{This break may result from sample redesign in 2004, which may have allowed a better
measurement of unemployment with different durations.}

\footnote{There was also an anomaly in the unemployed for 6-7 months data for 2003:Q2 and 2003:Q3,
which generated a level shift in the seasonally adjusted data. We replace the growth rates of
shares from 2003:Q1 to 2003:Q2 and from 2003:Q2 to 2003:Q3 with the average of the growth
rate of the same quarter of the two previous and the following years’ shares.}

\footnote{Expenditure based, in 1998 prices.}
10.5 percent over our sample period, rising from around 7.5 percent to more than 14 percent in the middle of the last recession. We are in a sense fortunate to have unemployment move around this much, as it helps to identify the movements in the trend and cycle components in the flow rates even within a short sample as we have here. Observed flow rate levels in Table 1 show that the Turkish labor market also features very low rates of turnover, similar to some OECD countries. As such, our approach to use more duration data to compute the average flow hazards is clearly warranted. Similar to the pattern we observe in other countries, outflow hazard, \( F_t \), is at least six times more volatile than the inflow hazard, \( S_t \).

We also look at how the computed flow rates move with the GDP. To compare our results with other studies, we use cyclical components extracted using HP filter. As expected, we see that unemployment is countercyclical and persistent (Table 2). The unemployment rate in Turkey is more countercyclical and less persistent compared with the U.S. data. The unemployment exit rate is persistent and procyclical while the entry rate is countercyclical and not as persistent. Shimer (2005) shows that unemployment and exit rates are negatively correlated with labor productivity while job-finding rate is positively correlated for the U.S. Though cyclical properties of flow rates for Turkey are qualitatively similar to those of the U.S., there is more persistence in the U.S. data compared to the flow rates in Turkey.

| Table 2: Business Cycle Properties |
|-----------------------------|----------------|----------------|----------------|
|                            | GDP | \( F \) | \( S \) | \( u \) |
| \( \sigma \)               | 0.024 | 0.127 | 0.2 | 0.06 |
| \( \sigma / \sigma_Y \)    | 1 | 5.28 | 8.29 | 2.50 |
| \( corr(x, y) \)           | 0.45* | -0.27*** | -0.75* |
| \( corr(x, x_{-1}) \)      | 0.58* | 0.67* | -0.05 | 0.68* |

Notes: All series are quarterly and are log-detrended with HP filter and a smoothing parameter of 98. Standard deviations are in absolute terms. * is significance at %1 and *** is significance at %10.

Using the flow rates described above, we estimate the model expressed in (4)-

\[ \text{We set the smoothing parameter to 98, as suggested by Alp et al. (2011).} \]

\[ \text{Shimer (2005) reports that the correlation between unemployment and productivity is} \]

\[ -0.408 \text{ and the quarterly autocorrelation of unemployment is} 0.93 \]
via maximum likelihood. The potential identification issue appears to be not a major one for the data at hand. The log-likelihood function turns out to be well behaved and quite variable such that we can avoid the normalization for the GDP components that Tasci (2012) relies on for the U.S. data. The same is not true for the flow rates, which implies that we estimate the process for both \( \varepsilon_t^{yn} \) and \( \varepsilon_t^{yc} \), but we resort to normalization for the flow rates. Our estimation results suggest that the drift term for the trend output for this time-period in Turkey was constant, that is \( \sigma_r = std(\varepsilon_t^r) = 0 \). Hence, we impose this restriction in our estimation, obtaining \( r = 0.012 \) for the sample period. This rate translates into an average of 4.9 percent annualized quarterly growth rate for the trend output. The normalization we find to be optimal for the flow rates in this baseline model estimation implies that \( \gamma_f = \frac{\sigma_f}{\sigma_{fc}} = 0.75 \) and \( \gamma_s = \frac{\sigma_s}{\sigma_{sc}} = 0.75 \). The procedure to choose parameter values for \( \gamma_s \) and \( \gamma_f \) follows Tasci (2012) and is explained in detail in Section 4.

In our estimation, we rely on the Kalman filter to generate the log-likelihood function and to obtain the smoothed unobserved states. Because we have several variables following a random walk, initiating the Kalman filter requires starting with a diffuse prior, which requires us to exclude some of the quarters at the beginning of the sample. We exclude the first eight quarters of the data in our estimation. We discuss the potential effects of this exclusion restriction in Section 4.

In Figure 1, we plot the estimated unobserved trend components as well as the data on the flow rates, unemployment rate, and the rate of convergence, \( \lambda_t \). The

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Std</th>
<th>Estimate</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_1 )</td>
<td>1.2959</td>
<td>(0.2379)</td>
<td>( \sigma_{yn} )</td>
</tr>
<tr>
<td>( \phi_2 )</td>
<td>-0.5498</td>
<td>(0.1881)</td>
<td>( \sigma_{yc} )</td>
</tr>
<tr>
<td>( \tau_1 )</td>
<td>0.2224</td>
<td>(0.1104)</td>
<td>( \sigma_{fn} )</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>0.1305</td>
<td>(0.1022)</td>
<td>( \sigma_{sn} )</td>
</tr>
<tr>
<td>( \tau_3 )</td>
<td>0.0523</td>
<td>(0.0879)</td>
<td>( r )</td>
</tr>
<tr>
<td>( \theta_1 )</td>
<td>-0.1413</td>
<td>(0.0448)</td>
<td>( \theta_2 )</td>
</tr>
</tbody>
</table>
| \( \theta_3 \) | -0.0217 | (0.0355) | \( \theta_4 \) | \n
Notes: Log-likelihood is 443.2050, \( \gamma_f = 0.75 \), and \( \gamma_s = 0.75 \). Standard deviations are in parentheses.
upper panel of Figure 1 shows interesting changes in the underlying trends for the flow rates. In particular, the outflow rate, at which an average unemployed would find a job in a month, has increased over the course of the decade by essentially doubling from 0.06 to 0.12, implying a monthly probability of roughly 11 percent by the end of the sample. In a somewhat similar fashion, the inflow rate also trended up over the sample period, tripling from its 0.005 level to 0.015. Since the end of the last recession, the trend changed course and has started to decline towards a level of 0.012.

Figure 1: Estimation Results (Constant Labor Force)

These trend changes together imply a relatively stable pattern for the unemployment rate trend early on in the sample period, with the exception of the first recessionary episode. Then, trend unemployment gradually declines from its recession era highs of 12 percent to around 9 percent at the end of the sample. In the first part of the sample, trend changes in $F$ and $S$ offset each other to some extent as they push trend unemployment in opposing directions. However, since the end of the last recession, changes in direction of the trend behavior of
S reinforced the decline in the unemployment rate trend that is implied by the gradual increase in the outflow rate over time.

A more important observation is that overall reallocation in the labor markets have experienced a steady increase in Turkey. The picture on the lower-right panel plots the reallocation measure we look at, \( \lambda_t \), which governs the rate at which unemployment approaches its flow steady state. The magnitude of the changes over time implies that the half-life of a cyclical gap in the unemployment rate declined from more than five quarters in early 2000s to around three quarters by the end of the sample. Hence, our results not only suggest a declining trend for the unemployment rate, but also more churning in the labor market implying faster adjustments in response to cyclical changes in the unemployment rate.

**Figure 2: Variance Decomposition (Constant Labor Force)**

![Decomposition of the High-Frequency Variation](image)

![Decomposition of the Low-Frequency Variation](image)

Note: In the lower panel, the solid line shows the movement of the natural rate, given the time series of trend flow rates. Dashed lines show the path the natural rate would have followed if the trend job-finding rate would have stayed constant at its mean and the trend separation rate would have followed its actual path. Similarly, the dotted line shows the contribution of the trend job-finding rate to the trend unemployment rate.

Our framework also lends itself to analyzing the contributions of different flows to the fluctuations in the unemployment rate, both at business cycle frequency and over the long-run. The flow model laid out in the previous section gives us the estimates of cyclical and trend components in the underlying flow rates,
thereby enabling us to tease out the particular flow that drives unemployment fluctuations over the business cycle, as well as in the long-run. Hence, in principle, one can use a similar decomposition used in Fujita and Ramey (2009) to study the contribution of each flow rate to variations in the unemployment rate, both at the high and the low frequency.

Figure 2 shows results of decomposing the variance of trend and cycle unemployment rate to variations from inflows and outflows. Trend unemployment is the unemployment rate computed using trend flow rates and equation (9). Contributions of each trend flow rate to variation in natural rate is computed using the steady state unemployment formula and the average of the trend of the other flow rate. Then, series are demeaned and plotted for ease of display purposes. The lower panel of Figure 2 shows the decomposition result for the trend unemployment rate. As discussed earlier, we observe that changes in the trend of the outflow rate pushes down the long-run unemployment trend throughout the sample period, though the effect is weaker in the latter parts. The separation rate, on the other hand, contributed towards increasing the natural rate of unemployment until the end of the last crisis, and then, through a decline in its trend, started to have a dampening effect on the unemployment rate trend. Hence, as a result of offsetting effects, we observe a relatively stable unemployment rate trend until the end of 2009, followed by a decline in the long-run rate. Analyzing the upper panel of Figure 2, we see that variations in the separation rate captures most of the small movements in the cyclical component of the unemployment rate. Hence, variations in inflows in the short-run are more relevant for the movements in cyclical unemployment, while trends in both flow rates are important in determining the underlying unemployment rate trend.

3 Model with Participation

We now extend the unobserved components model described in the previous section to allow for variations in the labor force participation rate. We rely on the aggregate data in this section as well, since micro household data for Turkey is only available annually. We are also limited in our ability to distinguish between exits from unemployment into employment or into inactivity, due to lack of data availability at a high frequency. However, since the focus of the paper is to measure and estimate the flows into and out of unemployment, we do not need to
have specific information about the nature of the exit from unemployment per se. In the next subsection, we describe how to incorporate the change in the labor force into the estimation of flow rates. Then, we describe the extended unobserved components model that now allows for time variation in the labor force participation.

### 3.1 Measurement of Flow Rates with Participation

Our first task is to construct the flow rates when one allows for potential changes in the participation rate. We follow the method used in [Sengul (2014)](Sengul2014), which extends the method used by [Elsby et al. (2013)](Elsby2013) (described in the previous section), to allow for changes in labor force. Let $N_{t+\tau}$ be the number of population and let the population grow at a rate $\rho_t$ and the participation rate (the ratio of the labor force to the population) grow at a rate $G_t$. Laws of motion for the population and the participation rate are

\[
\dot{N}_{t+\tau} = \rho_t N_{t+\tau}, \quad \dot{P}_{t+\tau} = G_t P_{t+\tau},
\]

respectively, where $P_{t+\tau}$ is the participation rate ($P_{t+\tau} = L_{t+\tau}/N_{t+\tau}$).

Furthermore, let $A_t$ denote the fraction of the inactive population ($N_{t+\tau} - L_{t+\tau}$) that decide to look for a job. We can write the law of motion for unemployment as follows:

\[
\dot{U}_{t+\tau} = (L_{t+\tau} - U_{t+\tau})S_t - U_{t+\tau}F_t + (N_{t+\tau} - L_{t+\tau})A_t. \tag{17}
\]

Note that the equation above is the same as equation (7), except for the last term. However, the interpretation of $F_t$ is different. In this extension of the model, $F_t$ captures the flows out of unemployment, regardless of their destination. Since some of the outflow may be due to the inactivity, $F_t$ is the unemployment exit rate, not necessarily the job-finding rate. In equation (7), $F_t$ was the job-finding rate, as exit from unemployment can only be into employment under the baseline model.

We solve the equation (17) and iterate it three months to get the evolution of
the unemployment rate based on observed data in discrete intervals as:

$$u_t = u_{t-3}(1 - \lambda_t) + \frac{\lambda_t(S_t - A_t)}{S_t + F_t + \rho_t + G_t} + \frac{A_t(1 - e^{-3(S_t + F_t + \rho_t)})}{P_t(S_t + F_t + \rho_t)}$$, \quad (18)

where $\lambda_t = (1 - e^{-3(S_t + F_t + \rho_t + G_t)})$ is the quarterly convergence rate. Note that if $G_t = 0$ and $\rho_t = 0$ (and hence $A_t = 0$), in other words if we assume that the labor force is constant, we get the original equations of Elsby et al. (2013), which is equation (8) in the previous section. Note further that the effect of participation on law of motion for unemployment has two channels. First is that now we have to account through $A_t$ for inactive population who start looking for a job, and hence become unemployed. Also, we have to take into account that participation also changes the size of the labor force.

One can use equation (18) and write the flow steady state unemployment rate as

$$u_t^{ss} = \frac{(S_t - A_t)}{S_t + F_t + \rho_t + G_t} + \frac{A_t(1 - e^{-3(S_t + F_t + \rho_t)})}{P_t(S_t + F_t + \rho_t)\lambda_t}$$, \quad (19)

Note that the law of motion for the short-term unemployed, that is unemployed for less than five weeks becomes

$$\dot{U}_t^{<1}(\tau) = (L_{t+\tau} - U_{t+\tau})S_t - U_t^{<1}(\tau)F_t + (N_{t+\tau} - L_{t+\tau})A_t$$, \quad (20)

Also note that subtracting equation (20) from equation (17) results in

$$\dot{U}_{t+\tau} = \dot{U}_t^{<1}(\tau) - (U_{t+\tau} - U_t^{<1}(\tau))F_t$$, \quad (21)

Hence, adding the inactivity state to the model does not change the law of motion for the number of the short-term unemployed, given the difference in interpretation of $F_t$. However, solving the differential equation above and the laws of motion for the population and the participation rate (and rewriting the equation in terms of rates) yields:

$$u_t = e^{-F_t - \rho_t - G_t}u_{t-1} + u_t^{<1}$$. \quad (22)

Assuming unemployment exit occurs with a Poisson process with parameter $F_t$, the probability of exiting unemployment within a month is $\hat{F}_t = 1 - e^{-F_t}$. There-
fore, equation (22) can be rewritten as

\[ \hat{F}_t = 1 - \frac{u_t - u_t^{<1}}{e^{-G_t - \rho_t u_{t-1}}}. \]  

(23)

Notice that \( \rho_t + G_t \) is the labor force growth rate, as labor force varies due to changes in population and the participation decisions. Hence, we modify our interpretation of the change in the pool of unemployed who are not short term unemployed, to take into account the change in the size of the labor force as well, in order to get the average outflow probability.

As stated previously, this last equation does not work well for countries with average unemployment durations that are long, like Turkey. We follow Sengul (2014) and use additional duration data to increase the precision of the estimate of \( \hat{F}_t \). Based on the unemployment data by duration, we can calculate the probability that an unemployed worker exits unemployment within \( d \) months as

\[ \hat{F}^d_t = 1 - \frac{u_t - u_t^{<d}}{e^{-\sum_{j=0}^{d-1} (G_t-j+\rho_t-j) u_{t-j}}} \].

(24)

As before, we can calculate the outflow rates as

\[ F_t^{<d} = -\ln(1 - \hat{F}^d_t)/d, \]

(25)

for \( d = 1, 3, 6, 9, 12 \).

Once again, before estimating the model, we formally test the duration dependence by testing the hypothesis that \( F_t^{<1} = F_t^{<3} = F_t^{<6} = F_t^{<9} = F_t^{<12} \). We use the same procedure as in the previous section and reject the hypothesis that there is no duration dependence. We discuss computation of \( A_t \) series below when we describe the data, as we infer the series directly from the data. Given \( F_t, u_t \) and \( A_t \) series, the equation (18) gives us the separation rate data.

### 3.2 Unobserved Components with Participation Margin

Due to the length of our sample and the additional number of parameters that arise with an additional variable in our unobserved components model, we cannot fully model all the rates that determine the steady state unemployment rate. Hence, we need to make some assumptions. We begin by assuming that the population
growth $\rho_t$ has a trend and a cycle that are independent of the GDP, and we identify these components using HP filter\textsuperscript{[11]}\footnote{We also fit an AR process to the population growth and see that trend we extract does not change much.}. We also subject $A_t$ series to the same procedure. Even though one expects the cyclical component of flows from inactivity to unemployment to depend on the overall cycle (GDP), we cannot model it together with the participation rate and its growth as we run out of degrees of freedom. Since $A_t$ is measured indirectly, we think including $P_t$ and $G_t$ in our model can be more informative.

We keep the way we model $Y_t$, $F_t$, and $S_t$ as in the previous section, described in equations (1) - (3). We complement the model with the participation rate as the fourth observable, where it has a cyclical component and potentially a stochastic growth component in its trend:

\begin{align}
\bar{P}_t &= \bar{P}_t + \rho_t \\
\rho_t &= \mu_1 y_t + \mu_2 y_{t-1} + \mu_3 y_{t-2} + \varepsilon_t^{pc} \\
\bar{P}_t &= \bar{P}_{t-1} + \varepsilon_t^{pn} \\
g_t &= g_{t-1} + \varepsilon_t^g
\end{align}

(26)

As in the previous section, all the error terms are independent white noise processes and we use the Kalman filter to find the trend components.

The full extended model described by equations (1), (2), (3), and (26) can be represented in a state-space representation in the following way:

\[
\begin{bmatrix}
Y_t \\
F_t \\
S_t \\
P_t
\end{bmatrix} = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \tau_1 & \tau_2 & \tau_3 & 0 & 0 & 0 & 1 & 0 \\
0 & \theta_1 & \theta_2 & \theta_3 & 0 & 0 & 0 & 0 & 1 \\
0 & \mu_1 & \mu_2 & \mu_3 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\bar{y}_t \\
y_t \\
y_{t-1} \\
y_{t-2} \\
r_t \\
g_t \\
\bar{p}_t \\
\bar{f}_t \\
\bar{s}_t
\end{bmatrix} + \begin{bmatrix}
0 \\
\varepsilon_t^{fc} \\
\varepsilon_t^{sc} \\
\varepsilon_t^{pc}
\end{bmatrix}, \tag{27}
\]
Hence, we compute $A_t$ less than one month. The ratio of this pool to the inactive population would be the model, but computed separately as the trend implied by the HP filter. In addition to the data described in the previous section, we make use of the flow steady state equation and evaluate at the current trend levels of the variables:

$$u_t = \frac{(\bar{s}_t - \bar{a}_t)}{\bar{s}_t + \bar{f}_t + \bar{p}_t + g_t} + \bar{a}_t(1 - e^{-3(\bar{s}_t + \bar{f}_t + \bar{p}_t)}) \bar{p}_t(\bar{s}_t + \bar{f}_t + \bar{p}_t)\lambda_t,$$

where $\lambda_t = 1 - e^{-3(\bar{s}_t + \bar{f}_t + \bar{p}_t + g_t)}$. Recall that $\bar{p}_t$ and $\bar{a}_t$ are not estimated through the model, but computed separately as the trend implied by the HP filter.

### 3.3 Data and Estimation Results

In addition to the data described in the previous section, we make use of the data on unemployment by reason to construct $A_t$ series. Ideal computation would require data on labor market transitions of entrants who will be unemployed for less than one month. The ratio of this pool to the inactive population would be $A_t$. However, data on the number of unemployed for less than one month by reason of unemployment is not available. Thus, we use data on unemployment by reason for a duration less than three months and assume that the fraction of entrants among unemployed for less than three months (the shortest duration for which we have data) is the same as the fraction of entrants among unemployed for less than one month. The assumption implies that $U_t^{e,<1} / U_t^{e,<3} \approx U_t^{e,<3} / U_t^{e,<3}$, where $U_t^{e,<d}$ denotes labor market entrants who are unemployed for less than $d$ months. Note that $U_t^{e,<1} \approx U_t^{<1} U_t^{e,<3} / U_t^{e,<3}$ and we have data for the right-hand side of this approximation.

Hence, we compute $A_t$ as $U_t^{<1} U_t^{e,<3} / (N_t - L_t)$. 

\[\begin{bmatrix}
\bar{y}_t \\
y_t \\
y_t-1 \\
y_t-2 \\
r_t \\
g_t \\
\bar{p}_t \\
\bar{f}_t \\
\bar{s}_t
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\bar{y}_{t-1} \\
y_{t-1} \\
y_{t-2} \\
r_{t-1} \\
g_{t-1} \\
\bar{p}_{t-1} \\
\bar{f}_{t-1} \\
\bar{s}_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{t}^{ym} \\
\varepsilon_{t}^{yc} \\
\varepsilon_{t}^{g} \\
\varepsilon_{t}^{pm} \\
\varepsilon_{t}^{fn} \\
\varepsilon_{t}^{s}
\end{bmatrix}.
\]
We begin with the description of the flow rates under the assumption that measurement takes into account variation in the labor force participation over time. Table 4 shows the average levels of flow rates for both cases; with constant and varying labor force assumptions. We observe that relaxing constant labor force assumption affects both the levels and the standard deviations of flow rates.

Table 4: Flow Rates

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>F</th>
<th>S</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Changing Labor Force</td>
<td></td>
<td>0.105</td>
<td>0.087</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.014)</td>
<td>(0.022)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Constant Labor Force</td>
<td></td>
<td>0.089</td>
<td>0.011</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.022)</td>
<td>(0.003)</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: Standard deviations are in parentheses.

We also document the cyclical properties of these flow rates in Table 5. With this measurement, we now interpret S and A together as flows into unemployment, whereas S is the separation from employment to unemployment. Inflow rates estimated under the extended model show different business cycle frequency features than the baseline. Due to the significantly procyclical nature of the inflows to unemployment, we obtain a somewhat less countercyclical S in the current measurement. We see that participation rate does not have a significant cyclical behavior. However, with longer data available at an annual frequency, Baskaya and Sengul (2014) show that the participation rate is countercyclical, which cautions the findings regarding the cyclical behavior with a relatively short sample.

Table 5: Cyclical Properties (Changing Labor Force)

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>F</th>
<th>S</th>
<th>A</th>
<th>g</th>
<th>ρ</th>
<th>P</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>σ</td>
<td>0.024</td>
<td>0.132</td>
<td>0.377</td>
<td>0.116</td>
<td>0.012</td>
<td>0.000</td>
<td>0.010</td>
<td>0.060</td>
</tr>
<tr>
<td>σ/σY</td>
<td>1</td>
<td>5.45</td>
<td>15.62</td>
<td>4.81</td>
<td>0.48</td>
<td>0.01</td>
<td>0.43</td>
<td>2.50</td>
</tr>
<tr>
<td>corr(x, y)</td>
<td>0.45*</td>
<td>-0.030</td>
<td>0.48*</td>
<td>0.29**</td>
<td>0.110</td>
<td>0.090</td>
<td>-0.76*</td>
<td></td>
</tr>
<tr>
<td>corr(x, x₋₁)</td>
<td>0.58*</td>
<td>0.67*</td>
<td>-0.100</td>
<td>0.090</td>
<td>-0.080</td>
<td>0.32**</td>
<td>0.38*</td>
<td>0.68*</td>
</tr>
</tbody>
</table>

Notes: y is the GDP while x is the variable of interest. Growth rate series are detrended while all other series are log-detrended with an HP filter. (*): significance at 1%, (**) significance at 5%.

Results for the estimation of the extended model with participation are displayed in Table 6. Some of the individual parameter estimates lose significance,
however, overall the model is preferable to the one with these parameters excluded and to the model with no participation, as the improvement in log-likelihood is significant. Contrary to the stochastic growth rate for the output trend, labor force participation indeed has a time-varying growth rate in its trend. Consistent with the cyclical behavior of $F$ and $S$, we observe that $\tau_1$ is positive while $\theta_1$ is negative. We see that $\tau_3$ is not independently significant, but the model is preferable to the one without $\tau_3$.

Table 6: Estimation Results: 2001:Q1-2012:Q4

<table>
<thead>
<tr>
<th></th>
<th>Estimate</th>
<th>Std</th>
<th>Estimate</th>
<th>Std</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td>1.6294</td>
<td>(0.1053)</td>
<td>$\mu_1$</td>
<td>-0.6691</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>-0.8198</td>
<td>(0.0978)</td>
<td>$\mu_2$</td>
<td>0.3620</td>
</tr>
<tr>
<td>$\tau_1$</td>
<td>1.2567</td>
<td>(0.6616)</td>
<td>$\mu_3$</td>
<td>-0.0311</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>-0.4243</td>
<td>(0.4839)</td>
<td>$\sigma_{ym}$</td>
<td>0.0218</td>
</tr>
<tr>
<td>$\tau_3$</td>
<td>-0.2793</td>
<td>(0.3220)</td>
<td>$\sigma_{yc}$</td>
<td>0.0027</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>-0.6044</td>
<td>(0.3081)</td>
<td>$\sigma_g$</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.9130</td>
<td>(0.5130)</td>
<td>$\sigma_{pm}$</td>
<td>0.0042</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>-0.3779</td>
<td>(0.2436)</td>
<td>$\sigma_{fn}$</td>
<td>0.0033</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\sigma_{sn}$</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

Note: Log likelihood is 594.97. Standard deviations are in parentheses. $\gamma_f = 0.75$, and $\gamma_s = 0.75$

Our estimates of the model with varying labor force suggest that the impact on the unemployment rate could be substantial. Figure 3 plots the unemployment rate trend from the baseline model together with the estimated trend from the extended model of this section. According to our estimates, for most of the early part of the sample, the difference between two models are quite substantial, and the difference is smaller towards the end of the sample. For instance, we observe as much as a 2 percentage point difference between two trend estimates in the middle of the sample and 0.5 percentage point difference at the end of the sample period. The main reason behind the divergence between two alternative trend estimates in the early part of the sample is the behavior of the flows from the inactive population directly into the unemployment pool, $A_t$. Our measurement of $A_t$ implies a level of 0.0016 at the beginning of the sample, tumbling later by more than 75 percent over the next 12 years, most of which happened in the first five quarters. One possible interpretation is that at the early parts of the sample period there is a movement from inactivity to unemployment, which implies a natural rate
Figure 3: Unemployment Rate Trends - Impact of the Variable Participation

with variable participation rate that is very different from the one with constant participation. As $A_t$ declines, that is as flows from inactivity to unemployment slow down, we see the gap between two natural rates closing. However, we suspect that part of the decline we observe in $A_t$ could be a measurement problem in the household survey, or an extraordinary response by the non-participants to the first major recession in our sample. We do not have a convincing way to isolate one or the other. In any case, the absence of the abnormal behavior in $A_t$ later on and the apparent convergence between the two alternatives suggest that this channel is no longer as important. Moreover, the implied natural rate with a varying participation rate is lower than the one implied by the baseline model. However, their overall pattern throughout the sample, including the turning points, align very closely with each other.

Figure 4 displays all of the important unobserved components for the extended model with variable labor force participation rate. Even though the implied trend estimates for $F$ and $S$ change somewhat, results confirm the secular trends we obtained from the baseline model. More importantly, the participation rate trend implied by the estimation (right figure in middle panel) shows that there has been an important trend growth change. The participation rate has been growing in Turkey over this period, and our model identifies part of this as a trend increase.
This is not unlike the behavior in the U.S. where participation hardly responds to the business cycle, if at all. Taken together, the convergence rate now reflects the added impact of an increasing growth rate in the labor force participation, which is pictured in the lower panel.

Figure 4: Estimation Results (Variable Labor Force)

Note: Dashed lines are trend and solid lines are original series.

When computing our estimate for the trend unemployment rate, we rely on equation (29) where we substituted the HP filter of the variables \( A_t \) and \( \rho_t \). We resort to this solution because of the data availability, but we are also mindful of its potential impact on our results. Therefore, we conducted a robustness check where we model the process that governs \( A_t \) and \( \rho_t \) in a more simple linear AR process and analyzed the effect on the trend unemployment. Note that this exercise still confines to the same model with participation but the process that determines the trend components of \( A_t \) and \( \rho_t \) are assumed to be a product of a process different from a basic HP filter. The actual estimate of the trend we back out assuming AR processes yields virtually the same result. We do not report them separately to save space here.

\[12\] Results are available upon request.
4 Discussion and Robustness

We have proposed and estimated a natural rate for Turkey using a relatively parsimonious model purely relying on the flow rates in and out of unemployment. We view this concept in line with Tasci (2012) and perceive it as the steady state unemployment rate that is implied by the current trend estimates of the flow rates. Practically, this means that it is the rate of unemployment in the long-run, to which the actual unemployment rate would converge.

This view comes as a stark contrast to the alternatives that the literature focuses on, such as Gordon (1997) and Staiger et al. (1997, 2001). These studies are concerned with a natural rate concept that relates price pressures to a level of unemployment that is consistent with constant inflation rate. As we argued in the introduction, there were some structural changes in the case of Turkey, that renders such a concept uninformative. In this section, we address this issue and compare our estimates to some alternatives, including a NAIRU. Furthermore, we address some of the robustness issues of the underlying estimation we employed, such as the normalization implied by $\gamma_s$ and $\gamma_f$, as well as the exclusion restrictions for the early part of the sample in the maximum likelihood estimation.

4.1 Alternative Natural Rates and Filters

In this section, we present a basic comparison between our measures of the natural rate and some alternatives proposed in the literature. One of these alternatives is a NAIRU. One can also take a different approach and use an unobserved components method without using the flow rates, but instead focusing on the unemployment rate. We will refer to this alternative as the bivariate unobserved components model with unemployment rate (UC-UR). Finally, we will also address whether purely statistical filters could be good substitutes for our proposed natural rate.

The NAIRU estimation takes a simple form, relating the current inflation to lagged inflation and the “unemployment gap” (Gordon (1997)), where we use quarterly changes in headline CPI at an annualized rate for the measure of inflation.\footnote{More specifically, we assume that, $\pi_t = \beta_\pi \pi_{t-1} + \beta_u [u_t - \bar{u}_t] + \varepsilon_\pi$, where $\pi_t$ and $u_t$ denote actual inflation and unemployment rate, respectively. The natural rate, $\bar{u}_t$, follows a random walk, whereas the “unemployment gap”, $u_t^* = u_t - \bar{u}_t$, is assumed to follow an AR (2) process; $u_t^* = \theta_1 u_{t-1}^* + \theta_2 u_{t-2}^* + \varepsilon_u$.} The bivariate model we have in mind is similar to the flow model, but
only uses data on the actual unemployment rate and real output as in Clark (1987, 1989) and Kim and Nelson (1999). In both frameworks, one can use the Kalman filter to infer the unobserved trends in the unemployment rate much like we do for the unobserved trends in the flow rates. Our comparison relies on these unobserved trends, which are interpreted as alternative natural rates.

Figure 5: Alternative Natural Rates

Figure 5 presents these alternatives along with the flow-based estimates of the natural rate from the baseline and the extended models. Both estimated NAIRU and UC-UR are almost constant over the sample period at around 10.5 percent. There is virtually no variation at all. For NAIRU, it is very easy to understand why this is the case. Turkey experienced a sharp drop in the consumer inflation over the early part of the sample period, caused by the aggressive efforts by the newly independent central bank that effectively instituted an inflation target. This will undoubtedly affect the statistical relationship between inflation and the unemployment rate, that any NAIRU estimate will rely on. Inflation tumbled

Output is modeled as in equation (1). The observed unemployment has cyclical and trend components such that the trend component follows a random walk and the cyclical component depends on the cyclical component of the real output, much like the flow rates.

Both alternative models are estimated using maximum likelihood estimation and results are available upon request.
from levels of more than 60 percent per year to single digits in a relatively short period, while unemployment only increased modestly and stayed at those levels for some time. This, in turn, renders the relative variation in inflation with respect to unemployment uninformative. Thus, we obtain a constant NAIRU.

The bivariate model, UC-UR, also implies a constant natural rate over our sample period. This model exploits the variation in the observed unemployment relative to the cyclical changes in the real GDP to identify the natural rate. First, we observe that there are two major episodes of business cycle contractions in our sample; the first one within the first year of the sample by 6 percent and the second one coinciding with the global recession by about 15 percent. Even though the output contractions were significantly different, unemployment rate increases were almost identical, by about 70 percent, in both episodes. Moreover, the unemployment rate did not decline at all following the first recession, showing a lot of persistence. These factors imply a constant natural rate in the UC-UR case. Our method, on the other hand, can address the persistence in the unemployment rate without implying a constant natural rate since we focus on the underlying flow rates, thereby easily accommodating the non-linearities.

One might argue that if our objective is to derive an empirically useful unemployment rate trend, a pure statistical trend of the unemployment rate might be more practical, if unemployment flows do not seem to provide us with any additional information. In order to address this issue, we focus on different statistical filtering methods with and without unemployment flows to distinguish the role they play. For the sake of exposition, we focus on the baseline model.

Taking an HP-filter of the unemployment rate itself has been one approach used in the literature to identify a trend for the unemployment rate in the context of the natural rate debate (see Rogerson (1997)). We compare our estimate of the long-run trend for the unemployment rate with those that could be obtained using an HP or a bandpass filter. Figure 6 presents the results of this exercise. When we omit the information on unemployment flows and filter the quarterly unemploy-

\footnote{Note that the first recession actually started right before the beginning of our sample, in 2000:Q4, with an overall peak-to-trough decline of 10 percent in real GDP.}

\footnote{Please see Ceritoğlu et al. (2012) for more on the comparison of the unemployment in two recessions.}

\footnote{Tasci (2012) also compares a variant of our baseline model with flows to these alternatives on some other dimensions, such as the precision of estimates, required retrospective revisions with additional data, and prediction accuracy for inflation and concludes that the flow-based approach has several desirable properties along those dimensions as well.}
ployment rate (top panel), we find a lot of variation in the trend and significant
diversion across different filters. For instance, applying an HP-filter with a high
smoothing parameter (1600) gives a relatively smooth trend that moves closely
with the preferred trend from the flow model. However, a bandpass filter or an
HP-filter with a smaller smoothing parameter (98) produces much more variation
in the trend. The top panel also shows the well-known problem related to the end
points of the sample in one-sided filters.

Figure 6: Alternative Filters - The Role of Flows

A relatively different picture emerges if we include information on unemploy-
ment flows and impute an unemployment rate trend, as we did in the paper, based
on the trends of these underlying flows. As the lower panel of Figure 6 shows,
unemployment trends imputed this way do not vary much across different filters
and are much smoother than the trend estimates based solely on unemployment
rate information. Moreover, the flow model, which puts a lot more structure on
the comovement of flows and real output, produces a trend that moves closely
with these other filters. We interpret this result as evidence of the importance
of unemployment flows in understanding the unemployment rate trend over the
long run. The obvious discrepancy between various estimates of the trend with different filters when flows data are ignored makes it harder to get an empirically consistent, and otherwise useful measure.

4.2 Robustness of the Estimation

In principle, the results of our estimation could be sensitive to the exact values of \( \gamma_f \) and \( \gamma_s \) that we use. In the benchmark estimation, we use values of 0.75 for both. These parameters control the relative variation in the cyclical components of the flow rates with respect to their estimated trends. Hence, it is reasonable to have different implied unemployment rate trends with different values. To pin down the exact numbers, we follow the approach proposed in Tasci (2012). This essentially means that we re-estimate the model over a fine grid for both \( \gamma_f \) and \( \gamma_s \); \( \gamma_f = \{0.25, 0.375, 0.5, \ldots, 3.375, 3.5\} \) and \( \gamma_s = \{0.5, 0.625, 0.75, \ldots, 3.875, 4\} \).

We target two moments to match: one is the maximum log-likelihood over this combination of points, the other is the maximum correlation between the implied natural rate from the estimation and the trend of the observed unemployment rate, calculated using a bandpass filter. Since we do not use the actual unemployment rate in the estimation, we are trying to impose some discipline on the estimation by not letting it diverge too much from the data\(^{19}\). The objective here is to maximize the likelihood of the model without getting an implied unemployment trend that is far from a statistical trend obtained by the bandpass filter.

Figure 7 shows how these two moments change across \( \gamma_f \) and \( \gamma_s \). The preferred benchmark values maximize the objective of high log-likelihood and high correlation, which is clear from Figure 7. For instance, we do not improve the likelihood of the model for higher values of \( \gamma_f \), whereas smaller values do not result in any reduction. The likelihood value seems more concave in \( \gamma_f \), and the preferred value of 0.75 is close to its global maximum. As \( \gamma_s \) declines, the trend of the separation converges to a straight line; hence, the natural rate will be determined more by the trend of the job-finding rate. The opposite is true when \( \gamma_f \) is small and its trend is close to a straight line. Hence, when one flow has a constant trend imposed (low \( \gamma_i \)), and the other flow has a very small cyclical variation (high \( \gamma_j, j \neq i \)), we miss the low-frequency movements in the observed unemployment

\(^{19}\)Note that with the flow rates themselves, the unemployment rate does not give any more information for our model, hence, it is not part of it.
rate by a significant margin. Any increase in $\gamma_s$ sharply reduces the correlation of the statistical filter with the trend estimate to the extent that the correlation potentially changes sign. The objective function determines the optimal trade-off between these two dimensions by putting more weight on the more informative moment, that is, by using the inverse of the covariance matrix as the weighting matrix. Finally, for almost all of the values of $\gamma_f$ and $\gamma_s$, the natural rate implied by the model varies between 9.5 percent and 11 percent at the end of the sample.

Another robustness issue arises with respect to the exclusion restrictions. Recall that, since we model most of the trend variables as random walks, we had to start with a diffuse prior for the Kalman filter. The impact of the diffuse prior sometimes can be substantial for the first few periods, as the Kalman filter does not converge on a reasonable unconditional variance for the unobserved states. This is usually handled by ignoring the initial several periods in the actual estimation - by not considering its contribution to the log-likelihood. Since we have a very short sample, this might be somewhat tricky and we are worried about potentially losing useful information that the Kalman filter can infer from the likelihood function for the initial data points, which in this case coincide with a recession. The tradeoff is between losing valuable information from the first
several quarters versus getting potentially noisy estimates for the unconditional variance due to the diffuse prior.

In order to address this, we have re-estimated the model several times, each time excluding a larger number of quarters from the initial part of the sample. Our results suggest that after 8 quarters, the estimates for the unconditional variance behave well. Figure 8 plots the estimated natural rates corresponding to each exclusion case and shows that with the exception of the excluded part of the sample, our results do not change much. Estimated parameters reported in Table 3 correspond to the case where the likelihood function ignores the first 8 quarters. Note that this does not mean that the smoothed unobserved variables we present do not include them. They include the first 8 data points, but the parameter estimates are only estimated using the rest of the data.

5 Near-Term Prospects

Using flow rates provides us with a measure of the natural rate for the Turkish economy, which in turn can help policymakers gauge the extent of the labor market slack. Beyond providing a simple way to measure the unemployment rate
trend in a theoretically meaningful way, another useful feature of this framework has recently been highlighted by Meyer and Tasci (2013): its forecasting accuracy. Meyer and Tasci (2013) argue that by essentially disciplining the long-run trends with the unobserved components method, this modeling framework does a remarkable job in forecasting the evolution of the unemployment rate in the short- and medium-run. Since the framework heavily relies on the flow rates more than the unemployment rate itself, it is especially very flexible in capturing the non-linearities around the turning points in the business cycle. We suspect that this is even more of a concern for Turkey, where reallocation rates are much lower relative to U.S. levels. Moreover, the absence of high frequency, timely information about the unemployment rate provides the necessary motivation to come up with a good forecasting framework for Turkey.

To evaluate the forecast performance of the framework, we estimate both the baseline model and the extended version with participation rate over time starting from 2007 fourth quarter and repeating the exercise for every quarter until the end of 2012. For every estimation sample, we produce two-period ahead forecasts for the unemployment rate using the predicted flows and the observed initial condition for the unemployment rate. Note that the models produce forecasts of the flow rates internally. However, we rely on the respective equation of motion for the unemployment rate, that is equations (8) and (18). In order to gauge forecasting performance of the framework, we report one-period and two-period ahead root mean squared forecast errors (RMSFE) relative to those generated from a simple time series process for the measured unemployment rate. In particular, we choose an AR(2) process. It is important to remember that we are not running this numerical exercise with real-time data. Given the changes in the data collection and methodology over the sample period and the sheer length of the data span (or lack thereof), repeating this experiment in real time seems like a futile effort.

Table 7: Forecast Performance: RMSFEs for 2007:Q4-2012:Q4

<table>
<thead>
<tr>
<th></th>
<th>AR (2) in UR</th>
<th>Baseline Model</th>
<th>Extended Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t + 1 )</td>
<td>0.6841</td>
<td>0.5334</td>
<td>0.5295</td>
</tr>
<tr>
<td>( t + 2 )</td>
<td>1.0526</td>
<td>0.9807</td>
<td>0.9814</td>
</tr>
</tbody>
</table>

\(^{20}\) Turkish Statistical Institute only releases unemployment rate data with more than two months of lag.

\(^{21}\) The AR process we assume takes the form \( u_t = \kappa_1 u_{t-1} + \kappa_2 u_{t-2} + \epsilon_t \), where data is quarterly.
Table 7 reports RMSFEs for one- and two-quarter ahead forecasts from the two models we used in the paper and the AR process that does not rely on flow rates at all. As forecast errors suggest, both models produce more accurate unemployment rate forecasts relative to the time series model for the forecast sample period we considered, especially at one-quarter ahead forecast horizon. This relative improvement in forecast accuracy over the near-term could provide a useful tool for policymakers in Turkey.

Figure 9: Forecasting Performance of Both Models

Having established a relative improvement in forecasting the unemployment rate with the unobserved components models we used in the paper, we finally provide the predictions of them conditional on the data we have for the whole sample; 2001:Q1-2012:Q4. Even though our sample ends by the end of 2012, we have actual unemployment rates until the end of August 2013. Hence, we have three quarters of data to compare the real-time forecasts from the models. Figure 9 presents the forecast paths for the baseline model as well as the extended model with participation. Regardless of the model we use, we predict a slight increase in the unemployment rate beyond 2012, which has been confirmed given the data for the first three quarters of 2013. Recall that the model with participation implies a lower natural rate in the long-run, therefore its higher levels of unemployment
rate in the first four quarters of the data, compared to the baseline forecast, are followed by a decline below the path implied by the baseline.

6 Conclusion

We use a parsimonious unobserved components model with unemployment flow rates, similar to the one used by Tasci (2012) for the U.S., to estimate a time-varying unemployment rate trend for Turkey that is grounded in the modern theory of labor market search. We believe that the specific challenges presented by the Turkish data makes it a worthwhile exercise. One of these challenges was the importance of the participation rate behavior, which we handled by extending the basic model to incorporate time-varying labor force participation.

Our results suggest that the natural rate for unemployment, or the underlying trend, is hovering around 9 percent by the end of 2012 for Turkey. Models with and without the participation margin imply substantially different estimates at the earlier parts of the sample period and the gap narrows over time, with the extended model featuring the participation rate predicting a level slightly below 9 percent. This is due to a slow down in the rate of flows from inactivity to unemployment. More importantly, we find that the reallocation rate, the sum of the inflow and outflow rates, has been gradually trending up for Turkey suggesting an increasingly dynamic labor market. Finally, we argue that the modeling framework we provide here can be used for near-term forecasting of the unemployment rate with relative ease and accuracy.

We are mindful of the main caveat of our paper: the sample size. Our data covers only 11 years at quarterly frequency. However, the fact that we have quite a bit of variation in the variables of interest over the sample period reassures us that the lack of longer time-series data does not undermine the usefulness of our approach. In future work, it would be interesting to focus on understanding the secular increase in the reallocation rate over time.
References


37