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Posterior-Predictive Evidence on US Inflation using Extended Phillips Curve Models with non-filtered Data*

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Abstract

Changing time series properties of US inflation and economic activity, measured as marginal costs, are modeled within a set of extended Phillips Curve (PC) models. It is shown that mechanical removal or modeling of simple low frequency movements in the data may yield poor predictive results which depend on the model specification used. Basic PC models are extended to include structural time series models that describe typical time varying patterns in levels and volatilities. Forward and backward looking expectation components for inflation are incorporated and their relative importance is evaluated. Survey data on expected inflation are introduced to strengthen the information in the likelihood. Use is made of simulation based Bayesian techniques for the empirical analysis. No credible evidence is found on endogeneity and long run stability between inflation and marginal costs. Backward-looking inflation appears stronger than forward-looking one. Levels and volatilities of inflation are estimated more precisely using rich PC models. The extended PC structures compare favorably with existing basic Bayesian vector autoregressive and stochastic volatility models in terms of fit and prediction. Tails of the complete predictive distributions indicate an increase in the probability of deflation in recent years.

Keywords: New Keynesian Phillips curve, unobserved components, time varying parameters, level shifts, inflation expectations, survey data
JEL Classification: C11, C32, E31, E37

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1 Introduction

Modeling the relation between inflation and fluctuations in economic activity has been one of the building blocks of macroeconomic policy analysis. Often, the analysis of this relation, denoted as Phillips Curve (PC) models, is conducted using the short-run variations in inflation and economic activity.[1] The conventional method for extracting this short run variation in the observed series is to demean and detrend the data prior to analysis, see Galí and Gertler (1999). However, mechanical removal of the low frequency movements in the data may lead to misspecification in the models, as suggested in Ferroni (2011) and Canova (2012) for DSGE models. The existence of complex low frequency movements, such as structural breaks and level shifts in the observed series, in particular, in the inflation series, is well documented in the literature, see McConnell and Perez-Quiros (2000) and Stock and Watson (2008). Distinct periods with different patterns can be observed for the non-filtered inflation series. The period between the early 1970s and the early 1980s is often labeled as a high inflationary period compared to earlier and later periods. A similar type of statement holds for economic activity. The real marginal cost series, often used as a proxy for economic activity, see Galí and Gertler (1999), follows a negative trend which is amplified in the recent decades. The importance of joint analysis of such high and low frequency movements in macroeconomic data has recently been documented, see Ferroni (2011), Delle Monache and Harvey (2011), Canova (2012), and Faust and Wright (2013).

In this paper we aim to contribute to this literature in four ways. We illustrate and discuss possible effects that simple prior filtering of the low frequencies in the data may have on posterior and predictive inference using a basic PC model. The issue is that the observed inflation and marginal cost data have more complex low frequency structures than just a simple constant mean and/or a basic linear or HP trend. We show that this

[1]For notational convenience we use the abbreviation 'PC' instead of the common abbreviation of the New Keynesian Phillips Curve models.

misspecification affects posterior inference of the structural PC parameters and gives poor forecasting results depending on the model specification. In the online appendix, we present extensive evidence on this feature using a set of simulated and real data and a range of PC model structures. Obviously, in well specified models and in series with relatively constant means and linear trends the misspecification effects are not severe. However, from the outset, the use of mechanical filters without properly examining the frequency features of the data is not advisable.

We extend the basic PC model by specifying structural time series models which allow for stochastic trends, structural breaks and stochastic volatility in inflation and log marginal cost series and integrate these with the basic model. The more complex model structure enables the identification of the relation between macroeconomic variables inherent in the PC model, together with possible long and short run dynamics in each series.

Next, we enrich the extended PC models to include both forward and backward looking expectation components. There is a debate in the literature on the relative weights of these two components in explaining and forecasting inflation patterns in the U.S.. Our combined model structure can provide valuable information on that point.

As a final contribution we make use of survey data on inflation expectations from the University of Michigan Research Center, which provides quarterly one year ahead inflation expectations. It is well known that the class of PC models including complex time series features and basic expectation mechanisms is not easy to estimate given the usually weak data information and the few available weak instrumental variables. The proposed richer expectation mechanism and making use of survey data strengthen the likelihood information and are expected to make inference more efficient and forecasting more accurate.

Several alternatives to structural time series models for efficiently combining the PC model with explicit low frequency movements in the data are available. One alternative is to focus only on the high frequencies by rewriting the likelihood in the frequency domain and maximizing the (log)likelihood only over a portion of fluctuations, see e.g. Christiano and Vigfusson (2003). Another alternative is to utilize multiple prior filters, to capture possibly incorrectly specified low frequency components, see Canova and Ferroni (2011). Here we focus on explicitly modeling the low frequency movements to improve the predictive performances of the structural form models while we keep the theoretical model at a simple tractable level.

We apply the proposed set of models to quarterly U.S. data over the period 1960-I until 2012-I. For all models considered, posterior and predictive results are obtained using a simulation based Bayesian approach. Our results indicate that PC structures with three additional components (structural time series features, expectation mechanisms and inflation survey data) capture time variation in the low and high frequency movements of both inflation and marginal cost data. For the inflation series, the extended model identifies distinct periods with different inflation levels and volatilities. In terms of marginal cost series, the local linear trend specification accommodates the smoothly changing trend observed in the series, specifically after 2000. We also find improved forecasting performance of the extended PC models when these are compared with basic PC models with demeaned and/or detrended data and with the standard stochastic volatility model proposed by Stock and Watson (2007) and, further, with an extended Bayesian Vector Autoregressive (BVAR) model which accounts for changing levels, trends and volatility in the data. The model comparison is based on predictive likelihood and Mean Squared Forecast Error (MSFE) comparisons. The Bayesian approach we adopt has additional appealing features for the models considered. In terms of inflation predictions, several measures of interest, such as deflation probabilities obtained from the lower tail of the complete predictive densities, are obtained as a by-product of Bayesian inference. Furthermore, for the most general model with good fit and forecasting features, weak endogeneity and almost non-existence of a stable long-run relationship between inflation and marginal cost series can easily be assessed using the posterior draws of the trends and levels.

The structure of this paper is as follows: Section 2 presents the three extensions to the standard Phillips curve model structure. Section 3 summarizes the likelihood, prior and the posterior sampling algorithm. Section 4 provides the application of the proposed models and the standard PC model on U.S. inflation and marginal cost data. Section 5 concludes. Additional illustrations, results, details of the posterior sampling algorithms and references are provided in the online appendix.

2 Extended Phillips Curve models

We start with a standard PC model based on a priori filtered data. Next, we extend this model with a structural time series model in order to deal with low and high frequencies that are present in U.S. inflation and the low frequency property in the U.S. log marginal cost series. Thirdly, we extend the latter PC model by introducing a Hybrid PC model (HPC) with both backward and forward looking inflation expectations making use of observed inflation expectations from survey data.

The standard PC can be derived by the approximation of the equilibrium conditions of the firms under staggered price setting using the Calvo formulation, see Calvo (1983). The Calvo model implies that a fraction of firms optimize their prices while the remaining fraction, i.e. non-optimizing firms, keep their prices unchanged. Assuming zero inflation at the steady state the basic PC model derived from the firm's price setting is given as

$$\begin{aligned} \tilde{\pi}_t &= \lambda \tilde{z}_t + \gamma_f E_t(\tilde{\pi}_{t+1}) + \epsilon_{1,t}, \\ \tilde{z}_t &= \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \epsilon_{2,t}, \end{aligned} \tag{1}$$

where $\tilde{\pi}_t$ is the filtered inflation and $\tilde{z}_t$ is the filtered (log) real marginal cost, $(\epsilon_{1,t}, \epsilon_{2,t})' \sim NID(0, \Sigma)$, λ is the slope of the Phillips curve, γ_f is the weight given to the forward looking inflation, and standard stationary restrictions hold for (ϕ_1, ϕ_2).

One way to estimate this model is to replace the expectation term with actual inflation values relying on rational expectations. Another option is to use survey data on expected inflation as 'observed' expectations. Still, direct substitution of survey data for expected inflation does not exploit the full model structure. In our modeling strategy, we use the full data generating process for real marginal costs together with the Phillips curve relation to form inflation expectations.[2] Iterating the model forward and computing future expected inflation, model (1) implies that inflation can be expressed as the sum of the current and future discounted stream of the real marginal costs. Given the AR(2) dynamics for the long run deviation of the marginal costs, one can compute this sum and obtain a closed-form solution of model (1). The PC model takes the form of an instrumental variable model with nonlinear parameters in the inflation equation[3]

$$\begin{aligned} \tilde{\pi}_t &= \frac{\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f}\tilde{z}_t + \frac{\phi_2\gamma_f\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f}\tilde{z}_{t-1} + \epsilon_{1,t} \\ \tilde{z}_t &= \phi_1\tilde{z}_{t-1} + \phi_2\tilde{z}_{t-2} + \epsilon_{2,t}. \end{aligned} \tag{2}$$

One way to estimate the structural parameters is by estimating the parameters of the unrestricted reduced form model using a uniform prior and solve for the structural form parameters, see the online appendix and Kleibergen and Mavroeidis (2011) for details. However, this parameter transformation involves a complex Jacobian determinant that may seriously obscure posterior inference on the structural parameters. Hence we opt for estimating structural parameters directly.

Extended PC models: low frequency components, non-filtered data

We depart from the standard PC model by avoiding a priori data filtering and emphasize that data filtering is an integral part of modeling from an econometric point of view. Specifically, we make use of models with time varying levels as well as volatility for capturing both the low and high frequency changes in the U.S. inflation and marginal cost series. Furthermore, estimating data filters together with other model parameters

[2] We also estimate the model by inserting the survey expectations directly in model (1). The results are provided in the online appendix.

[3] The model in (2) can be written as a triangular simultaneous equations model (SEM).

concerns the uncertainty related to long run specifications. Modeling the data filters explicitly takes this uncertainty into account while the use of filtered data does not. Finally, prior data filtering also has important effects on the predictive performance of the models as shown in section 4.

There exists a substantial literature on the connection between actual inflation and target inflation and the firms' pricing behavior. We summarize the major issues here. In full equilibrium DSGE models with explicit monetary policy modeling, the mean level of the inflation is related to the target inflation rate. In these specifications, the target inflation rate is either assumed to be constant or is allowed to change to accommodate variation in inflation level. Prominent examples include Woodford (2003) and Sargent et al. (2006), who fix the target inflation and Erceg and Levin (2003), Schorfheide (2005), Ireland (2007) and Liu et al. (2011), who allow for discrete or continuous changes in the target inflation level.

In our partial equilibrium PC models, the specification of the steady state inflation level and the firms' decision process are of key importance for the final model structure. In the standard PC models as in Galí and Gertler (1999), assuming zero steady state inflation and keeping the prices fixed for the non-optimizing firms results in the standard form of PC as in model (1). Ascari (2004) and Ascari and Sbordone (2013), extend these models to allow for constant positive trend inflation and they analyze the implications of the trend inflation on the PC structure. Cogley and Sbordone (2008) take one-step further and derive the PC model with time-varying trend inflation modeled as a driftless random walk. Adding a trend inflation to standard PC assumptions, while preserving the assumption that non-optimizing firms keep their prices fixed causes the resulting PC coefficients to depend on the trend inflation. Thus, the interpretation of the coefficients differs from the standard model in these extended models.

Other assumptions on non-optimizing firm's pricing behaviour include indexation on past inflation (i.e. non-optimizing firms change their prices based on past inflation), see Smets and Wouters (2003) and Christiano et al. (2005). Alternatively, Smets and Wouters (2007) and Liu et al. (2011) make use of steady state inflation. As discussed in Ascari (2004) and Levin and Yun (2007) the structure of the PC remains as in model (1) with constant parameters if the non-optimizing firms adjust their price by the steady state inflation. Indeed, this is the route taken in Yun (1996), Jeanne (1998) and Schorfheide (2005). Moreover, Nason and Smith (2008) provide empirical evidence in favor of stable structural parameters. In our extended PC models with non-filtered data we follow this assumption and keep the structural parameters constant focusing on short and long run inflation levels.

The proposed joint modeling of data filters and other model parameters is also motivated by the stylized facts for the non-filtered U.S. inflation and log marginal cost data, shown in Figure 1 over the period between 1960-I and 2012-I.[4] The left panel displays distinct periods with differing inflation patterns. The period between the early 1970s and the early 1980s can be labeled as a high inflationary period with high volatility compared with the remaining periods. Existing evidence shows that the decline in inflation level and volatility is due to credible monetary policy that stabilized inflationary expectations since the early eighties, see McConnell and Perez-Quiros (2000) and Stock and Watson (2007). One way to model this changing inflation behavior is to allow for regime changes in parameters, see Sims and Zha (2006) and Cogley and Sbordone (2008). We consider two cases for the inflation process. In the first case, we assume continuous level shifts in inflation using a random walk process

$$c_{\pi,t+1} = c_{\pi,t} + \eta_{1,t+1}, \quad \eta_{1,t} \sim NID(0, \sigma^2_{\eta_1}). \tag{3}$$

-Insert Figure 1 about here-

Alternatively, we consider an inflation level subject to occasional and discrete shifts.

[4]Inflation is computed as the continuously compounded growth rate of the implicit GDP deflator and for the real marginal cost series we use labor share in non-farm business sector obtained from http://research.stlouisfed.org/fred2/, see Galí and Gertler (1999) for details. The right panel in Figure 1 displays real marginal cost series, in natural logarithms and multiplied by 100.

Such level shifts are modeled as follows

$$c_{\pi,t+1} = c_{\pi,t} + \kappa_t \eta_{1,t+1}, \quad \eta_{1,t} \sim NID(0, \sigma^2_{\eta_1}), \tag{4}$$

where κ_t is a binary variable taking the value of 1 with probability p_κ if there is level shift and the value 0 with probability $1 - p_\kappa$ if the level does not change. This model structure allows for level shifts to depend on p_κ while preserving a parsimonious model structure with only a single additional parameter. Occasional and large level shifts correspond to low values of p_κ together with high values of σ_{η_1}. When p_κ is 1, the model becomes the local level model of (3). We use both specifications (3) and (4) in the empirical analysis.

The real marginal cost series, shown in the right panel of Figure 1, does not exhibit discrete changes as observed in the inflation series. These data instead have a continuously changing pattern around a negative trend, which can be attributed to technology shocks. Since this trend is more prominent in the second half of the sample period, we allow for a changing trend using a local linear trend specification

$$\begin{aligned} c_{z,t+1} &= \mu_{z,t} + c_{z,t} + \eta_{2,t+1}, \quad \eta_{2,t} \sim NID(0, \sigma^2_{\eta_2}) \\ \mu_{z,t+1} &= \mu_{z,t} + \eta_{3,t+1}, \qquad\qquad \eta_{3,t} \sim NID(0, \sigma^2_{\eta_3}). \end{aligned} \tag{5}$$

This specification is flexible enough to encompass many types of filters used for detrending, see Delle Monache and Harvey (2011) and Canova (2012) for a similar specification in the more general context of DSGE models. When $\sigma^2_{\eta_3} = 0$, the level of the real marginal costs follow a random walk with a drift, μ_z. Additionally, when $\sigma^2_{\eta_2} = 0$, a deterministic trend is obtained. Note that, setting only $\sigma^2_{\eta_2} = 0$ but allowing $\sigma^2_{\eta_3}$ to be positive results in an integrated random walk process which can approximate nonlinear trends including the Hodrick-Prescott (HP) trend.

Together with the level specifications of the inflation and real marginal cost series, the PC model in (2) using (4) and (5) takes the following form

$$\begin{aligned}
\pi_t - c_{\pi,t} &= \tfrac{\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f}\left(z_t - c_{z,t}\right) + \tfrac{\phi_2\gamma_f\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f}\left(z_{t-1} - c_{z,t-1}\right) + \epsilon_{1,t}, \\
z_t - c_{z,t} &= \phi_1\left(z_{t-1} - c_{z,t-1}\right) + \phi_2\left(z_{t-2} - c_{z,t-2}\right) + \epsilon_{2,t}, \\
c_{\pi,t+1} &= c_{\pi,t} + \kappa_t\eta_{1,t+1}, \\
c_{z,t+1} &= \mu_{z,t} + c_{z,t} + \eta_{2,t+1}, \\
\mu_{z,t+1} &= \mu_{z,t} + \eta_{3,t+1},
\end{aligned} \tag{6}$$

where $(\epsilon_{1,t},\ \epsilon_{2,t})' \sim NID\left(0, \left(\begin{smallmatrix} \sigma^2_{\epsilon_1} & \rho\sigma_{\epsilon_1}\sigma_{\epsilon_2} \\ \rho\sigma_{\epsilon_1}\sigma_{\epsilon_2} & \sigma^2_{\epsilon_2} \end{smallmatrix}\right)\right)$, $(\eta_{1,t},\ \eta_{2,t},\ \eta_{3,t})' \sim NID\left(0, \left(\begin{smallmatrix} \sigma^2_{\eta_1} & 0 & 0 \\ 0 & \sigma^2_{\eta_2} & 0 \\ 0 & 0 & \sigma^2_{\eta_3} \end{smallmatrix}\right)\right)$ and the disturbances $(\epsilon_{1,t},\ \epsilon_{2,t})'$ and $(\eta_{1,t},\ \eta_{2,t},\ \eta_{3,t})'$ are independent for all t.

Adding stochastic volatility as high frequency component

A further refinement in the PC model can be achieved allowing for time variation in the variances of the disturbances. This extension is particularly appealing for the inflation series, as the inflation variance changes over time substantially, see e.g. Stock and Watson (2007) for a reduced form model with a stochastic volatility component. The following state equation extends the PC model with a stochastic volatility process for inflation

$$h_{t+1} = h_t + \eta_{4,t+1},\ \eta_{4,t+1} \sim NID(0, \sigma^2_{\eta_4}), \tag{7}$$

where we specify a time-varying volatility, $\sigma_{\epsilon_{1,t}} = \exp(h_t/2)$, in the first equation in (6). We follow Stock and Watson (2007) by fixing the value of $\sigma^2_{\eta_4}$ prior to analysis to facilitate inference. We set $\sigma_{\eta_4} = 0.5$, which seems to work well for U.S. inflation.

An important estimation challenge in this extended model is the close relation between the changing inflation levels and volatilities. These changing data patterns can be captured by either of these model components which makes it hard to identify these components unless one makes strong prior restrictions. We fix the value of $\sigma^2_{\eta_4}$ prior to analysis to facilitate inference and in order to impose smoothness in the volatility process. It is straightforward to extend the analysis with a more flexible, strong, stochastic prior so that the parameter $\sigma^2_{\eta_4}$ is estimated together with the rest of the parameters. We report on this in section 4.

Hybrid PC: forward and backward expectations using survey data

The specification of the HPC can be derived using an assumption on the firm's behavior, where a fraction ω of the firms, that are unable to reset their prices, adjust their price by the lagged inflation rate. The HPC model takes then the form of

$$\begin{aligned} \tilde{\pi}_t &= \lambda^H \tilde{z}_t + \gamma_f^H E_t(\tilde{\pi}_{t+1}) + \gamma_b^H \tilde{\pi}_{t-1} + \epsilon_{1,t}, \\ \tilde{z}_t &= \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \epsilon_{2,t}, \end{aligned} \tag{8}$$

where parameters of the HPC model, indicated by a superscript H are functions of the price stickiness parameter, a discount factor and the fraction of firms with backward looking pricing behavior. We note that the HPC has the same forward looking inflation expectation term in the model as the PC but the HPC has both a backward and forward looking component due to the specification of the lagged inflation deviation.

As in the PC case, we opt for using the full information approach by exploiting the information in the data generating process for real marginal costs.[5] Iterating the first equation forward and solving for the expected inflation, the HPC implies the triangular simultaneous equations model which is nonlinear in parameters

$$\begin{aligned} \tilde{\pi}_t &= \frac{\lambda^H}{(1-\gamma_b^H\gamma_f^H)(1-(\phi_1+\phi_2\gamma_f^H)\gamma_f^H)}\tilde{z}_t + \frac{\phi_2\gamma_f^H\lambda^H}{(1-\gamma_b^H\gamma_f^H)(1-(\phi_1+\phi_2\gamma_f^H)\gamma_f^H)}\tilde{z}_{t-1} \\ &+ \frac{\gamma_b^H\gamma_f^H}{(1-\gamma_b^H\gamma_f^H)}\sum_{k=1}^{\infty}(\gamma_f^H)^k E_t(\tilde{\pi}_{t+k}) + \frac{\gamma_b^H}{(1-\gamma_b^H\gamma_f^H)}\tilde{\pi}_{t-1} + \frac{1}{(1-\gamma_b^H\gamma_f^H)}\epsilon_{1,t} \\ \tilde{z}_t &= \phi_1\tilde{z}_{t-1} + \phi_2\tilde{z}_{t-2} + \epsilon_{2,t}. \end{aligned} \tag{9}$$

Unlike the PC solution, this system has a lagged inflation term and an infinite sum of inflation expectations. A closed form solution for the latter expression only exists under certain assumptions such as rational expectations.

We do not follow this route but proceed differently. Consider $E_t(\tilde{\pi}_{t+k}) = E_t(\pi_{t+k}) - E_t(c_{\pi,t+k})$ which is the difference between expected future inflation and the expected future value of the low frequency component of inflation that we modeled in (4) as a

[5] We also estimate the model by inserting the survey expectations directly in model (8). The results are provided in the online appendix.

process that is similar to a random walk but subject to occasional and discrete level shifts and it has a bounded variance. One can interpret this difference as the difference between short and long run inflation expectations. As a next step we substitute the observed survey data on next period's expected inflation, denoted by μ_t, for the expected inflation in period $t+1$, i.e. $\mu_t = E_t(\pi_{t+1})$ and we assume the following partial adjustment mechanism

$$\mu_t - c_{\pi,t+1} = \beta(\mu_{t-1} - c_{\pi,t}) + \eta_{5,t+1}, \tag{10}$$

where $|\beta| < 1$ and $\eta_{5,t+1}$ is iid and $E_t(\eta_{5,t+1}) = 0$. Iterating this equation forward and taking expectations one obtains $E_t(\mu_{t+k-1} - c_{\pi,t+k}) = \beta^{k-1}(\mu_t - c_{\pi,t+1})$. That is, the partial adjustment mechanism described in (10) implies that the further one gets into the future the smaller will be the difference between short and long run inflation expectations. Estimates of β will indicate the empirical speed of adjustment. For instance, for a value of the posterior mean of β equal to 0.5 it follows that within a few periods one has almost complete adjustment. Given the restriction on β one can solve (10) for μ_t and obtain $\mu_{t-1} = c_{\pi,t} + \sum_{j=0}^{\infty} \beta^j \eta_{5,t-j}$. That is, the observed survey inflation expectations are equal to the long run unobserved inflation pattern and an infinite moving average of errors with declining weights that are determined by the adjustment mechanism given in (10). This adaptive mechanism has a Bayesian learning and updating interpretation on the difference between short and long run expected inflation. Using this mechanism, the term $\sum_{k=1}^{\infty}(\gamma_f^H)^k E_t(\tilde{\pi}_{t+k})$ in (9) can be rewritten and the HPC model becomes

$$\begin{aligned}
\pi_t - c_{\pi,t} = \;& \tfrac{\lambda^H}{(1-\gamma_b^H\gamma_f^H)(1-(\phi_1+\phi_2\gamma_f^H)\gamma_f^H)}\left(z_t - c_{z,t}\right) + \tfrac{\phi_2\gamma_f^H\lambda^H}{(1-\gamma_b^H\gamma_f^H)(1-(\phi_1+\phi_2\gamma_f^H)\gamma_f^H)}\left(z_{t-1} - c_{z,t-1}\right), \\
& + \tfrac{\gamma_b^H\gamma_f^H}{(1-\gamma_b^H\gamma_f^H)}\tfrac{\gamma_f^H}{1-\gamma_f^H\beta}(\mu_t - c_{\pi,t}) + \tfrac{\gamma_b^H}{(1-\gamma_b^H\gamma_f^H)}\left(\pi_{t-1} - c_{\pi,t-1}\right) + \tfrac{1}{(1-\gamma_b^H\gamma_f^H)}\epsilon_{1,t}, \\
z_t - c_{z,t} = \;& \phi_1\left(z_{t-1} - c_{z,t-1}\right) + \phi_2\left(z_{t-2} - c_{z,t-2}\right) + \epsilon_{2,t}.
\end{aligned} \tag{11}$$

We emphasize that alternative models on inflation expectations exist, see Mankiw et al. (2003). For instance, a skew density for $\eta_{5,t}$ allows systematic under- or overoptimism. This is an interesting topic for further research but outside the scope of the present paper.[6]

Note that the model-implied expectation is for GDP inflation while the overlaid data is CPI inflation expectations. For this reason we subtract the average difference between CPI and GDP inflation from the survey data.[7] Furthermore, since the survey data provide four-steps-ahead (one-year) expectations, we divide the survey data by 4, assuming constant expectations over the year.

The PC model in (6) is a special case of (11) when $\gamma_b^H = 0$. Then the model becomes purely forward looking. Similar to the PC model, we consider three case of the HPC model with different specifications for inflation: (i) continuous level changes; (ii) discrete occasional level changes; and (iii) discrete occasional level changes and stochastic volatility.

3 Bayesian inference

In this section we summarize the prior specifications, our use of prior predictive likelihoods, and the posterior sampling algorithms for the extended PC and HPC models.

Prior specification for parameters and prior predictive likelihood

The extended PC and HPC models contain several additional parameters compared to the standard PC model. We classify the model parameters in five groups, and assign independent priors for each group. The first group includes the common parameters in the PC and HPC models, $\theta_N = \{\lambda, \gamma_f, \phi_1, \phi_2, \Sigma\}$, in (1). For the structural parameters $\{\lambda, \gamma_f, \phi_1, \phi_2\}$ we define flat priors on restricted regions, which also ensure that the

[6]Alternatively, survey expectations may be measured with an error. In this case one can specify unobserved inflation expectations anchored around observed survey expectations. We consider this possibility and report these estimation results in the online appendix. Such extensions do not seem to alter the results.

[7]We thank an anonymous referee for pointing this out. Our approach of recalculating the inflation expectations is similar to Del Negro and Schorfheide (2013).

autoregressive parameters, ϕ_1 and ϕ_2, are in the stationary region and the (observation) variance priors are of inverse-Wishart type[8]

$$\begin{aligned} p(\lambda, \gamma_f, \phi_1, \phi_2 | \Sigma) &\propto \text{ constant for } |\lambda| < 1, \ |\gamma_f| < 1, \ |\phi_1| + \phi_2 < 1, \ |\phi_2| < 1, \\ \Sigma &\sim IW(1, 20 \times \tilde{\Sigma}), \end{aligned} \tag{12}$$

where $IW(\nu, \Psi)$ is the inverse Wishart density with scale Ψ and degrees of freedom ν. It is possible to use economic theory or steady state relationships to construct priors for these parameters, see Del Negro and Schorfheide (2008). We do not follow this approach but let the data information dominate our relatively weak prior information. For the same reason, we perform a prior-predictive analysis and investigate the sensitivity of our posterior results with respect to the prior.

Note that the prior specifications of the observation and state covariances are important in this class of models and for macroeconomic data. Since the sample size is typically small, differentiating the short-run variation in series (the observation variances) from the variation in the long-run (the state variation) can be cumbersome, see Canova (2012). We therefore impose a data based prior on the observation covariances. We first estimate an unrestricted reduced form VAR model using demeaned inflation series and (linear) detrended (log) real marginal cost series, and base the observation variance prior on this covariance estimate, $\tilde{\Sigma}$. This specification imposes smoothness for the estimated levels and trends, and ensures that the state errors do not capture all variation in the observed variables. Second, prior distributions for the extra model parameters stemming from the hybrid models, $\theta_H = \{\gamma_b^H, \beta\}$ are defined as uniform priors on restricted regions $|\gamma_b^H| < 1$, $|\beta| < 1$. Third, we define independent inverse-Gamma priors for the state variances in (3)–(5)

$$\sigma_{\eta_1} \sim IG(20, 20 \times 10^{-2}), \quad \sigma_{\eta_2} \sim IG(20, 20 \times 10^{-3}), \quad \sigma_{\eta_3} \sim IG(1, 1 \times 10^{-5}), \tag{13}$$

[8]We experimented with wider truncated uniform densities for the λ and γ_f parameters. The prior truncation does not seem to have a substantial affect on the posterior results.

where $IG(\alpha, \alpha\xi)$ is the inverse-Gamma distribution with shape α and scale $\alpha\xi$. Parameters α and ξ are the a priori number and variance of dummy observations.

Similar to the standard counterparts, the extended PC and HPC models may also suffer from flat likelihood functions. We therefore set weakly informative priors for the state parameters, such that not all variation in inflation and marginal cost series are captured by the time-varying trends and levels. For example, the number of prior dummy observations for σ_{η_1} and σ_{η_2} is much less than the number of observations to limit the prior information.

The fourth prior distribution we consider is applicable to the PC and HPC models with level shifts. For these models, we consider a fixed level shift probability of 0.04. This choice leads to an a priori expected number of shifts of 8 for 200 observations in the sample. Alternatively, this parameter can be estimated together with other model parameters. However, often the limited number of level shifts plague the inference of this parameter. Hence, we set this value, obtained trough an extensive search over intuitive values of this parameter, prior to analysis.

Finally, for the stochastic volatility models, we specify an inverse-gamma prior for the marginal cost variances. For the correlation coefficient, ρ, we take an uninformative prior $p(\rho) \propto (1-\rho^2)^{-3/2}$, see Çakmaklı et al. (2011).

In the proposed models, it is important to assess the effects of the specified prior distributions on the predictive likelihoods. Due to the nonlinear structure of the models, assessing the amount of prior information on the predictive results is not trivial. We present a prior-predictive analysis as in Geweke (2010). For each of the extended PC and HPC model, we consider 1000 parameter draws from the joint prior distribution and compute the prior predictive likelihoods for the period between 1973-II and 2012-I. Hence a comparison of the resulting prior predictions will indicate which model is preferred by the priors. We provide these results in section 4.

Posterior existence and the sampling algorithm

We summarize the Bayesian inference for the proposed models. An important point regarding the posterior of the structural parameters is the existence of a posterior distribution and its moments, which depends on the number of instruments and the prior. Given one relatively weak instrument (the second lag of the marginal cost series) the posterior will have very fat tails and the existence of the posterior distribution is ensured through priors defined on a bounded region, see Zellner et al. (2013) for a detailed analysis of a linear IV model with small numbers of weak instruments.

The MCMC sampler for the full conditional posterior distribution is based on Gibbs sampling with a Metropolis-Hastings step and data augmentation, combining the methodologies in Geman and Geman (1984); Tanner and Wong (1987); Gerlach et al. (2000) and Çakmaklı et al. (2011). Details are provided in the online appendix.

4 Posterior and Predictive Evidence

In this section we present posterior and predictive evidence on several features of the extended PC models using U.S. data on inflation and marginal costs. We compare the results with those obtained from alternative reduced form models like BVAR models and the stochastic volatility model from Stock and Watson (2007). Specifically, we estimate two PC models with demeaned inflation series and with detrended real marginal costs using a linear trend or the HP filter, which are labeled PC-LT and PC-HP, respectively. In six extended PC models we make use of structural time series models to specify low and high frequencies. The first three of these models allow for continuous changes in the level of inflation (PC-TV), in addition discrete occasional level shifts (PC-TV-LS), and in further addition stochastic volatility for inflation (PC-TV-LS-SV). The final three models use the HPC framework with forward and backward looking expectations and using survey data. The corresponding extensions are denoted as HPC-TV, HPC-TV-LS and HPC-TV-LS-SV. All six models use the local linear trend specification in (6) for the real marginal cost series. A summary of the eight models used in this paper is given in Table 1. As a robustness check, we considered several other model specifications in order to obtain a smooth transition from a basic PC model with a mechanical filter to an extended PC model with low and high frequencies in levels and volatilities, rich expectation mechanisms and survey data. These results, together with a detailed discussion are reported in the online appendix.

Posterior evidence

We display the estimation results in Table 2 and focus on four features: slope of the Phillips Curve; weight of forward and backward inflation expectations; degree of endogeneity and persistence in survey expectations. First, the slope of the PC ($\lambda^{(H)}$) is estimated around 0.07 and 0.09 which is slightly higher than the conventional estimates of the Phillips curve slope, that indicate an almost flat curve, see e.g. Galí and Gertler (1999); Galí et al. (2005); Nason and Smith (2008). When we model the levels of the series explicitly, $\lambda^{(H)}$ drops to values around 0.05 for both PC and HPC models. A possible explanation for this difference is the departure from the zero steady state inflation assumed in the traditional PC models. As shown in Ascari (2004) and Ascari and Ropele (2007) among others, when firms that cannot re-optimize their prices keep their prices fixed, trend inflation can affect the slope of the PC. In this case, this slope is a decreasing function of the trend inflation. Still, in both PC and HPC models, the estimated slopes are substantially different from zero as point 0 is outside the 95% Highest Posterior Density Interval (HPDI) for most cases.

-Insert Table 2 about here-

Second, with respect to inflation expectations, the coefficient of the short-run inflation expectations in Table 2, $\gamma_f^{(H)}$, is much lower than the conventional estimates, which are above 0.9 in most cases. A potential reason for this finding is the methodology used. Conventional Bayesian analyses often impose dogmatic priors on this parameter unlike our uninformative prior specification. When we consider the PC model with the subjective discount factor γ_f, the (implied) prior for the discount factor (either directly or through other parameter's priors in the steady state relations) is either fixed to the values around 0.99, see Smets and Wouters (2003) for example, or it is tightly centered around 0.99, see for example Schorfheide (2005); An and Schorfheide (2007). We also notice a relatively higher posterior standard deviation for this parameter, hence another potential cause of this finding is the relatively low information content in the data about this parameter. This is in accordance with the discussion in the section 3 on the shape of the likelihood in these macro-models. Note that the more conventional values of this parameter are still inside the 95% HPDI.

Another reason might be the fact that, even if the models are estimated without a restriction, in most cases inflation expectations are replaced by the real leading value of the inflation relying on the rational expectations hypothesis, see e.g. Galí and Gertler (1999) and Sims (2002). However, we opt for explicitly solving for expectations resulting in a highly nonlinear system of simultaneous equations.

A striking result from Table 2 is the relative importance of the forward and backward looking components of the HPC, measured by parameters γ_f^H and γ_b^H. On the one hand, the evidence in Galí et al. (2005) suggests a dominant forward looking effect. Cogley and Sbordone (2008) document that the forward looking component of the HPC model dominates once the trend variation in inflation is taken into account. Similarly, Benati (2008) shows that under stable monetary regimes with clearly defined nominal anchors, inflation appears to be (nearly) forward looking. On the other hand, many studies including Fuhrer and Moore (1995); Rudd and Whelan (2005) document a dominant backward looking effect. Our results favor the latter view since the effect of the backward looking component of inflation estimated by the HPC models in the bottom panel of Table 2 are substantially higher than those of the forward looking components. More specifically, Table 2 shows that the HPC and PC model results differ in terms of the forward looking components' coefficient $\gamma_f^{(H)}$. From an economic point of view, these results maybe driven by the model assumptions on firm behavior that differs from those of Cogley and Sbordone (2008) and Benati (2008). From an econometric point of view, as in the PC case, the specification of the prior distribution is crucial. In many analyses, the implied prior on these parameters suggests a support of the distribution in the interval [0.5,1] ([0,0.5]) for γ_f^H (γ_b^H), see Smets and Wouters (2003, 2007); Benati (2008); Del Negro and Schorfheide (2008) and Del Negro and Schorfheide (2013). Hence, the difference may be partly due to the presence of only one weak instrument (second order lagged marginal costs), see Nason and Smith (2008) for further empirical results and a discussion on this topic.

Third, the contemporaneous correlation between the observation disturbances determines the degree of endogeneity of the log real marginal costs in the PC. The estimates of this correlation parameter, ρ, are displayed in the fifth column of Table 2. Posterior means of ρ from all PC models are negative and close to 0, with high standard deviations and point 0 is inside the 95% HPDI. For the HPC models, posterior means of ρ are mostly positive with an even smaller magnitude. Therefore, the endogeneity problem does not seem to be severe and single equation inference may yield credible results for inflation and marginal costs. Still, we refrain from doing so since one neglects several cross-equation restrictions in that case.

Fourth, the β parameter, which indicates the adaptation of the short run survey expectations to the long run inflation, has posterior means given in the fifth column of Table 2. All HPC models indicate relatively quick adjustment, as the posterior means are around 0.5.

Estimated Levels, Volatilities and Breaks

We present estimated levels, trends, inflation volatilities and break probabilities for the proposed HPC models in Figures 2, 3 and 4, respectively. Estimates for the PC counterparts are similar, and are provided in the online appendix.

-Insert Figures 2, 3, 4 about here-

The top-left panel of Figure 2 shows estimated levels for the HPC-TV-LS-SV model.

We first stress that models that only allow for discrete and occasional level shifts lead to smoother inflation levels compared to the model that allows for continuous level changes. Detailed results on this issue are provided in the online appendix. In DSGE models, mean inflation is generally connected to the inflation target in the central bank's policy rule. Hence movements in trend inflation reflect to a large extent changes in the monetary policy target (see also Schorfheide (2005); Cogley and Sbordone (2008)). Adding stochastic volatility to the model with level shifts creates more frequent discrete changes in the inflation level, possibly reflecting the uncertainty in monetary policy target captured by volatility changes. Estimated marginal cost levels for the HPC-TV-LS-SV are given in the top-right panel of Figure 2 and indicate a slightly nonlinear trend during the sample period.

Figure 3 presents estimated volatility levels for the (H)PC model with level shifts and the stochastic volatility component. The stochastic volatility pattern coincides nicely with data features of the Great Moderation. The decline in inflation level and volatility after the 1980s is linked to credible monetary policy that stabilized inflationary expectations at a low level via commitment to a nominal anchor since the early eighties, see Ahmed et al. (2004); Stock and Watson (2007). The effect of this is also seen in the inflation levels presented in Figure 2. This period of low volatility is replaced by a volatile period after 2005 and during the recent financial crisis. A slight difference between PC and HPC models occurs during the volatility peaks around 1975. High volatility is distributed more evenly in the HPC model with stochastic volatility, whereas for the PC counterpart, high volatility is concentrated around 1975. Peak points of estimated volatilities coincide with rapid and substantial changes in inflation.

Estimated break probabilities for the PC and HPC models with and without the stochastic volatility component are presented in Figure 4. The estimated level shift probabilities for the PC-TV-LS model identify four major shifts in the inflation level around 1966, 1973, 1982 and 2005. Note that the estimated shift probabilities in the

PC-TV-LS-SV model demonstrate the complementarity of level shifts and changing volatility. The probabilities follow a similar pattern with the PC-TV-LS model but the periods subject to level shifts are much longer. During the highly volatile periods of the 1970s, the model produces clear signals of changing inflation levels, as high volatility causes rapid changes in inflation. Accordingly, low volatility periods are characterized by mild but significant changes in inflation. This shows the complementarity of the stochastic volatility component and level shifts.

Predictive Performance

Predictive performances of the models are reported using predictive likelihoods, MSFEs and predictive densities which enable us to report the deflation probabilities.

The first metric we consider is the predictive likelihoods of all models in order to compare the density forecasts of the models. The one-step ahead predictive likelihood of the observation at t_0+1, y_{t_0+1}, conditional on the previous observations $y_{1:t_0}$, is

$$f(y_{t_0+1}|y_{1:t_0}) = \int p(y_{t_0+1}|X_{t_0+1},\theta)p(X_{t_0+1},\theta|y_{1:t_0})dX_{t_0+1}d\theta, \tag{14}$$

which can be computed by first generating $\{X_{t_0+1}\}_{m=1}^{M}$ for M posterior draws, using the corresponding state equations. Next, the predictive likelihood of the observation at t_0+1 can be approximated by $\frac{1}{M}\sum_{m=1}^{M} p(y_{t_0+1}|X_{t_0+1}^{m},\theta^{m})$, where $p(y_{t_0+1}|X_{t_0+1}^{m},\theta_{1:t_0}^{m})$ is a multivariate normal density and M is a sufficiently large number.

We base the MSFE and predictive likelihood comparisons on the inflation predictions. For the general case of $h \geq 1$ period ahead forecasts, the predictive density of inflation at time t is calculated conditional on the inflation and marginal cost data up to time t, the estimated mean marginal cost values for the periods $t+1,\dots,t+h$ and, if $h>1$, on the estimated mean inflation levels for the periods $t+1,\dots,t+h-1$. For all models using survey expectations, predictive likelihoods are also conditioned on the observed survey expectations up to time t.

A feature of the predictive likelihoods is that these can be evaluated by $p(y_{t_0+1:T}) =$

$\prod_{t=t_0}^{T} f(y_{t+1}|y_{1:t})$, which provides a tool to analyze the contribution of each observation at time period t, see Geweke and Amisano (2010). For the models with a priori demeaned and detrended data predictive likelihoods do not take into account the parameter uncertainty arising from this a priori step. We choose to calculate the predictive likelihoods this way, which is a fair replication of the literature.

Accurate point predictions of inflation are of key importance to economic agents such as investors and central banks. Therefore, we consider MSFE, computed as the mean of the sum of squares of the prediction errors. Point forecasts for inflation are defined as the mean of the predictive distribution, which is consistent with a quadratic loss function. We report MSFE for one and four period ahead forecasts in order to examine the forecasting ability of the models for longer horizons.

As a third performance criteria, we report the deflation risk indicated by each model, which are computed as the lower tail probability of the one step ahead predictive distributions.

Apart from the models considered so far, we include alternative reduced form models that are proven to have good predictive abilities. The first model is the unobserved component model proposed by Stock and Watson (2007), henceforth denoted as SW2007. This model captures the unobserved trend in inflation where both inflation and trend volatility follow a stochastic process, see SW2007 for details. The second model is an unrestricted Bayesian VAR (BVAR-SV) model with two lags and with stochastic volatility for inflation. BVAR models are one of the workhorse models used for forecasting macroeconomic series. For the sake of brevity, we do not provide details of this class of models and refer to Del Negro and Schorfheide (2013). We use the proposed 'TV' model extension in the BVAR-SV model, which allows for continuous changes in the level of inflation and a smoothly changing trend for the marginal cost series. Both SW2007 and BVAR-TV-SV models are strong competitors for the extended PC and HPC models we propose. In all considered models, the data based prior distributions from section 3, calculated using the full sample data, are used.

Predictive likelihoods and MSFE of the alternative models are presented in Table 3. The likelihood contribution of each observation and the corresponding cumulative predictive likelihoods are displayed in Figure 5. We present the log predictive likelihoods of the competing models in the first column of Table 3. These values together with Figure 5 indicate three groups of models in terms of their predictive performances. The first group of models include the conventional PC models with demeaned and detrended data (PC-LT and PC-HP). The second group consists of the PC models with time variation in inflation levels (PC-TV, PC-TV-LS) together with BVAR-TV-SV and the SW2007 model. The models in the second group have much superior performance in terms of the predictive likelihood values. A second increase in the predictive likelihood values can be observed when we consider the models in the third group, namely the HPC models (HPC-TV, HPC-TV-LS, HPC-TV-LS-SV) and the PC model together with discrete level shifts and stochastic volatility for inflation (PC-TV-LS-SV).

-Insert Table 3 and Figure 5 about here-

A similar clustering of models is observed when we compare model performances using the one period ahead MSFE with the exception of the BVAR-TV-SV model. BVAR-TV-SV model performs considerably better in terms of point prediction compatible with the HPC models.

Three main conclusions can be drawn from these findings. First, the conventional PC models with demeaned and detrended data (PC-LT and PC-HP) perform worse than the competing models both in terms of MSFE and in terms of the cumulative predictive likelihood metric. The difference between HPC and PC models in terms of point forecasts is less pronounced compared to the increase in precision when switching from models using demeaned and detrended data to the models that use the raw data. Hence it is important to estimate levels and trends together with the structural model parameters.

Second, the difference between the PC model with level shifts and stochastic volatility and the basic PC models is substantial. Thus, models with level shifts and stochastic volatility deliver more accurate point predictions considering MSFE and predictive likelihood values. These results pinpoint to the importance of incorporating both high and low frequency movements in structural models. This performance increases further in the HPC models, which incorporate the survey data and the backward looking component.

Third, structural models perform at least as well as the strong reduced form candidates, the SW2007 and BVAR-TV-SV models. These findings are crucial since structural models deliver both structural macroeconomic information and predictive performance, whereas the reduced form models are solely designed for improving the predictive performance.

The evolution of the model performance over the forecast sample is shown in Figure 5. An important finding from the figure is the increasing predictive performance of the HPC models and the models with stochastic volatility components after mid 1980s. This period is characterized by a decrease in inflation volatility during the Great Moderation, which the stochastic volatility component captures accurately. Moreover, the effect of the level shifts can be observed when we compare the PC-TV-LS-SV model with the SW2007 model. Much of the difference in the performance of these models can be attributed to the changes in inflation levels. This shows that the inflation process exhibits several regime changes.

The last metric we use for model comparison considers the implied deflationary risk. The left panel in Figure 6 shows the entire density of the inflation predictions for the HPC-LS-SV model where the levels and trends are estimated together with the structural parameters. The mean predicted inflation is represented by the solid line, and the width of the predictive distribution is indicated by the white area under the inflation density. As expected, inflation predictions are concentrated around high (low) values during the high (low) inflationary periods. The uncertainty around the inflation predictions are also high for these periods, together with the periods when inflation is subject to a transition to low values around 1980s. When the observed inflation values are close to the zero bound, the predictive densities indicate deflationary risk.

-Insert Figure 6 about here-

The right panel in Figure 6 displays this deflationary risk, which is of key importance especially for policy making. The figure shows that PC models with a priori demeaned and detrended data do not signal any pronounced deflation risk except for the low deflation probabilities during mid 1970s and mid 1980s. However, extended PC and HPC models exploiting the high and low frequency movements produce clear signals of deflation risk and deflationary pressure during the recent recession.

Note that actual deflation only occurs around 2009 in this sample period and the models signal deflationary risk slightly later than this period. This result can be explained by the agents' learning process. As indicated in Schorfheide (2005), if agents learn about the monetary policy changes later than the inflation level changes, the perceived target inflation in general equilibrium happens only gradually. In Schorfheide (2005), this is incorporated as Bayesian learning of the agents which is in line with the econometric assumption underlying our models. As the modeled state-space updating incorporates Bayesian learning, changes in the inflation level occur gradually and deflationary risk signals are delayed. Our models are still able to capture this deflationary pressure successfully.

Prior predictive likelihoods of proposed models

Due to the complex model structures in the proposed models, it is important to address the effects of the specified prior distributions on the predictive performances. We therefore perform the prior predictive analysis outlined in section 3 for the extended PC models, for the forecast sample analyzed earlier, covering the period between 1973-II and 2012-I. Table 4 presents the average and cumulative prior predictive likelihoods for the forecast sample. Prior predictive likelihoods, not using the data information and also using weak prior information, naturally perform worse than the predictive results reported in Table 3. Table 4 also shows that the adopted prior distributions clearly favor the less parameterized model, PC-TV. Moreover, the priors clearly do not favor models with stochastic volatility components. Most importantly, the 'best performing model' according to the predictive results in Table 3, HPC-TV-LS-SV, is the least favorable one according to the adopted prior distributions using the same forecast sample. We therefore conclude that data information is dominant, and the superior predictive performance of the HPC-TV-LS-SV model is not driven by the prior distribution.

-Insert Table 4 about here-

Robustness analysis

The proposed models extend the standard models in several ways and the predictive improvements in section 4 stem from these extensions jointly. However, it is important to address predictive gains from each proposed model extension. We therefore estimate several alternative models, in order to find which distinct gains from each model extension can be identified. For space limitations, detailed results are provided in the online appendix and here we briefly summarize the main findings.

Predictive gains obtained from including the survey expectations in the models are substantial and incorporating the low and high frequency data movements in the model is crucial. These two conclusions are in line with Faust and Wright (2013), who consider a large set of alternative models for inflation forecasting, including unrestricted reduced form models, and compare their forecast performances based on MSFE. Our model incorporates both these features in the PC model structure. Third, once survey data and time variation are included in the model, there are still additional predictive gains from the backward looking component in the hybrid models. We therefore conclude that the superiority of the most extensive model, HPC-TV-LS-SV, is based on all proposed model extensions jointly.

We conclude this section with a remark on the possible existence of a long run stable relation between inflation and marginal costs. The models we considered so far rely on the implicit assumption of the absence of a long-run cointegrating relationship. We assess whether this assumption is plausible for the U.S. data considering the HPC-TV-LS-SV model, and find credible evidence that the existance of such a cointegrating relationship is very unlikely. Details on this result are provided in the online appendix.

5 Conclusion

Phillips curve models constitute an integral part of macroeconomic models used for forecasting and policy analysis. These models are often estimated after demeaning and/or detrending the data. In this paper it is shown that mechanical removal of the low frequency movements in the data may lead to poor forecasts. Potential structural breaks and level shifts as well as changing volatility in the observed series require more complex models, which can handle these time variation together with the standard PC parameters. We have proposed a set of models where levels and trends of the series together with the volatility process are integrated with a structural PC model. Furthermore, we consider richer expectational mechanisms for the inflation series in enlarged Hybrid-PC models using survey data for inflation expectations.

The proposed models capture time variation in the low frequency movements of both inflation and marginal cost data. For the inflation series we identify three distinct periods with high and low inflation. The high inflationary period corresponds to 1970s, following a low inflationary period of 1960s. The last period starting with 1980s is characterized by low inflation levels corresponding to an annual inflation level around 2%. When this model is blended with the stochastic volatility component, the level shifts can be identified even more precisely.

The use of macroeconomic information in the structural models together with the remaining high and low frequency movements in the data improves the predictive ability also compared to celebrated reduced form models, including the Bayesian VAR and the stochastic volatility model, see Stock and Watson (2007). Furthermore, modeling inflation expectations using survey data and adding stochastic volatility to the PC model structure improves in sample fit and out of sample predictive performance substantially. We also analyze deflation probabilities indicated by each competing model. The complete predictive densities, most notably from the enlarged models, indicate an increase in the probability of deflation in the U.S. in recent years.

Modeling forward and backward looking components of inflation has important effects on empirical results. Endogeneity and persistence do not appear to be very important empirical issues in PC model structures. Finally, we analyze the existence of a long-run relation between the low frequency movements of both series. No credible evidence is found on such a long run stable cointegrating relation for the U.S. series.

Given that incorporating low and high frequency movements explicitly in macroeconomic models provides additional insights for both policy analysis and more accurate predictions, we plan to enlarge the proposed model to a more general DSGE framework in future work. Another interesting possibility of future research is to combine different PC models using their predictive performances, which seem to be time varying.

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Tables and Figures

Table 1: Standard and extended Phillips curve models

model structure / low/high frequencies	Phillips curve	Hybrid Phillips curve
Inf: constant level RMC: linear trend	PC-LT	HPC-LT*
Inf: constant level RMC: Hodrick-Prescott filter	PC-HP	HPC-HP*
Inf: time varying levels RMC: local linear trend	PC-TV (2)-(3)-(5)	HPC-TV (3)-(5)-(10)-(11)
Inf: time varying levels and switching RMC: local linear trend	PC-TV-LS (2)-(4)-(5)	HPC-TV-LS (4)-(5)-(10)-(11)
Inf: ... and stochastic volatility RMC: local linear trend	PC-TV-LS-SV (2)-(4)-(5)-(7)	HPC-TV-LS-SV (4)-(5)-(7)-(10)-(11)

Note: Results for the models indicated by (*) are provided in the online appendix. 'Inf' ('RMC') stands for Inflation (Real Marginal Cost).

Table 2: Posterior results of alternative Phillips curve models

Model	$\lambda^{(H)}$	$\gamma_f^{(H)}$	γ_b^H	β	ρ	ϕ_1	ϕ_2
PC-LT	0.07 (0.03)	0.36 (0.24)	–	–	-0.01 (0.02)	0.84 (0.05)	0.08 (0.05)
PC-HP	0.10 (0.05)	0.43 (0.27)	–	–	-0.05 (0.04)	0.66 (0.05)	-0.01 (0.05)
PC-TV	0.06 (0.03)	0.39 (0.25)	–	–	-0.09 (0.06)	0.82 (0.05)	0.06 (0.05)
PC-TV-LS	0.05 (0.02)	0.36 (0.24)	–	–	-0.06 (0.05)	0.82 (0.05)	0.07 (0.05)
PC-TV-LS-SV	0.06 (0.02)	0.32 (0.23)	–	–	-0.02 (0.07)	0.87 (0.05)	0.10 (0.05)
HPC-TV	0.04 (0.02)	0.01 (0.01)	0.42 (0.12)	0.52 (0.29)	0.01 (0.06)	0.81 (0.05)	0.07 (0.05)
HPC-TV-LS	0.04 (0.02)	0.01 (0.01)	0.47 (0.10)	0.50 (0.19)	0.02 (0.01)	0.81 (0.06)	0.16 (0.07)
HPC-TV-LS-SV	0.06 (0.02)	0.03 (0.05)	0.21 (0.11)	0.56 (0.21)	-0.01 (0.01)	0.87 (0.05)	0.10 (0.05)

Note: The table presents posterior means and standard deviations (in parentheses) of parameters for the competing New Keynesian Phillips Curve (PC) type models estimated for quarterly inflation and real marginal costs over the period 1960-I until 2012-I. λ (λ^H) and γ_f (γ_f^H) are the slope of the Phillips curve and the coefficient of inflation expectations in PC (HPC) model in (2) ((11)). γ_b^H is the coefficient of the backward looking component in the HPC model in (11). H superscript denotes the parameters of the hybrid models while these parameters without H superscript correspond to the PC model counterparts. β is the autoregressive parameter for the deviation of the short run expectations from the long run, as defined in (10). ρ is the correlation coefficient of the residuals ϵ_1 and ϵ_2. ϕ_1 and ϕ_2 are the autoregressive parameters for the real marginal cost specification in model (2). Posterior results are based on 40000 simulations of which the first 20000 are discarded for burn-in. Model abbreviations are as in Table 1.

Table 3: Predictive performance of Phillips curve models and reduced form alternatives

Model	Cumulative (Log) Pred. Likelihood	MSFE 1 period ahead	MSFE 4 period ahead
SW2007	-78.03	0.17	0.25
BVAR-TV-SV	-97.98	0.10	0.25
PC-LT	-139.33	0.35	0.36
PC-HP	-157.19	0.46	0.37
PC-TV	-46.16	0.14	0.26
PC-TV-LS	-61.97	0.14	0.28
PC-TV-LS-SV	-33.48	0.13	0.21
HPC-TV	-36.38	0.12	0.28
HPC-TV-LS	-35.05	0.11	0.24
HPC-TV-LS-SV	-18.15	0.09	0.18

Note: The table reports the predictive performances of all competing models for the prediction sample over the period 1973-II until 2012-I. 'Cumulative (Log) Pred. Likelihood' stands for the sum of the natural logarithms of predictive likelihoods. 'MSFE' stands for the Mean Squared Forecast Error. Results are based on 10000 simulations of which the first 5000 are discarded for burn-in. 'SW2007' stands for the model proposed by Stock and Watson (2007), and 'BVAR-TV-SV' stands for the Bayesian VAR model with time varying levels and trends and a stochastic volatility component for the inflation equation. Remaining abbreviations are as in Table 1.

Table 4: Prior-predictive results for the PC model structures

Model	Average (Log) Pred. Likelihood	Cumulative (Log) Pred. Likelihood
PC-TV	-1.16	-180.88
PC-TV-LS	-1.36	-210.91
PC-TV-LS-SV	-1.45	-224.66
HPC-TV	-1.28	-199.22
HPC-TV-LS	-1.27	-197.68
HPC-TV-LS-SV	-2.04	-318.77

Note: The table reports the prior-predictive performances of all competing models for the prediction sample over the period 1973-II until 2012-I. 'Average (Cumulative) Log Pred. Likelihood' stands for the average (sum) of the natural logarithms of predictive likelihoods. Results are based on 1000 simulations from the joint priors of model parameters. Model abbreviations are as in Table 1.

Figure 1: Inflation, inflation expectations and log real marginal cost ($\times 100$) series over the period 1960-I until 2012-I

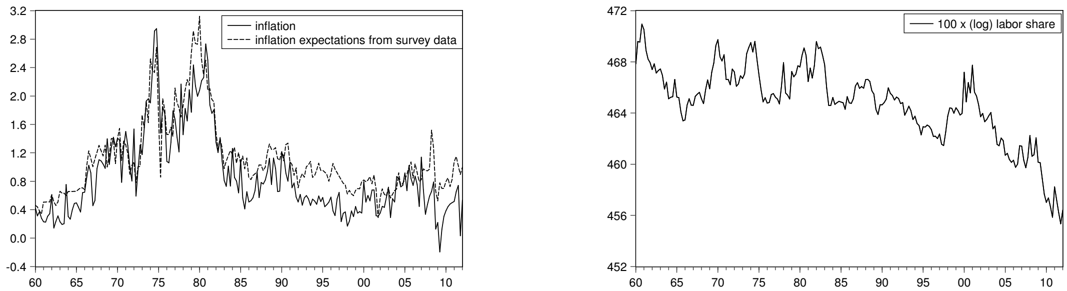
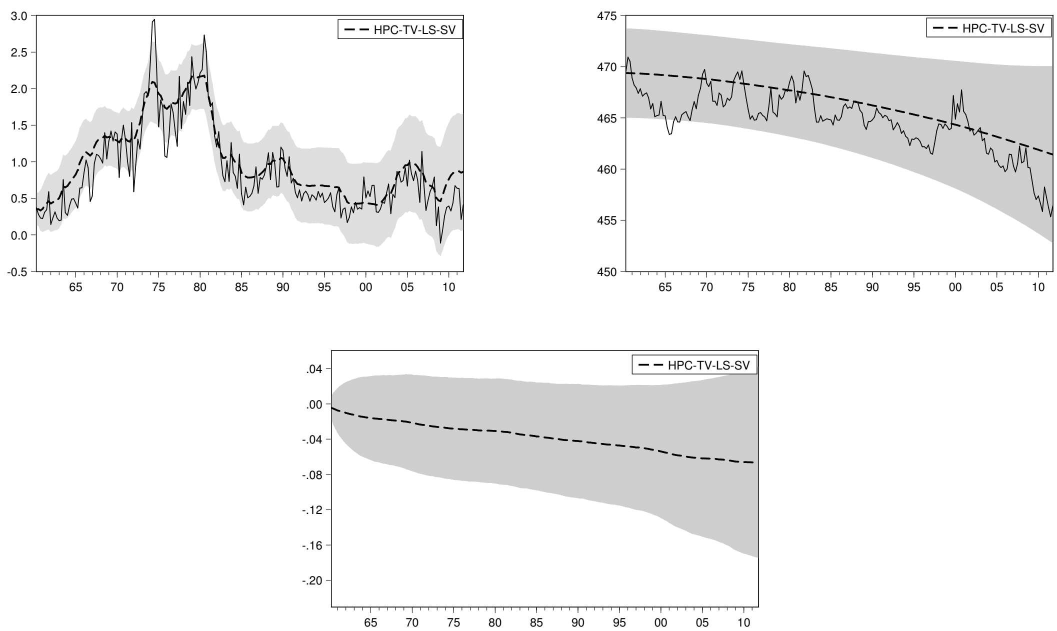
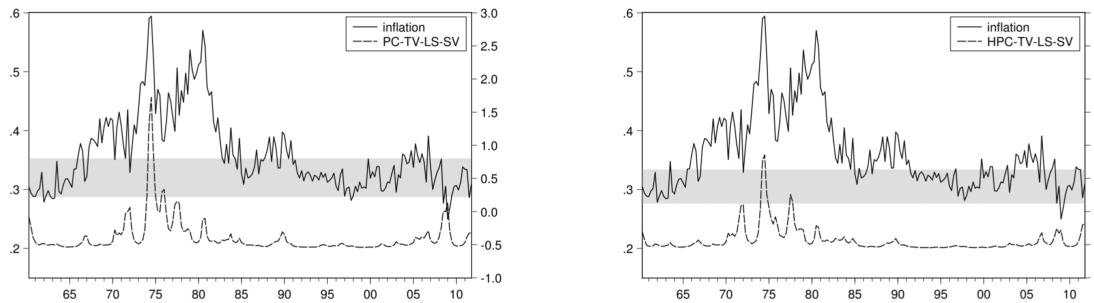


Figure 2: Level, trend and slope estimates from the HPC-TV-LS-SV model



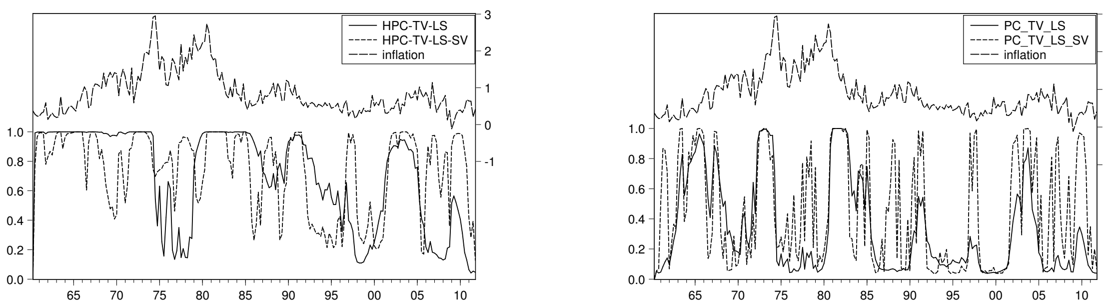
Note: The top-left panel exhibits estimated inflation levels, $c_{\pi,t}$ in model (6). The top-right and bottom panels show estimated (log) real marginal cost levels and the slopes, $c_{z,t}$ and $\mu_{z,t}$ in model (6), respectively. Grey shaded areas correspond to the 95% HPDI. Model abbreviations are as in Table 1. Results are based on 40000 simulations of which the first 20000 are discarded for burn-in.

Figure 3: Estimated inflation volatility from the (H)PC-TV-LS-SV models



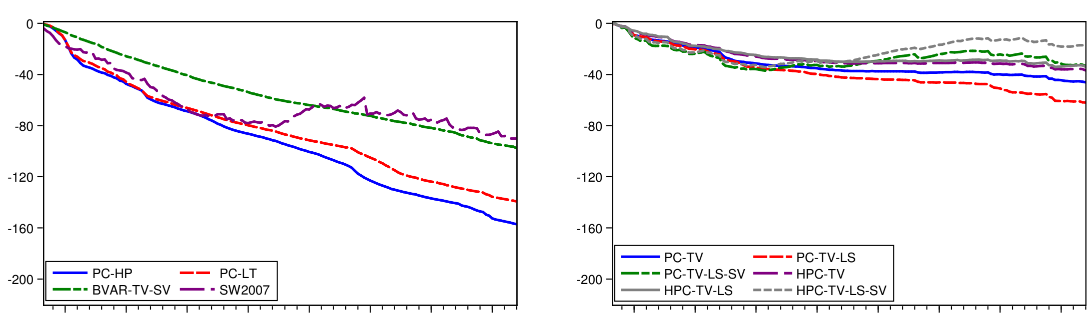
Note: The dashed and solid lines show the posterior mean of the time varying inflation volatility and the observed inflation level. The shaded areas are the 90% HPDI of inflation volatility estimated by the equivalent models without the stochastic volatility components. Results are based on 40000 simulations of which the first 20000 are discarded for burn-in.

Figure 4: Estimated level shift probabilities for the PC and HPC models



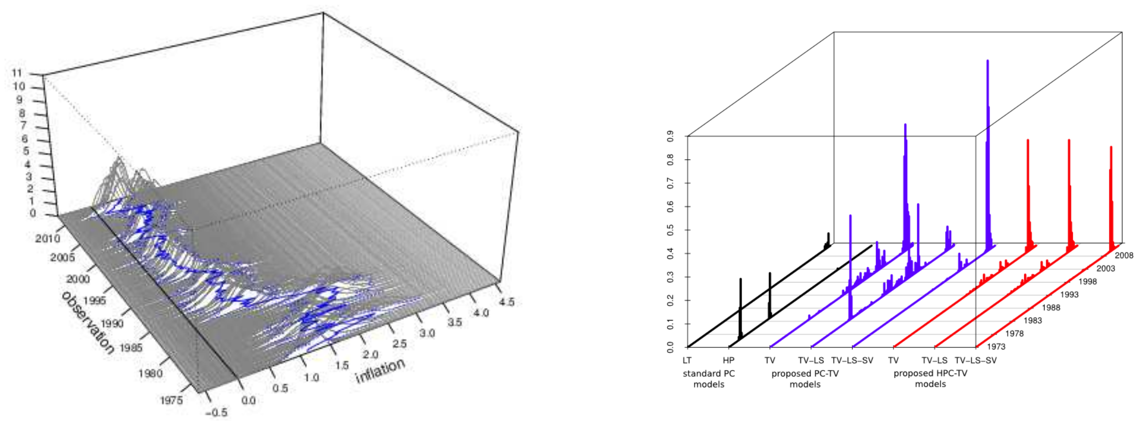
Note: The solid and dotted lines are the posterior means of the estimated level shift probabilities from the (H)PC-TV-LS model and the (H)PC-TV-LS models, respectively. The dashed line is the observed inflation level. Results are based on 40000 simulations with the first 20000 discarded for burn-in.

Figure 5: Predictive likelihoods from competing models



Note: The figure displays the evolution of the (log) predictive likelihoods for the computing models over the period 1973-II until 2012-I. Model abbreviations are as in Table 1. Results are based on 5000 simulations of which the first 10000 are discarded for burn-in.

Figure 6: Predicted inflation densities from HPC-LS-SV model and deflation probabilities implied by different Phillips curve models



Note: The left figure presents one period ahead predictive distribution of inflation from the HPC-LS-SV model, over the period 1973-II until 2012-I. The right figure presents deflation probabilities computed using these predictive distributions of inflation over the same period. Model abbreviations are as in Table 1. Results are based on 5000 simulations of which the first 10000 are discarded for burn-in.