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Risk, Uncertainty, and Expected Returns*

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Abstract

A conditional asset pricing model with risk and uncertainty implies that the time-varying exposures of equity portfolios to the market and uncertainty factors carry positive risk premiums. The empirical results from the size, book-to-market, and industry portfolios as well as individual stocks indicate that the conditional covariances of equity portfolios (individual stocks) with market and uncertainty predict the time-series and cross-sectional variation in stock returns. We find that equity portfolios that are highly correlated with economic uncertainty proxied by the variance risk premium (VRP) carry a significant, annualized 6 to 8 percent premium relative to portfolios that are minimally correlated with VRP.

JEL classification: G10, G11, C13.
Keywords: Risk, Uncertainty, Expected Returns, ICAPM, Time-Series and Cross-Sectional Stock Returns, Variance Risk Premium, Conditional Asset Pricing Model.

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1 Introduction

This paper investigates whether the market price of risk and the market price of uncertainty are significantly positive and whether they predict the time-series and cross-sectional variation in stock returns. Although the literature has so far shown how uncertainty impacts optimal allocation decisions and asset prices, the results have been provided based on a theoretical model.[1] Earlier studies do not pay attention to empirical testing of whether the exposures of equity portfolios and individual stocks to uncertainty factors predict their future returns. We extend the original intertemporal capital asset pricing model (ICAPM) of Merton (1973) to propose a conditional ICAPM with time-varying market risk and economic uncertainty. According to our model, the premium on equity is composed of two separate terms; the first term compensates for the standard market risk and the second term represents additional premium for variance risk. We use the conditional ICAPM to test whether the time-varying conditional covariances of equity returns with market and uncertainty factors predict the time-series and cross-sectional variation in future stock returns.

In this paper, economic uncertainty is proxied by the variance risk premia in the U.S. equity market. Following Britten-Jones and Neuberger (2000), Jiang and Tian (2005), and Carr and Wu (2009), we define the variance risk premium (VRP) as the difference between expected variance under the risk-neutral measure and expected variance under the objective measure.[2] We generate several proxies for financial and economic uncertainty and then compute the correlations between uncertainty variables and VRP. The first set of measures can be viewed as macroeconomic uncertainty proxied by the conditional variance of the U.S.

[1] Although formal understanding of uncertainty and uncertainty aversion is poor, there exists a definition of uncertainty aversion originally introduced by Schmeidler (1989) and Epstein (1999). In recent studies, uncertainty aversion is defined for a large class of preferences and in different economic settings by Epstein and Wang (1994), Epstein and Zhang (2001), Chen and Epstein (2002), Klibanoff, Marinacci, and Mukerji (2005), Maccheroni, Marinacci, and Rustichini (2006), and Ju and Miao (2012). In addition to these theoretical papers, Ellsberg's (1961) experimental evidence demonstrates that the distinction between risk and uncertainty is meaningful empirically because people prefer to act on known rather than unknown or ambiguous probabilities.

[2] Other studies (e.g., Rosenberg and Engle (2002), Bakshi and Madan (2006), Bollerslev, Gibson, and Zhou (2011), and Bekaert, Hoerova, and Duca (2012)) interpret the difference between the implied and expected volatilities as an indicator of the representative agent's risk aversion. Bollerslev, Tauchen, and Zhou (2009) and Drechsler and Yaron (2011) relate the variance risk premia to economic uncertainty risk.

output growth and the conditional variance of the Chicago Fed National Activity Index (CFNAI). The second set of uncertainty measures is based on the extreme downside risk of financial institutions obtained from the left tail of the time-series and cross-sectional distribution of financial firms' returns. The third uncertainty variable is related to the health of the financial sector proxied by the credit default swap (CDS) index. The last uncertainty variable is based on the aggregate measure of investors' disagreement about individual stocks trading at NYSE, AMEX, and NASDAQ. We find that the variance risk premium is strongly and positively correlated with all measures of uncertainty considered in the paper. Our results indicate that VRP can be viewed as a sound proxy for financial and economic uncertainty.[3]

Anderson, Ghysels, and Juergens (2009) introduce a model in which the volatility, skewness and higher order moments of all returns are known exactly, whereas there is uncertainty about mean returns. In other words, asset returns are uncertain only because mean returns are not known. In their model, investors' uncertainty in mean returns is defined as the dispersion of predictions of mean market returns obtained from the forecasts of aggregate corporate profits. They find that the price of uncertainty is significantly positive and explains the cross-sectional variation in stock returns. Bekaert, Engstrom, and Xing (2009) investigate the relative importance of economic uncertainty and changes in risk aversion in the determination of equity prices. Different from Knightian uncertainty or uncertainty originated from disagreement of professional forecasters, Bekaert, Engstrom, and Xing (2009) focus on economic uncertainty proxied by the conditional volatility of dividend growth, and find that both the conditional volatility of cash flow growth and time-varying risk aversion are important determinants of equity returns.

Different from the aforementioned studies, we propose a conditional asset pricing model in which economic uncertainty (proxied by VRP) plays a significant role along with the standard market risk. After introducing a two-factor model with risk and uncertainty, we

[3]Knight (1921) draws a distinction between risk and true uncertainty and argues that uncertainty is more common in decision-making process. Knight (1921) points out that risk occurs where the future is unknown, but the probability of all possible outcomes is known. Uncertainty occurs where the probability distribution is itself unknown. We use the variance risk premium as a proxy for economic uncertainty, which is different from Knightian uncertainty.

investigate the significance of risk-return and uncertainty-return coefficients using the time-series and cross-sectional data. Our empirical analyses are based on the size, book-to-market, and industry portfolios as well as individual stocks. We first use the dynamic conditional correlation (DCC) model of Engle (2002) to estimate equity portfolios' (individual stocks') conditional covariances with the market portfolio and then test whether the conditional covariances predict future returns on equity portfolios (individual stocks). We find the risk-return coefficients to be positive and highly significant, implying a strongly positive link between expected return and market risk. Similarly, we use the DCC model to estimate equity portfolios' (individual stocks') conditional covariances with the variance risk premia and then test whether the conditional covariances with VRP predict future returns on equity portfolios (individual stocks). The results indicate a significantly positive market price of uncertainty. Equity portfolios (individual stocks) that are highly correlated with uncertainty (proxied by VRP) carry a significant premium relative to portfolios (stocks) that are uncorrelated or minimally correlated with VRP.

We also examine the empirical validity of the conditional asset pricing model by testing the hypothesis that the conditional alphas on the size, book-to-market, and industry portfolios are jointly zero. The test statistics fail to reject the null hypothesis, indicating that the two-factor model explains the time-series and cross-sectional variation in equity portfolios. Finally, we investigate whether the model explains the return spreads between the high-return (long) and low-return (short) equity portfolios (Small-Big for the size portfolios; Value-Growth for the book-to-market portfolios; and HiTec-Telcm for the industry portfolios). The results from testing the equality of conditional alphas for high-return and low-return portfolios provide no evidence for a significant alpha for Small-Big, Value-Growth, and HiTec-Telcm arbitrage portfolios, indicating that the two-factor model proposed in the paper provides both statistical and economic success in explaining stock market anomalies. Overall, the DCC-based conditional covariances capture the time-series and cross-sectional variation in returns on size, book-to-market, and industry portfolios because the essential tests of the model are passed: (i) significantly positive risk-return and uncertainty-return tradeoffs; (ii) the conditional alphas are jointly zero; and (iii) the conditional alphas for high-return and low-return portfolios are not statistically different from each other.[4] These results are robust to using an alternative specification of the time-varying conditional covariances with an asymmetric GARCH model, using a larger cross-section of equity portfolios in asset pricing tests, and after controlling for a wide variety of macroeconomic variables, market illiquidity, and credit risk.

Finally, we investigate the cross-sectional asset pricing performance of our model based on the 25 and 100 size and book-to-market portfolios. Using the long-short equity portfolios and the Fama and MacBeth (1973) regressions, we test the significance of a cross-sectional relation between expected returns on equity portfolios and the portfolios' conditional covariances (or betas) with VRP. Quintile portfolios are formed by sorting the 25 and 100 Size/BM portfolios based on their VRP-beta. The results indicate that the equity portfolios in highest VRP-beta quintile generate 6 to 8 percent more annual raw returns and alphas compared to the equity portfolios in the lowest VRP-beta quintile. These economically and statistically significant return differences are also confirmed by the Fama-MacBeth cross-sectional regressions, which produce positive and significant average slope coefficients on VRP-beta.

The rest of the paper is organized as follows. Section 2 defines the variance risk premium and provides its empirical measurement. Section 3 presents the conditional asset pricing model with risk and uncertainty. Section 4 describes the data. Section 5 outlines the estimation methodology. Section 6 presents the empirical results. Section 7 provides a battery of robustness checks. Section 8 investigates the cross-sectional asset pricing performance of our model. Section 9 concludes the paper.

2 Variance Risk Premium and Empirical Measurement

The central empirical variable of this paper, as a proxy for economic uncertainty, is the market variance risk premium (VRP)—which is not directly observable but can be esti-

[4] Alternatively, our empirical result on VRP may be interpreted as compensating for the rare disaster risk (Gabaix, 2011), jump risk (Todorov, 2010; Drechsler and Yaron, 2011), or tail risk (Bollerslev and Todorov, 2011; Kelly, 2011). Alternatively, VRP can be generated from a habit-formation model with sophisticated consumption dynamics (Bekaert and Engstrom, 2010). The finding may also be related to the expected business conditions (Campbell and Diebold, 2009) and its cross-sectional implications for stock returns (Goetzmann, Watanabe, and Watanabe, 2009).

mated from the difference between model-free option-implied variance and the conditional expectation of realized variance.

2.1 Variance Risk Premium: Definition and Measurement

In order to define the model-free implied variance, let $C_t(T, K)$ denote the price of a European call option maturing at time T with strike price K, and $B(t, T)$ denote the price of a time t zero-coupon bond maturing at time T. As shown by Carr and Madan (1998) and Britten-Jones and Neuberger (2000), among others, the market's risk-neutral Q expectation of the return variance σ^2_{t+1} conditional on the information set Ω_t, or the implied variance IV_t at time-t, can be expressed in a "model-free" fashion as a portfolio of European calls,

$$IV_t \;\equiv\; \mathrm{E}^Q\left[\sigma^2_{t+1}|\Omega_t\right] \;=\; 2\int_0^\infty \frac{C_t\left(t+1, \frac{K}{B(t,t+1)}\right) - C_t\left(t, K\right)}{K^2}\, dK, \tag{1}$$

which relies on an ever increasing number of calls with strikes spanning from zero to infinity.[5] This equation follows directly from the classical result in Breeden and Litzenberger (1978), that the second derivative of the option call price with respect to strike equals the risk-neutral density, such that all risk neutral moments payoff can be replicated by the basic option prices (Bakshi and Madan, 2000).

In order to define the actual return variance, let p_t denote the logarithmic price of the asset. The realized variance over the discrete t to $t+1$ time interval can be measured in a "model-free" fashion by

$$RV_{t+1} \equiv \sum_{j=1}^{n} \left[p_{t+\frac{j}{n}} - p_{t+\frac{j-1}{n}}\right]^2 \longrightarrow \sigma^2_{t+1}, \tag{2}$$

where the convergence relies on $n \to \infty$; i.e., an increasing number of within period price observations. As demonstrated in the literature (see, e.g., Andersen, Bollerslev, Diebold, and Ebens, 2001; Barndorff-Nielsen and Shephard, 2002), this "model-free" realized variance measure based on high-frequency intraday data offers a much more accurate ex-post

[5] Such a characterization is accurate up to the second order when there are jumps in the underlying asset (Jiang and Tian, 2005; Carr and Wu, 2009), though Martin (2011) has refined the above formulation to make it robust to jumps.

observation of the true (unobserved) return variance than the traditional ones based on daily or coarser frequency returns.

Variance risk premium (VRP) at time t is defined as the difference between the ex-ante risk-neutral expectation and the objective or statistical expectation at time t of the return variance at time $t+1$,

$$VRP_t \equiv \mathrm{E}^Q\left[\sigma^2_{t+1}|\Omega_t\right] - \mathrm{E}^P\left[\sigma^2_{t+1}|\Omega_t\right], \tag{3}$$

which is not directly observable in practice.[6] To construct an empirical proxy for such a VRP concept, one needs to estimate various reduced-form counterparts of the risk neutral and physical expectations. In practice, the risk-neutral expectation $\mathrm{E}^Q\left[\sigma^2_{t+1}|\Omega_t\right]$ is typically replaced by the CBOE implied variance ($\mathrm{VIX}^2/12$) and the true variance σ^2_{t+1} is replaced by realized variance RV_{t+1}.

To estimate the objective expectation, $\mathrm{E}^P\left[\sigma^2_{t+1}|\Omega_t\right]$, we use a linear forecast of future realized variance as $RV_{t+1} = \alpha + \beta IV_t + \gamma RV_t + \epsilon_{t+1}$, with current implied and realized variances. The model-free implied variance from options market is an informationally more efficient forecast for future realized variance than the past realized variance (see, e.g., Jiang and Tian, 2005, among others), while realized variance based on high-frequency data also provides additional power in forecasting future realized variance (Andersen, Bollerslev, Diebold, and Labys, 2003). Therefore, a joint forecast model with one lag of implied variance and one lag of realized variance seems to capture the most forecasting power based on time-t available information (Drechsler and Yaron, 2011).

3 Conditional ICAPM with Economic Uncertainty

The time-varying conditional version of the Sharpe (1964) and Lintner (1965) capital asset pricing model (CAPM) relates the conditionally expected excess returns on risky assets to

[6] The difference between option implied and GARCH type filtered volatilities has been associated in existing literature with notions of aggregate market risk aversion (Rosenberg and Engle, 2002; Bakshi and Madan, 2006; Bollerslev, Gibson, and Zhou, 2011).

the conditionally expected excess return on the market portfolio:

$$E\left[R_{i,t+1}|\Omega_t\right] = \frac{E\left[R_{m,t+1}|\Omega_t\right]}{\operatorname{var}\left[R_{m,t+1}|\Omega_t\right]} \cdot \operatorname{cov}\left[R_{i,t+1}, R_{m,t+1}|\Omega_t\right], \tag{4}$$

where $R_{i,t+1}$ and $R_{m,t+1}$ are, respectively, the return on risky asset i and the market portfolio m in excess of the risk-free interest rate, Ω_t denotes the information set at time t that investors use to form expectations about future returns, $E\left[R_{i,t+1}|\Omega_t\right]$ and $E\left[R_{m,t+1}|\Omega_t\right]$ are the expected excess return on the risky asset and the market portfolio at time $t+1$ conditional on the information set at time t, $\operatorname{var}\left[R_{m,t+1}|\Omega_t\right]$ is the time-t expected conditional variance of excess returns on the market at time $t+1$, and $\operatorname{cov}\left[R_{i,t+1}, R_{m,t+1}|\Omega_t\right]$ is the time-t expected conditional covariance between excess returns on the risky asset and the market portfolio at time $t+1$.

In equation (4), the ratio of $\operatorname{cov}\left[R_{i,t+1}, R_{m,t+1}|\Omega_t\right]$ to $\operatorname{var}\left[R_{m,t+1}|\Omega_t\right]$ is the asset's time-t expected conditional beta $E\left[\beta_{i,t+1}|\Omega_t\right] = \frac{\operatorname{cov}[R_{i,t+1}, R_{m,t+1}|\Omega_t]}{\operatorname{var}[R_{m,t+1}|\Omega_t]}$, and the ratio $\frac{E[R_{m,t+1}|\Omega_t]}{\operatorname{var}[R_{m,t+1}|\Omega_t]}$ is known as the reward-to-risk ratio that represents the compensation the investor must receive for a unit increase in the conditional variance of the market. As pointed out by Merton (1980), the reward-to-risk ratio can also be interpreted as the relative risk aversion coefficient.

Merton (1973) intertemporal capital asset pricing model (ICAPM) implies the following equilibrium relation between expected return and risk for any risky asset i:

$$\mu_i = A \cdot \sigma_{im} + B \cdot \sigma_{ix}, \tag{5}$$

where μ_i denotes the unconditional expected excess return on risky asset i, σ_{im} denotes the unconditional covariance between the excess returns on the risky asset i and the market portfolio m, and σ_{ix} denotes a $(1 \times k)$ row of unconditional covariances between the excess returns on the risky asset i and the k-dimensional state variables x. A is the relative risk aversion of market investors and B measures the market's aggregate reaction to shifts in a k-dimensional state vector that governs the stochastic investment opportunity set. Equation (5) states that in equilibrium, investors are compensated in terms of expected return for bearing market risk and for bearing the risk of unfavorable shifts in the investment opportunity set.

In the original Merton (1973) model, the parameters of expected returns and covariances are all interpreted as constant but the ability to model time variation in expected returns and covariances makes it natural to include time-varying parameters directly in the analysis (see Bali and Engle, 2010). In principle, if the covariances are stochastic, they would represent additional sources of variation in the investment opportunity set and potential hedging demand terms. In this paper, we provide a time-series and cross-sectional investigation of the conditional ICAPM with time-varying covariances:

$$E\left[R_{i,t+1}|\Omega_t\right] = A \cdot \text{cov}\left[R_{i,t+1}, R_{m,t+1}|\Omega_t\right] + B \cdot \text{cov}\left[R_{i,t+1}, X_{t+1}|\Omega_t\right], \tag{6}$$

where A is the reward-to-risk ratio and interpreted as the Arrow-Pratt relative risk-aversion coefficient. The difference between the conditional CAPM and the conditional ICAPM is the intertemporal hedging demand component, $B \cdot \text{cov}\left[R_{i,t+1}, X_{t+1}|\Omega_t\right]$, in equation (6). Note that $\text{cov}\left[R_{i,t+1}, X_{t+1}|\Omega_t\right]$ measures the time-t expected conditional covariance between the excess returns on risky asset i and a state variable X. The parameter B represents the price of risk for the state variable X.

The unconditional (static) CAPM is built on an implausible assumption that investors care only about the mean and variance of single-period portfolio returns. However, in practice, investors make decisions for multiple periods and they revise their portfolio and risk management decisions over time based on the expectations about future investment opportunities. In Merton's (1973) ICAPM, investors are concerned not only with the terminal wealth that their portfolio produces, but also with the investment and consumption opportunities that they will have in the future. Hence, when choosing a portfolio at time t, ICAPM investors consider how their wealth at time $t+1$ might vary with future state variables. This implies that like CAPM investors, ICAPM investors prefer high expected return and low return variance, but they are also concerned with the covariances of portfolio returns with state variables that affect future investment opportunities.

Fama (1996) point out that Merton's (1973) ICAPM generalizes the logic of the CAPM. Since ICAPM investors are risk averse, they are concerned with the mean and variance of their portfolio return. ICAPM investors are, however, also concerned with hedging more specific state-variable (consumption-investment) risks. As a result, their optimal portfolios are multifactor efficient, i.e., ICAPM investors have the largest possible expected returns, given their return variances and the covariances of their returns with the relevant state variables.

Bloom (2009) develops a structural model with time-varying volatility to investigate the effects of economic uncertainty shocks. His empirical analyses indicate that a macroeconomic uncertainty shock produces a sharp decline and rebound in aggregate output and employment because higher uncertainty causes firms to temporarily pause their investment and hiring. He finds that productivity growth also falls because the drop in investment and hiring reduces the rate of reallocation from low to high productivity firms. Overall, his results provide evidence that economic uncertainty shocks generate short sharp recessions and recoveries. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) show that recessions appear in periods of significantly higher economic uncertainty, suggesting an uncertainty approach to modeling business cycles. Bloom, Bond, and Van Reenen (2007) provide a link between economic uncertainty and investment dynamics. Stock and Watson (2012) indicate that the decline in aggregate output and employment during the recent crisis period are driven by financial and economic uncertainty shocks. Allen, Bali, and Tang (2012) show that downside risk in the financial sector predicts future economic downturns, linking financial uncertainty to future investment opportunity set.

Hence, we assume that economic uncertainty is a relevant state variable that affects investors' expectations about future consumption and investment opportunities. We will also show that the variance risk premia significantly covary with alternative measures of financial and economic uncertainty factors. Hence, the state variable X_{t+1} in equation (6) is proxied by VRP. Based on this two-factor conditional ICAPM, we investigate whether the market price of risk and the market price of uncertainty are significantly positive and whether they predict returns in a panel data setting:

$$E\left[R_{i,t+1}|\Omega_t\right] = A \cdot \mathrm{cov}\left[R_{i,t+1}, R_{m,t+1}|\Omega_t\right] + B \cdot \mathrm{cov}\left[R_{i,t+1}, VRP_{t+1}|\Omega_t\right], \tag{7}$$

where the time-varying exposure of asset i to changes in the market portfolio is measured by the conditional covariance between the excess return on asset i and the excess return on the aggregate stock market, denoted by $\text{cov}\,[R_{i,t+1}, R_{m,t+1}|\Omega_t]$, and the time-varying exposure of asset i to economic uncertainty is proxied by the conditional covariance between the excess return on asset i and the variance risk premia, denoted by $\text{cov}\,[R_{i,t+1}, VRP_{t+1}|\Omega_t]$.[7]

Maio and Santa-Clara (2012) study the restrictions associated with the ICAPM for a time-series of the market return and a cross-section of portfolios. By using Merton's ICAPM, they identify three main conditions for a multifactor model to be justifiable by the ICAPM. First, the candidates for ICAPM state variables must predict the first or second moments of stock market returns. Second, and most importantly, the state variables should forecast expected market return with the same sign as its innovation prices the cross-section. Specifically, if a given state variable forecasts positive expected returns, it should earn a positive risk price in the cross-sectional test of the respective multifactor model. The third restriction associated with the ICAPM is that the market price of risk estimated from the cross-sectional tests must be economically plausible as an estimate of the relative risk aversion coefficient.

As shown by Bollerslev, Tauchen, and Zhou (2009), the variance risk premia (VRP) predicts future returns on the stock market portfolio. Specifically, there is a significantly positive intertemporal relation between VRP and expected market returns. As will be presented in this paper, VRP earns a positive risk price in the cross-sectional test of the two-factor conditional ICAPM model. Finally, the market price of VRP from the cross-sectional tests provides economically sensible estimates of relative risk aversion coefficient. Hence, the conditional ICAPM introduced in the paper meets three conditions proposed by Maio and Santa-Clara (2012).

[7]In the internet appendix (Section A), we show that the two-factor conditional ICAPM specification in equation (7) can be obtained in a consumption-based asset pricing model with time-varying volatility of the consumption growth and the volatility uncertainty in the consumption growth process (e.g., as in Bollerslev, Tauchen, and Zhou, 2009). Alternatively, we can motivate such a risk-return and uncertainty-return specification using the habit formation model of Campbell and Cochrane (1999), similar to the approach taken by Bekaert, Engstrom, and Xing (2009) and Bekaert, Hoerova, and Duca (2012).

3.1 Variance Risk Premia and Economic Uncertainty Measures

For the option-implied variance of the S&P500 market return, we use the end-of-month Chicago Board of Options Exchange (CBOE) volatility index on a monthly basis ($\text{VIX}^2/12$). Following earlier studies, the daily realized variance for the S&P500 index is calculated as the summation of the 78 intra-day five-minute squared log returns from 9:30am to 4:00pm including the close-to-open interval. Along these lines, we compute the monthly realized variance for the S&P500 index as the summation of five-minute squared log returns in a month. As shown in equation (3), variance risk premium (VRP) at time t is defined as the difference between the ex-ante risk-neutral expectation and the objective or statistical expectation of the return variance over the $[t, t+1]$ time interval. The monthly VRP data are available from January 1990 to December 2010.

To give a visual illustration, Figure 1 plots the monthly time series of variance risk premium (VRP), implied variance, and expected variance. The VRP proxy is moderately high around the 1990 and 2001 economic recessions but much higher during the 2008 financial crisis and to a lessor degree around 1997-1998 Asia-Russia-LTCM crisis. The variance spike during October 2008 already surpasses the initial shock of the Great Depression in October 1929. The huge run-up of VRP in the fourth quarter of 2008 leads the equity market bottom reached in March 2009. The sample mean of VRP is 18.75 (in percentages squared, monthly basis), with a standard deviation of 22.15. Notice that the extraordinary skewness (3.81) and kurtosis (27.46) signal a highly non-Gaussian process for VRP.

According to the conditional ICAPM specification in equation (7), VRP is viewed as a proxy for uncertainty. To test whether VRP is in fact associated with alternative measures of uncertainty, we generate some proxies for financial and economic uncertainty. We obtain monthly values of the U.S. industrial production index from G.17 database of the Federal Reserve Board and monthly values of the Chicago Fed National Activity Index (CFNAI) from the Federal Reserve Bank of Chicago for the period January 1990 – December 2010.[8] We

[8]The CFNAI is a monthly index that determines increases and decreases in economic activity and is designed to assess overall economic activity and related inflationary pressure. It is a weighted average of 85 existing monthly indicators of national economic activity, and is constructed to have an average value of zero and a standard deviation of one. Since economic activity tends toward a trend growth rate over time,

use the GARCH(1,1) model of Bollerslev (1986) to estimate the conditional variance of the growth rate of industrial production and the conditional variance of the CFNAI index. These two measures can be viewed as macroeconomic uncertainty. The sample correlation between VRP and economic uncertainty variables is positive and significant; sample correlation is 33.20% with the variance of output growth and 31.82% with the variance of CFNAI index.

Our second set of uncertainty measures is based on the downside risk of financial institutions obtained from the left tail of the time-series and cross-sectional distribution of financial firms' returns. Specifically, we obtain monthly returns for financial firms (6000 $\leq$ SIC code $\leq$ 6999) for the sample period January 1990 to December 2010. Then, the 1% nonparametric Value-at-Risk (VaR) measure in a given month is measured as the cut-off point for the lower one percentile of the monthly returns on financial firms.[9] For each month, we determine the one percentile of the cross-section of returns on financial firms, and obtain an aggregate 1% VaR measure of the financial system for the period 1990-2010. In addition to the cross-sectional distribution, we use the time-series daily return distribution to estimate 1% VaR of the financial system. For each month from January 1990 to December 2010, we first determine the lowest daily returns on financial institutions over the past 1 to 12 months. The catastrophic risk of financial institutions is then computed by taking the average of these lowest daily returns obtained from alternative measurement windows. The estimation windows are fixed at 1 to 12 months, and each fixed estimation window is updated on a monthly basis. These two downside risk measures can be viewed as a proxy for uncertainty in the financial sector. The sample correlations between VRP and financial uncertainty variables are positive and significant: 47.37% with the cross-sectional VaR measure and 37.01% with the time-series VaR measure.

The third uncertainty variable is related to the health of the financial sector proxied by the credit default swap (CDS) index. We download the monthly CDS data from Bloomberg. For the sample period January 2004 – December 2010, we obtain monthly CDS data for Bank of

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a positive index reading corresponds to growth above trend and a negative index reading corresponds to growth below trend.

[9] Assuming that we have 900 financial firms in month t, the nonparametric measure of 1% VaR is the 9th lowest observation in the cross-section of monthly returns.

America (BOA), Citigroup (CICN), Goldman Sachs (GS), JP Morgan (JPM), Morgan Stanley (MS), Wells Fargo (WFC), and American Express (AXP). Then, we standardized all CDS data to have zero mean and unit standard deviation. Finally, we formed the standardized CDS index (EWCDS) based on the equal-weighted average of standardized CDS values for the 7 major financial firms. For the common sample period 2004-2010, the correlation between VRP and EWCDS is positive, 42.99%, and highly significant.

The last uncertainty variable is based on the aggregate measure of investors' disagreement about individual stocks trading at NYSE, AMEX, and NASDAQ. Following Diether, Malloy, and Scherbina (2002), we use dispersion in analysts' earnings forecasts as a proxy for divergence of opinion. It is likely that investors partly form their expectations about a particular stock based on the analysts' earnings forecasts. If all analysts are in agreement about expected returns, uncertainty is likely to be low. However, if analysts provide very different estimates, investors are likely to be unclear about future returns, and uncertainty is high. The sample correlation between VRP and the aggregate measure of dispersion is about 14.92%. Overall, these results indicate that the variance risk premia is strongly and positively correlated with all measures of uncertainty considered here. Hence, VRP can be viewed as a sound proxy for financial and economic uncertainty.

4 Data

4.1 Equity Portfolios

We use the monthly excess returns on the value-weighted aggregate market portfolio and the monthly excess returns on the 10 value-weighted size, book-to-market, and industry portfolios. The aggregate market portfolio is represented by the value-weighted NYSE-AMEX-NASDAQ index. Excess returns on portfolios are obtained by subtracting the returns on the one-month Treasury bill from the raw returns on equity portfolios. The data are obtained from Kenneth French's online data library.[10] We use the longest common sample period available, from January 1990 to December 2010, yielding a total of 252 monthly

[10] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

observations.

Table I of the internet appendix presents the monthly raw return and CAPM Alpha differences between high-return (long) and low-return (short) equity portfolios. The results are reported for the size, book-to-market (BM), and industry portfolios for the period January 1990 – December 2010.[11] The OLS t-statistics are reported in parentheses. The Newey and West (1987) t-statistics are given in square brackets.

For the ten size portfolios, "Small" (Decile 1) is the portfolio of stocks with the smallest market capitalization and "Big" (Decile 10) is the portfolio of stocks with the biggest market capitalization. For the 1990-2010 period, the average return difference between the Small and Big portfolios is 0.40% per month with the OLS t-statistic of 1.22 and the Newey-West (1987) t-statistic of 1.13, implying that small stocks on average do not generate higher returns than big stocks. In addition to the average raw returns, Table I of the internet appendix presents the intercept (CAPM alpha) from the regression of Small-Big portfolio return difference on a constant and the excess market return. The CAPM Alpha (or abnormal return) for the long-short size portfolio is 0.35% per month with the OLS t-statistic of 1.06 and the Newey-West t-statistic of 0.98. This economically and statistically insignificant Alpha indicates that the static CAPM does explain the size effect for the 1990-2010 period.

For the ten book-to-market portfolios, "Growth" is the portfolio of stocks with the lowest book-to-market ratios and "Value" is the portfolio of stocks with the highest book-to-market ratios. For the sample period January 1990 – December 2010, the average return difference between the Value and Growth portfolios is economically and statistically insignificant; 0.29% per month with the OLS t-statistic of 0.92 and the Newey-West t-statistic of 0.79, implying that value stocks on average do not generate higher returns than growth stocks. Similar to our findings for the size portfolios, the unconditional CAPM can explain the value premium for the 1990-2010 period; the CAPM Alpha (or abnormal return) for the long-short book-to-market portfolio is only 0.28% per month with the OLS t-statistic of 0.86 and the Newey-West t-statistic of 0.71.

[11]Since the monthly data on variance risk premia (VRP) start in January 1990, our empirical analyses with equity portfolios and VRP are based on the sample period January 1990 - December 2010.

Interestingly, industry effects in the U.S. equity market are economically and statistically strong over the past two decades, although size and value premiums are not. The average raw and risk-adjusted return differences between the high-return and low-return industry portfolios are significant for the sample period 1990-2010. The high-return and low-return portfolios of 48 and 49 industries generate highly significant return differences, 30 and 38 industry portfolios generate marginally significant return differences, whereas the average return differences and Alphas for the high-return and low-return portfolios of 10 and 17 industries are insignificant. Specifically, for 30-, 48- and 49-industry portfolios of Kenneth French, "Coal" industry has the highest average monthly return, whereas "Other" industry has the lowest return, yielding an average raw and risk-adjusted return differences of 1.54% to 1.79% per month and statistically significant. The static CAPM cannot explain these economically and statistically strong industry effects either.

Earlier studies starting with Fama and French (1992, 1993) provide evidence for the significant size and value premiums for the post-1963 period. Some readers may find the insignificant size and value premiums for the 1990-2010 period controversial. Hence, in internet appendix (Section B), we examine the significance of size and book-to-market effects for the longest sample period July 1926 – December 2010 and the subsample period July 1963 – December 2010. The results indicate significant raw return difference between the Value and Growth portfolios for both sample periods and significant risk-adjusted return difference (Alpha) only for the post-1963 period. Consistent with the findings of earlier studies, we find significant raw return difference between the Small and Big stock portfolios for the 1926-2010 period, which becomes very weak for the post-1963 period. The CAPM Alpha (or abnormal return) for the long-short size portfolio is economically and statistically insignificant for both sample periods.

5 Estimation Methodology

Following Bali (2008) and Bali and Engle (2010), our estimation approach proceeds in steps.

1) We take out any autoregressive elements in returns and VRP and estimate univariate

GARCH models for all returns and VRP.

2) We construct standardized returns and compute bivariate DCC estimates of the correlations between each portfolio and the market and between each portfolio and VRP using the bivariate likelihood function.

3) We estimate the expected return equation as a panel with the conditional covariances as regressors. The error covariance matrix specified as seemingly unrelated regression (SUR). The panel estimation methodology with SUR takes into account heteroskedasticity and autocorrelation as well as contemporaneous cross-correlations in the error terms.

The following subsections provide details about the estimation of time-varying covariances and the estimation of time-series and cross-sectional relation between expected returns and risk and uncertainty.

5.1 Estimating Time-Varying Conditional Covariances

We estimate the conditional covariance between excess returns on equity portfolio i and the market portfolio m based on the mean-reverting dynamic conditional correlation (DCC) model:

$$R_{i,t+1} = \alpha_0^i + \alpha_1^i R_{i,t} + \varepsilon_{i,t+1} \tag{8}$$

$$R_{m,t+1} = \alpha_0^m + \alpha_1^m R_{m,t} + \varepsilon_{m,t+1} \tag{9}$$

$$E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \beta_0^i + \beta_1^i \varepsilon_{i,t}^2 + \beta_2^i \sigma_{i,t}^2 \tag{10}$$

$$E_t\left[\varepsilon_{m,t+1}^2\right] \equiv \sigma_{m,t+1}^2 = \beta_0^m + \beta_1^m \varepsilon_{m,t}^2 + \beta_2^m \sigma_{m,t}^2 \tag{11}$$

$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1} \tag{12}$$

$$\rho_{im,t+1} = \frac{q_{im,t+1}}{\sqrt{q_{ii,t+1} \cdot q_{mm,t+1}}}, \; q_{im,t+1} = \bar{\rho}_{im} + a_1 \cdot (\varepsilon_{i,t} \cdot \varepsilon_{m,t} - \bar{\rho}_{im}) + a_2 \cdot (q_{im,t} - \bar{\rho}_{im}) \tag{13}$$

where $R_{i,t+1}$ and $R_{m,t+1}$ denote the time $(t+1)$ excess return on equity portfolio i and the market portfolio m over a risk-free rate, respectively, and $E_t[\cdot]$ denotes the expectation operator conditional on time t information. $\sigma^2_{i,t+1}$ is the time-t expected conditional variance of $R_{i,t+1}$, $\sigma^2_{m,t+1}$ is the time-t expected conditional variance of $R_{m,t+1}$, and $\sigma_{im,t+1}$ is the time-t expected conditional covariance between $R_{i,t+1}$ and $R_{m,t+1}$. $\rho_{im,t+1} = q_{im,t+1}/\sqrt{q_{ii,t+1} \cdot q_{mm,t+1}}$ is the time-t expected conditional correlation between $R_{i,t+1}$ and $R_{m,t+1}$, and $\bar{\rho}_{im}$ is the unconditional correlation. To ease the parameter convergence, we use correlation targeting assuming that the time-varying correlations mean reverts to the sample correlations $\bar{\rho}_{im}$.

We estimate the conditional covariance between each equity portfolio i and the variance risk premia VRP, $\sigma_{i,VRP}$, using an analogous DCC model:

$$R_{i,t+1} = \alpha^i_0 + \alpha^i_1 R_{i,t} + \varepsilon_{i,t+1} \tag{14}$$

$$VRP_{t+1} = \alpha^{VRP}_0 + \alpha^{VRP}_1 VRP_t + \varepsilon_{VRP,t+1} \tag{15}$$

$$E_t\left[\varepsilon^2_{i,t+1}\right] \equiv \sigma^2_{i,t+1} = \beta^i_0 + \beta^i_1 \varepsilon^2_{i,t} + \beta^i_2 \sigma^2_{i,t} \tag{16}$$

$$E_t\left[\varepsilon^2_{VRP,t+1}\right] \equiv \sigma^2_{VRP,t+1} = \beta^{VRP}_0 + \beta^{VRP}_1 \varepsilon^2_{VRP,t} + \beta^{VRP}_2 \sigma^2_{VRP,t} \tag{17}$$

$$E_t\left[\varepsilon_{i,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{i,VRP,t+1} = \rho_{i,VRP,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{VRP,t+1} \tag{18}$$

$$\begin{aligned} \rho_{i,VRP,t+1} &= \frac{q_{i,VRP,t+1}}{\sqrt{q_{ii,t+1} \cdot q_{VRP,t+1}}}, \\ q_{i,VRP,t+1} &= \bar{\rho}_{i,VRP} + a_1 \cdot \left(\varepsilon_{i,t} \cdot \varepsilon_{VRP,t} - \bar{\rho}_{i,VRP}\right) + a_2 \cdot \left(q_{i,VRP,t} - \bar{\rho}_{i,VRP}\right) \end{aligned} \tag{19}$$

where $\sigma_{i,VRP,t+1}$ is the time-t expected conditional covariance between $R_{i,t+1}$ and VRP_{t+1}. $\rho_{i,VRP,t+1}$ is the time-t expected conditional correlation between $R_{i,t+1}$ and VRP_{t+1}. We use the same DCC model to estimate the conditional covariance between the market portfolio m and the variance risk premia VRP, $\sigma_{m,VRP}$.[12]

We estimate the conditional covariances of each equity portfolio with the market portfolio and with VRP using the maximum likelihood method described in the internet appendix (Section C). Then, as discussed in the following section, we estimate the time-series and

[12]We assume that the excess returns on equity portfolios and the market portfolio as well as the variance risk premia follow an autoregressive of order one, AR(1) process, given in equations (8), (9), and (15). At an earlier stage of the study, we consider alternative specifications of the conditional mean. More specifically, the excess returns are assumed to follow a moving average of order one, MA(1) process, ARMA(1,1) process, and a constant. Our main findings are not sensitive to the choice of the conditional mean specification.

cross-sectional relation between expected return and risk and uncertainty as a panel with the conditional covariances as regressors.

As pointed out by earlier studies, estimating Multivariate GARCH-in-mean models with time-varying conditional correlations is a difficult task, especially if the number of cross-sections gets bigger. Early work on time-varying covariances in large dimensions was carried out by Bollerslev (1990) in his constant correlation model, where the volatilities of each asset were allowed to vary through time but the correlations were time invariant. Recently, the DECO model of Engle and Kelly (2012) and the MacGyver estimation method of Engle (2009) deal with the computation of correlations for a large number of assets with an assumption that the correlation amongst assets changes through time but is constant across the cross-section of assets. We should note that estimating time-varying correlations based on a Multivariate GARCH model with a constant mean is easier than estimating time-varying correlations based on a Multivariate GARCH-in-mean model with time-varying mean.

At an earlier stage of the study, we use 10 equity portfolios and estimate in one step the time-varying conditional correlations as well as the parameters of time-varying conditional mean in a Multivariate GARCH-in-mean framework. To ease the parameter convergence, we use correlation targeting assuming that the time-varying correlations mean reverts to the sample correlations. To reduce the overall time of maximizing the conditional log-likelihood, we first estimate all pairs of bivariate GARCH-in-mean model and then use the median values of A, B, a_1 and a_2 as starting values along with the bivariate GARCH-in-mean estimates of variance parameters $(\beta_0, \beta_1, \beta_2)$. Even after going through these steps to increase the speed of parameter convergence, it takes a long time to obtain the full set of parameters in the Multivariate GARCH-in-mean model. Similar to the findings of Bali and Engle (2010), the results from the one-step estimation of 10 equity portfolios turned out to be similar to those obtained from the two-step estimation procedure described in Section 5.[13]

[13] Bali and Engle (2010) also estimate the risk aversion coefficient in two steps; first they obtain the conditional covariances with DCC and then they use the covariance estimates in the panel regression with a common slope coefficient. In this setting, since the covariance matrices implied by the DCC model are not used in estimating risk premia or in computing their standard errors, a common worry in testing asset pricing models is that time-varying covariances are measured with error. Using different samples, they show that the significance of measurement errors in covariances is small. Hence, the one-step and two-step estimation

5.2 Estimating Risk-Uncertainty-Return Tradeoff

Given the conditional covariances, we estimate the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$R_{i,t+1} = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) + \varepsilon_{i,t+1} \tag{20}$$

$$R_{m,t+1} = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) + \varepsilon_{m,t+1} \tag{21}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the variance risk premia (VRP_{t+1}), $Cov_t\left(R_{m,t+1}, VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and the variance risk premia (VRP_{t+1}), and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio.

We estimate the system of equations in (20)-(21) using a weighted least square method that allows us to place constraints on coefficients across equations. We compute the t-statistics of the parameter estimates accounting for heteroskedasticity and autocorrelation as well as contemporaneous cross-correlations in the errors from different equations. The estimation methodology can be regarded as an extension of the seemingly unrelated regression (SUR) method, the details of which are in the internet appendix (Section D).

6 Empirical Results

In this section we first present results from the 10 decile portfolios of size, book-to-market, and industry. Second, we discuss the economic significance of risk and uncertainty compensations. Finally, we compare the relative performances of conditional CAPM and ICAPM with both risk and uncertainty.

procedures generate similar slope coefficients and standard errors.

6.1 Ten Decile Portfolios of Size, Book-to-Market, and Industry

The common slopes and the intercepts are estimated using the monthly excess returns on the 10 value-weighted size, book-to-market, and industry portfolios for the sample period January 1990 to December 2010. The aggregate stock market portfolio is measured by the value-weighted CRSP index. Table 1 reports the common slope estimates (A, B), the abnormal returns or conditional alphas for each equity portfolio (α_i) and the market portfolio (α_m), and the t-statistics of the parameter estimates. The last two rows, respectively, show the Wald statistics; Wald_1 from testing the joint hypothesis $H_0 : \alpha_1 = ... = \alpha_{10} = \alpha_m = 0$, and Wald_2 from testing the equality of conditional alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

The risk aversion coefficient is estimated to be positive and highly significant for all equity portfolios: $A = 3.96$ with the t-statistic of 3.12 for the size portfolios, $A = 2.51$ with the t-statistic of 2.53 for the book-to-market portfolios, and $A = 3.41$ with the t-statistic of 2.35 for the industry portfolios.[14] These results imply a positive and significant relation between expected return and market risk.[15] Consistent with the conditional ICAPM specification in equation (7), the uncertainty aversion coefficient is also estimated to be positive and highly significant for all equity portfolios: $B = 0.0058$ with the t-statistic of 2.97 for the size portfolios, $B = 0.0050$ with the t-statistic of 2.27 for the book-to-market portfolios, and $B = 0.0060$ with the t-statistic of 2.78 for the industry portfolios. These results indicate a significantly positive market price of uncertainty in the aggregate stock market. Equity portfolios with higher sensitivity to increases in the variance risk premia are expected to generate higher returns next period.

One implication of the conditional asset pricing model in equation (7) is that the intercepts (α_i, α_m) are not jointly different from zero assuming that the conditional covariances of

[14] Our risk aversion estimates ranging from 2.51 to 3.41 are very similar to the median level of risk aversion, 2.52, identified by Bekaert, Engstrom, and Xing (2009) in a different model.

[15] Although the literature is inconclusive on the direction and significance of a risk-return tradeoff, some studies do provide evidence supporting a positive and significant relation between expected return and risk (e.g., Bollerslev, Engle, and Wooldridge (1988), Ghysels, Santa-Clara, and Valkanov (2005), Guo and Whitelaw (2006), Guo and Savickas (2006), Lundblad (2007), Bali (2008), and Bali and Engle (2010)).

equity portfolios with the market portfolio and the variance risk premia have enough predictive power for expected future returns. To examine the empirical validity of the conditional asset pricing model, we test the joint hypothesis $H_0 : \alpha_1 = ... = \alpha_{10} = \alpha_m = 0$. As presented in Table 1, the Wald_1 statistics for the size, book-to-market, and industry portfolios are, respectively, 16.74, 8.88, and 14.35 with the corresponding p-values of 0.12, 0.63, and 0.21. The significantly positive risk and uncertainty aversion coefficients and the insignificant Wald_1 statistics indicate that the two-factor model in equation (7) is empirically sound.

We also investigate whether the model explains the return spreads between Small and Big; Value and Growth; and HiTec and Telcm portfolios. The last row in Table 1 reports Wald_2 statistics from testing the equality of conditional alphas for high-return and low-return portfolios ($H_0 : \alpha_1 = \alpha_{10}$). These intercepts capture the monthly abnormal returns on each portfolio that cannot be explained by the conditional covariances with the market portfolio and the variance risk premia.

The first column of Table 1 shows that the abnormal return on the small-stock portfolio is $\alpha_1 = 0.41\%$ per month with a t-statistic of 0.94, whereas the abnormal return on the big-stock portfolio is $\alpha_{10} = 0.01\%$ per month with a t-statistic of 0.01. The Wald_2 statistic from testing the equality of alphas on the Small and Big portfolios is 1.56 with a p-value of 0.21, indicating that there is no significant risk-adjusted return difference between the small-stock and big-stock portfolios. The second column provides the conditional alphas on the Value and Growth portfolios: $\alpha_1 = 0.36\%$ per month with a t-statistic of 0.90, and $\alpha_{10} = 0.82\%$ per month with a t-statistic of 1.90. The Wald_2 statistic from testing $H_0 : \alpha_1 = \alpha_{10}$ is 1.79 with a p-value of 0.18, implying that the conditional asset pricing model explains the value premium, i.e., the risk-adjusted return difference between value and growth stocks is statistically insignificant. The last column shows that the conditional alphas on HiTec and Telcm portfolios are, respectively, 0.26% and -0.05% per month, generating a risk-adjusted return spread of 31 basis points per month. As reported in the last row, the Wald_2 statistic from testing the significance of this return spread is 0.40 with a p-value of 0.53, yielding insignificant industry effect over the sample period 1990-2010.

The differences in conditional alphas are both economically and statistically insignificant, indicating that the two-factor model proposed in equation (7) provides both statistical and economic success in explaining stock market anomalies. Overall, the DCC-based conditional covariances capture the time-series and cross-sectional variation in returns on size, book-to-market, and industry portfolios because the essential tests of the conditional ICAPM are passed: (i) significantly positive risk-return and uncertainty-return tradeoffs; (ii) the conditional alphas are jointly zero; and (iii) the conditional alphas for high-return and low-return portfolios are not statistically different from each other.[16]

6.2 Economic Significance of Uncertainty-Return Tradeoff

In this section, we test whether the risk-return (A) and uncertainty-return (B) coefficients are sensible and whether the uncertainty measure is associated with macroeconomic state variables.

Specifically, we rely on equation (21) and compute the expected excess return on the market portfolio based on the estimated prices of risk and uncertainty as well as the sample averages of the conditional covariance measures:

$$E_t\left[R_{m,t+1}\right] = \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) \tag{22}$$

where $\alpha_m = 0.0008$, $A = 3.96$, and $B = 0.0058$ for the 10 size portfolios; $\alpha_m = 0.0032$, $A = 2.51$, and $B = 0.0050$ for the 10 book-to-market portfolios; and $\alpha_m = 0.0019$, $A = 3.41$, and $B = 0.0060$ for the 10 industry portfolios (see Table 1). The sample averages of $Var_t\left(R_{m,t+1}\right)$ and $Cov_t\left(R_{m,t+1}, VRP_{t+1}\right)$ are 0.002187 and -0.7026, respectively.[17] These values produce $E_t\left[R_{m,t+1}\right] = 0.54\%$ per month when the parameters are estimated using the 10 size portfolios, $E_t\left[R_{m,t+1}\right] = 0.52\%$ per month when the parameters are estimated using the 10 book-to-market portfolios, and $E_t\left[R_{m,t+1}\right] = 0.51\%$ when the parameters are estimated using the 10 industry portfolios.

[16] As discussed in Section E of the internet appendix, we estimate the DCC-based conditional covariances using the Asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993). Table III of the internet appendix shows that our main findings from the Asymmetric GARCH model are very similar to those reported in Table 1.

[17] The negative value for the conditional covariance of the market return with the VRP factor is consistent with the consumption-based asset pricing model and the negative contemporaneous correlation between the market return and the VRP factor reported by Bollerslev, Tauchen, and Zhou (2009).

To evaluate the performance of our model with risk and uncertainty, we calculate the sample average of excess returns on the market portfolio, which is a standard benchmark for the market risk premium. The sample average of $R_{m,t+1}$ is found to be 0.52% per month for the period January 1990 – December 2010, indicating that the estimated market risk premiums of 0.51% – 0.54% are very close to the benchmark. This again shows outstanding performance of the two-factor model introduced in the paper.

To further appreciate the economics behind the apparent connection between the variance risk premium (VRP) and the time-series and cross-sectional variations in expected stock returns, Figure 2 plots the VRP together with the quarterly growth rate in GDP. As seen from the figure, there is a tendency for VRP to rise in the quarter before a decline in GDP, while it typically narrows ahead of an increase in GDP. Indeed, the sample correlation equals -0.17 between lag VRP and current GDP (as first reported in Bollerslev et al., 2009). In other words, VRP as a proxy for economic uncertainty does seem to negatively relate to future macroeconomic performance.

Thus, not only the difference between the implied and expected variances positively covaries with stock returns, it also covaries negatively with future growth rates in GDP. Intuitively, when VRP is high (low), it generally signals a high (low) degree of aggregate economic uncertainty. Consequently agents tend to simultaneously cut (increase) their consumption and investment expenditures and shift their portfolios from more (less) to less (more) risky assets. This in turn results in a rise (decrease) in expected excess returns for stock portfolios that covaries more (less) with the macroeconomic uncertainty, as proxied by VRP.

As mentioned earlier, in the internet appendix (Section A), we provide a two-factor consumption-based asset pricing model in which the consumption growth and its volatility follow the joint dynamics and hence VRP affects expected future returns. In essence, our finding of a positive significant relation between economic uncertainty measure and stock expected returns, is consistent with the consumption-based model's implication that the intertemporal elasticity of substitution (IES) is larger than one, i.e., agents prefer an earlier resolution of uncertainty, hence uncertainty (proxied by VRP) carries a positive premium, and heightened VRP does signal the worsening of macroeconomic fundamentals.

6.3 Relative Performance of Conditional ICAPM with Uncertainty

We now assess the relative performance of the newly proposed model in predicting the cross-section of expected returns on equity portfolios. Specifically, we test whether the conditional ICAPM with the market and uncertainty factors outperforms the conditional CAPM with the market factor in terms of statistical fit. The goodness of fit of an asset pricing model describes how well it fits a set of realized return observations. Measures of goodness of fit typically summarize the discrepancy between observed values and the values expected under the model in question. Hence, we focus on the cross-section of realized average returns on equity portfolios (as a benchmark) and the portfolios' expected returns implied by the two competing models.

Using equation (20), we compute the expected excess return on equity portfolios based on the estimated prices of risk and uncertainty (A, B) and the sample averages of the conditional covariance measures, $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ and $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$:

$$E_t\left[R_{i,t+1}\right] = \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right). \tag{23}$$

Table 2 presents the realized monthly average excess returns on the size, book-to-market, and industry portfolios and the cross-section of expected excess returns generated by the Conditional CAPM and the Conditional ICAPM models. Clearly the newly proposed model with risk and uncertainty provides much more accurate estimates of expected returns on the size, book-to-market, and industry portfolios. Especially for the size and industry portfolios, expected returns implied by the Conditional ICAPM with the market and VRP factors are almost identical to the realized average returns. The last row in Table 2 reports the Mean Absolute Percentage Errors (MAPE) for the two competing models:

$$MAPE = \frac{|\text{Realized} - \text{Expected}|}{\text{Expected}}, \tag{24}$$

where "Realized" is the realized monthly average excess return on each equity portfolio and "Expected" is the expected excess return implied by equation (23). For the conditional

CAPM with the market factor, MAPE equals 5.20% for the size portfolios, 5.37% for the book-to-market portfolios, and 6.32% for the industry portfolios. Accounting for the variance risk premium improves the cross-sectional fitting significantly: MAPE reduces to 0.61% for the size portfolios, 1.66% for the book-to-market portfolios, and 0.55% for the industry portfolios.

Figure 3 provides a visual depiction of the realized and expected returns for the size, book-to-market, and industry portfolios. It is clear that the conditional ICAPM with uncertainty nails down the realized returns of the size, book-to-market, and industrial portfolios, while the conditional CAPM systematically over-predicts these portfolio returns. Overall, the results indicate superior performance of the conditional asset pricing model introduced in the paper.

7 Robustness Check

In this section we first examine whether the model's performance changes when we use a larger cross-section of equity portfolios. Second, we provide robustness analysis when controlling for popular macroeconomic and financial variables. Third, we provide results from individual stocks. Finally, we test whether the predictive power of the variance risk premia is subsumed by the market illiquidity and/or credit risk.

7.1 Results from Larger Cross-Section of Industry Portfolios

Given the positive risk-return and positive uncertainty-return coefficient estimates from the three data sets and the success of the conditional asset pricing model in explaining the industry, size, and value premia, we now examine how the model performs when we use a larger cross-section of equity portfolios.

The robustness of our findings is investigated using the monthly excess returns on the value-weighted 17-, 30-, 38-, 48-, and 49-industry portfolios for the sample period January 1990 – December 2010. Table 3 reports the common slope estimates (A, B), their t-statistics in parentheses, and the Wald_1 and Wald_2 statistics along with their p-values in square brackets. For the industry portfolios, the risk aversion coefficients (A) are estimated to be positive, in the range of 2.20 to 2.78, and highly significant with the t-statistics ranging from 2.31 to 3.34. Consistent with our earlier findings from the 10 size, 10 book-to-market, and 10 industry portfolios, the results from the larger cross-section of industry portfolios (17 to 49) imply a positive and significant relation between expected return and market risk. Again similar to our findings from 10 decile portfolios, the uncertainty aversion coefficients are estimated to be positive, in the range of 0.0036 to 0.0041, and highly significant with the t-statistics ranging from 2.44 to 4.21. These results provide evidence for a significantly positive market price of uncertainty and show that assets with higher correlation with the variance risk premia generate higher returns next month.

Not surprisingly, the Wald_1 statistics for all industry portfolios have p-values in the range of 0.20 to 0.75, indicating that the two-factor asset pricing model explains the time-series and cross-sectional variation in larger number of equity portfolios. The last row shows that the Wald_2 statistics from testing the equality of conditional alphas on the high-return and low-return industry portfolios have p-values ranging from 0.44 to 0.80, implying that there is no significant risk-adjusted return difference between the extreme portfolios of 17, 30, 38, 48, and 49 industries. The differences in conditional alphas are both economically and statistically insignificant, showing that the two-factor model introduced in the paper provides success in explaining industry effects.

7.2 Controlling for Macroeconomic Variables

A series of papers argue that the stock market can be predicted by financial and/or macroeconomic variables associated with business cycle fluctuations. The commonly chosen variables include default spread (DEF), term spread (TERM), dividend price ratio (DIV), and the de-trended riskless rate or the relative T-bill rate (RREL).[18] We define DEF as the difference between the yields on BAA- and AAA-rated corporate bonds, and TERM as the difference between the yields on the 10-year Treasury bond and the 3-month Treasury bill. RREL is

[18]See, e.g., Campbell (1987), Fama and French (1989), and Ferson and Harvey (1991) who test the predictive power of these variables for expected stock returns.

defined as the difference between 3-month T-bill rate and its 12-month backward moving average.[19] We obtain the aggregate dividend yield using the CRSP value-weighted index return with and without dividends based on the formula given in Fama and French (1988). In addition to these financial variables, we use some fundamental variables affecting the state of the U.S. economy: Monthly inflation rate based on the U.S. Consumer Price Index (INF); Monthly growth rate of the U.S. industrial production (IP) obtained from the G.17 database of the Federal Reserve Board; and Monthly US unemployment rate (UNEMP) obtained from the Bureau of Labor Statistics.

According to Merton's (1973) ICAPM, state variables that are correlated with changes in consumption and investment opportunities are priced in capital markets in the sense that an asset's covariance with those state variables affects its expected returns. Merton (1973) also indicates that securities affected by such state variables (or systematic risk factors) should earn risk premia in a risk-averse economy. Macroeconomic variables used in the literature are excellent candidates for these systematic risk factors because innovations in macroeconomic variables can generate global impact on firm's fundamentals, such as their cash flows, risk-adjusted discount factors, and/or investment opportunities. Following the existing literature, we use the aforementioned financial and macroeconomic variables as proxies for state variables capturing shifts in the investment opportunity set.

We now investigate whether incorporating these variables into the predictive regressions affects the significance of the market prices of risk and uncertainty. Specifically, we estimate the portfolio-specific intercepts and the common slope coefficients from the following panel regression:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{m,t+1}
\end{aligned}
$$

where X_t denotes a vector of lagged control variables; default spread (DEF), term spread (TERM), relative T-bill rate (RREL), aggregate dividend yield (DIV), inflation rate (INF),

[19] The monthly data on 10-year T-bond yields, 3-month T-bill rates, BAA- and AAA-rated corporate bond yields are available from the Federal Reserve statistics release website.

growth rate of industrial production (IP), and unemployment rate (UNEMP). The common slope coefficients (A, B, and λ) and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten size, book-to-market, and industry portfolios.

As presented in Table 4, after controlling for a wide variety of financial and macroeconomic variables, our main findings remain intact for all equity portfolios. The common slope estimates on the conditional covariances of equity portfolios with the market factor (A) remain positive and highly significant, indicating a positive and significant relation between expected return and market risk. Similar to our earlier findings, the common slopes on the conditional covariances of equity portfolios with the uncertainty factor (B) remain significantly positive as well, showing that assets with higher correlation with the variance risk premium generate higher returns next month. Among the control variables, the growth rate of industrial production is the only variable predicting future returns on equity portfolios; λ_{IP} turns out to be positive and significant—especially for the industry portfolios. The positive relation between expected stock returns and innovations in output makes economic sense. Increases in real economic activity (proxied by the growth rate of industrial production) increase investors' expectations of future growth. Overall, the results in Table 4 indicate that after controlling for variables associated with business conditions, the time-varying exposures of equity portfolios to the market and uncertainty factors carry positive risk premiums.[20]

7.3 Results from Individual Stocks

We have so far investigated the significance of risk, uncertainty, and return tradeoffs using equity portfolios. In this section, we replicate our analyses using individual stocks trading at NYSE, AMEX, and NASDAQ. First, we generate a dataset for the largest 500 common stocks (share code = 10 or 11) traded at NYSE/AMEX/NASDAQ. Following Shumway (1997), we adjust for stock de-listing to avoid survivorship bias.[21] Firms with missing observations on

[20] We also used "expected business conditions" variable of Campbell and Diebold (2009) and our main findings remain intact for all equity portfolios. To save space, we do not report these results in the paper. They are available upon request.

[21] Specifically, the last return on an individual stock used is either the last return available on CRSP, or the de-listing return, if available. Otherwise, a de-listing return of -100% is included in the study, except that

beginning-of-month market cap or monthly returns over the period January 1990 – December 2010 are eliminated. Due to the fact that the list of 500 firms changes over time as a result of changes in firms' market capitalizations, we obtain more than 500 firms over the period 1990-2010. Specifically, the largest 500 firms are determined based on their end-of-month market cap as of the end of each month from January 1990 to December 2010. There are 738 unique firms in our first dataset. In our second dataset, the largest 500 firms are determined based on their market cap at the end of December 2010. Our last dataset contains stocks in the S&P 500 index. Since the stock composition of the S&P 500 index changes through time, we rely on the most recent sample (as of December 2010). We also restrict our S&P 500 sample to 318 stocks with non-missing monthly return observations for the period January 1990 – December 2010.

Table 5 presents the common slope estimates (A, B) and their t-statistics for the individual stocks in the aforementioned data sets. The risk aversion coefficient is estimated to be positive and highly significant for all stock samples considered in the paper: $A = 6.42$ with the t-statistic of 8.04 for the first dataset containing 738 stocks (largest 500 stocks as of the end of each month from January 1990 to December 2010); $A = 6.80$ with the t-statistic of 8.70 for the second dataset containing largest 500 stocks as of the end of December 2010; and $A = 6.02$ with the t-statistic of 6.79 for the last dataset containing 318 stocks with non-missing monthly return observations for the period 1990-2010. Confirming our findings from equity portfolios, the results from individual stocks imply a positive and significant relation between expected return and market risk. Similarly, consistent with our earlier findings from equity portfolios, the uncertainty aversion coefficient is also estimated to be positive and highly significant for all data sets: $B = 0.0043$ with the t-statistic of 3.61 for the first dataset, $B = 0.0044$ with the t-statistic of 3.67 for the second dataset, and $B = 0.0046$ with the t-statistic of 3.52 for the last dataset. These results indicate a significantly positive market price of uncertainty for large stocks trading in the U.S. stock market.

the deletion reason is coded as 500 (reason unavailable), 520 (went to OTC), 551-573, 580 (various reason), 574 (bankruptcy), and 584 (does not meet exchange financial guidelines). For these observations, a return of -30% is assigned.

7.4 Controlling for Market Illiquidity and Default Risk

Elevated variance risk premia during economic recessions and market downturns often correspond to the periods in which market illiquidity and default risk are both higher. Thus, it is natural to think that the conditional covariances of equity portfolios with market illiquidity and credit risk factors are positively linked to expected returns. In this section, we test whether the covariances with VRP could be picking up covariances with illiquidity and default risk.

Following Amihud (2002), we measure market illiquidity in a month as the average daily ratio of the absolute market return to the dollar trading volume within the month:

$$ILLIQ_t = \frac{1}{n}\sum_{d=1}^{n}\frac{|R_{m,d}|}{VOLD_m}$$

where $R_{m,d}$ and $VOLD_{m,d}$ are, respectively, the daily return and daily dollar trading volume for the S&P 500 index on day d, and n is the number of trading days in month t.

First, we generate the DCC-based conditional covariances of portfolio returns with market illiquidity and then estimate the common slope coefficients (A, B_1, B_2) from the following panel regressions:

$$\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}$$

where $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ and $Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right)$ are the time-t expected conditional covariance between the change in market illiquidity and the excess return on portfolio i and market portfolio m, respectively.

Table 6, Panel A, presents the common slope coefficients and their t-statistics estimated using the monthly excess returns on the market portfolio and the 10 size, book-to-market, and industry portfolios for the sample period January 1990 - December 2010. The slope on $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ is found to be positive but statistically insignificant for all equity portfolios considered in the paper. A notable point in Table 6 is that the slopes on $Cov_t(R_{i,t+1}, R_{m,t+1})$ and $Cov_t(R_{i,t+1}, VRP_{t+1})$ remain positive and highly significant after controlling for the covariances of equity portfolios with market illiquidity.

Next, we test whether the variance risk premium is proxying for default or credit risk. We use the TED spread as an indicator of credit risk and the perceived health of the banking system. The TED spread is the difference between the interest rates on interbank loans and short-term U.S. government debt (T-bills). TED is an acronym formed from T-Bill and ED, the ticker symbol for the Eurodollar futures contract.[22] The size of the spread is usually denominated in basis points (bps). For example, if the T-bill rate is 5.10% and ED trades at 5.50%, the TED spread is 40 bps. The TED spread fluctuates over time but generally has remained within the range of 10 and 50 bps (0.1% and 0.5%) except in times of financial crisis. A rising TED spread often presages a downturn in the U.S. stock market, as it indicates that liquidity is being withdrawn. The TED spread is an indicator of perceived credit risk in the general economy. This is because T-bills are considered risk-free while LIBOR reflects the credit risk of lending to commercial banks. When the TED spread increases, that is a sign that lenders believe the risk of default on interbank loans (also known as counterparty risk) is increasing. Interbank lenders therefore demand a higher rate of interest, or accept lower returns on safe investments such as T-bills. When the risk of bank defaults is considered to be decreasing, the TED spread decreases.

We first estimate the DCC-based conditional covariances of portfolio returns with the TED spread and then estimate the common slope coefficients from the following SUR re-

[22] Initially, the TED spread was the difference between the interest rates for three-month U.S. Treasuries contracts and the three-month Eurodollars contract as represented by the London Interbank Offered Rate (LIBOR). However, since the Chicago Mercantile Exchange dropped T-bill futures, the TED spread is now calculated as the difference between the three-month T-bill interest rate and three-month LIBOR.

gressions:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}
$$

where $Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right)$ and $Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right)$ are the time-t expected conditional covariance between the changes in TED spread and the excess returns on portfolio i and market portfolio m, respectively.

Table 6, Panel A, shows the common slope coefficients and their t-statistics estimated using the monthly excess returns on the market portfolio and the size, book-to-market, and industry portfolios. The slope on $Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right)$ is found to be positive for the size and book-to-market portfolios, and negative for the industry portfolios. Aside from yielding an inconsistent predictive relation with future returns, the slopes on the conditional covariances with the change in TED spread are statistically insignificant for all equity portfolios. Similar to our earlier findings, the slopes on the conditional covariances with the market risk and uncertainty factors remain positive and highly significant after controlling for the covariances with default risk.

Finally, we investigate the significance of risk and uncertainty coefficients after controlling for liquidity and credit spread simultaneously:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) \\
&\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}
$$

As shown in Panel A of Table 6, for the extended specification above, the common slope coefficient, B_2 on $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ is estimated to be positive and marginally significant for the book-to-market and industry portfolios, whereas B_2 is insignificant for the size portfolios. The covariances of equity portfolios with the change in TED spread do not predict future returns as B_3 is insignificant for all equity portfolios. Controlling for the market illiquidity and credit risk does not affect our main findings: the market risk-return and uncertainty-return coefficients (A and B_1) are both positive and highly significant for all equity portfolios. Equity portfolios that are highly correlated with VRP carry a significant premium relative to portfolios that are uncorrelated or minimally correlated with VRP.

We have so far provided evidence from the individual equity portfolios (10 size, 10 book-to-market, and 10 industry portfolios). We now investigate whether our main findings remain intact if we use a joint estimation with all test assets simultaneously (total of 30 portfolios). Panel B of Table 6 reports the parameter estimates and the t-statistics that are adjusted for heteroskedasticity and autocorrelation for each series and the cross-correlations among the error terms. As shown in the first row of Panel B, the risk aversion coefficient is estimated to be positive and highly significant for the pooled dataset: $A = 2.31$ with the t-statistic of 2.64, implying a positive and significant relation between expected return and market risk. Similar to our earlier findings, the uncertainty aversion coefficient is also estimated to be positive and highly significant for the joint estimation: $B = 0.0053$ with the t-statistic of 3.72. These results indicate a significantly positive market price of uncertainty when all portfolios are combined together. Equity portfolios with higher sensitivity to increases in VRP are expected to generate higher returns next period.

The last three rows in Panel B of Table 6 provide evidence for a positive and marginally significant relation between $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ and future returns, indicating that the conditional covariances of equity portfolios with the market illiquidity are positively linked to expected returns. However, the insignificant relation between $Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right)$ and portfolio returns remains intact for the joint estimation as well. A notable point in Panel B is that controlling for the market illiquidity and default risk individually and simultaneously does not influence the significant predictive power of the conditional covariances of portfolio returns with the market risk and VRP factors.

8 Cross-Sectional Relation between VRP-beta and Expected Returns

In this section, we investigate the cross-sectional asset pricing performance of our model by testing the significance of a cross-sectional relation between expected returns on equity portfolios and the portfolios' conditional covariances with VRP. Following Bali (2008) and Campbell, Giglio, Polk, and Turley (2012), we use the 25 and 100 Size and Book-to-Market (BM) portfolios of Kenneth French as test assets.[23] First, we estimate the DCC-based conditional covariances of 25 and 100 Size/BM portfolios with VRP and then for each month we form quintile portfolios sorted based on the portfolios' conditional covariances (or betas) with VRP. Since the conditional variance of VRP is the same across portfolios, we basically sort equity portfolios based on their VRP-beta:

$$VRP_{i,t}^{beta} = \frac{\text{cov}\left[R_{i,t+1}, VRP_{t+1}|\Omega_t\right]}{\text{var}\left[VRP_{t+1}|\Omega_t\right]}, \tag{25}$$

where $VRP_{i,t}^{beta}$ is the VRP-beta of portfolio i in month t, $\text{cov}\left[R_{i,t+1}, VRP_{t+1}|\Omega_t\right]$ is the conditional covariance of portfolio i with VRP estimated using equation (18), and $\text{var}\left[VRP_{t+1}|\Omega_t\right]$ is the conditional variance of VRP which is constant in the cross-section of equity portfolios.

Table 7 presents the average excess monthly returns of quintile portfolios that are formed by sorting the 25 and 100 Size/BM portfolios based on their VRP-beta. When we use the 25 Size/BM portfolios as test assets, each quintile arbitrage portfolio has a total of five Size/BM portfolios. Similarly, when we use the 100 Size/BM portfolios as test assets, each arbitrage portfolio has a total of 20 Size/BM portfolios. The results are presented for the sample period January 1990 to December 2010.

In Table 7, Q1 (Low VRP^{beta}) is the quintile portfolio of Size/BM portfolios with the lowest VRP-beta during the past month, and Q5 (High VRP^{beta}) is the quintile portfolio of Size/BM portfolios with the highest VRP-beta during the previous month. As shown in the left panel of Table 7, when the 25 Size/BM portfolios are used, the average excess return increases from 0.38% per month to 0.96% per month as we move from Q1 to Q5, generating

[23] 25 Size/BM and 100 Size/BM portfolios are described in and obtained from Kenneth French's data library.

an average return difference of 0.58% per month between Quintile 5 (High VRP^{beta}) and Quintile 1 (Low VRP^{beta}). This return difference is statistically significant with a Newey-West (1987) t-statistic of 2.51. In addition to the average excess returns, Table 7 also presents the intercepts (Fama-French three-factor alphas, denoted by FF3) from the regression of the average excess portfolio returns on a constant, the excess market return, a size factor (SMB), and a book-to-market factor (HML), following Fama and French (1993).[24] As shown in the last row of Table 7, the difference in FF3 alphas between the High VRP^{beta} and Low VRP^{beta} portfolios is 0.69% per month with a Newey-West t-statistic of 3.33. These results indicate that an investment strategy that goes long Size/BM portfolios in the highest VRP^{beta} quintile and shorts Size/BM portfolios in the lowest VRP^{beta} quintile produces average raw and risk-adjusted returns of 6.96% to 8.28% per annum, respectively. These return and alpha differences are economically and statistically significant at all conventional levels.

To determine whether the cross-sectional predictive power of VRP-beta is driven by the outperformance of High VRP^{beta} portfolios and/or the underperformance of Low VRP^{beta} portfolios, we compute the FF3 alpha of each quintile portfolio as well. As reported in Table 7, FF3 alpha of Q1 is -0.42% per month with a t-statistic of -2.66, and FF3 alpha of Q5 is 0.27% per month with a t-statistic of 2.46. These statistically significant FF3 alphas indicate that the significantly positive link between VRP-beta and the cross-section of portfolio returns is driven by both the outperformance of High VRP^{beta} and the underperformance of Low VRP^{beta} portfolios.

The right panel of Table 7 shows that similar results are obtained from the 100 Size/BM portfolios. The average excess return increases from 48 to 97 basis points per month as we move from the Low VRP^{beta} to High VRP^{beta} quintile portfolios. The last row of Table 7 presents an average return difference of 49 basis points per month between Q5 and Q1, with a Newey-West t-statistic of 2.14. Similar to our earlier findings, the difference in FF3 alphas between the High VRP^{beta} and Low VRP^{beta} portfolios is positive, 0.65% per month, and

[24]SMB (small minus big) and HML (high minus low) factors are described in and obtained from Kenneth French's data library.

highly significant with a t-statistic of 2.70. These results indicate that the equity portfolios in highest VRP^{beta} quintile generate 5.88% to 7.80% more annual raw and risk-adjusted returns compared to the equity portfolios in the lowest VRP^{beta} quintile. As shown in the last column of Table 7, FF3 alpha of Q1 is -0.37% per month with a t-statistic of -2.61, and FF3 alpha of Q5 is 0.28% per month with a t-statistic of 2.10, implying that the significantly positive link between VRP-beta and the cross-section of expected returns on the 100 Size/BM portfolios is driven by both the outperformance of High VRP^{beta} and the underperformance of Low VRP^{beta} portfolios.

We now examine the cross-sectional relation between VRP-beta, Market-beta and expected returns using Fama and MacBeth (1973) regressions. We calculate the time-series averages of the slope coefficients from the regressions of one-month ahead portfolio returns on the conditional covariances of portfolios with the market and VRP factors, $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ and $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$. The average slopes provide standard Fama-MacBeth tests for determining whether the market and/or uncertainty factors on average have non-zero premiums. Monthly cross-sectional regressions are run for the following asset pricing specification:

$$R_{i,t+1} = \lambda_{0,t} + \lambda_{1,t} \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + \lambda_{2,t} \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) + \varepsilon_{i,t+1}$$

where $R_{i,t+1}$ is the excess return on portfolio i in month $t+1$, $\lambda_{1,t}$ and $\lambda_{2,t}$ are the monthly slope coefficients on $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ and $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$, respectively. The predictive cross-sectional regressions of $R_{i,t+1}$ are run on the time-t expected conditional covariances of portfolios with the market and VRP factors.

We compute the time series averages of the slope coefficients ($\bar{\lambda}_1$, $\bar{\lambda}_2$) over the 252 months from January 1990 to December 2010 for both the 25 and 100 Size/BM portfolios. The bivariate regression results produce a positive and statistically significant relation between $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ and the cross-section of portfolios returns. The average slope, $\bar{\lambda}_2$, is estimated to be 0.0603 with a Newey-West t-statistic of 2.25 for the 25 Size/BM portfolios, and 0.0176 with a Newey-West t-statistic of 2.15 for the 100 Size/BM portfolios. Although we find a robust, significantly positive link between VRP-beta and expected returns from the Fama-MacBeth regressions, the cross-sectional relation between market beta and expected returns turns out to be sensitive to the choice of test assets. Specifically, the average slope, $\bar{\lambda}_1$, is found to be 7.78 with a t-statistic of 1.94 for the 25 Size/BM portfolios, whereas it is positive, but statistically insignificant for the 100 Size/BM portfolios.

The economic significance of the monthly slope coefficients from the Fama-MacBeth regressions can be interpreted based on the long-short equity portfolios. First, we compute the average values of $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ for the Size/BM portfolios sorted into the quintile portfolios. For the 25 Size/BM portfolios, the average $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ values are -1.0150 for Quintile 1, -0.8049 for Quintile 2, -0.7104 for Quintile 3, -0.6478 for Quintile 4, and -0.5655 for Quintile 5.[25] Hence, the difference in $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ values between equity portfolios in the Low VRP^{beta} and High VRP^{beta} quintiles is 0.4495 ($= -0.5655 - (-1.0150)$). To be consistent with our univariate portfolio results in Table 7, we run a univariate regression of $R_{i,t+1}$ on $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$, and the average slope of 0.0149 implies that the equity portfolios in highest VRP^{beta} quintile generate 0.67% ($0.0149 \times 0.4495 = 0.67\%$) more monthly returns compared to the equity portfolios in the lowest VRP^{beta} quintile.

To determine the economic significance of the slope coefficients for the 100 Size/BM portfolios, we calculate the average $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ values for each quintile portfolio as well: -1.0872 for Quintile 1, -0.8206 for Quintile 2, -0.7150 for Quintile 3, -0.6308 for Quintile 4, and -0.5053 for Quintile 5. Hence, the difference in $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ values between equity portfolios in the Low VRP^{beta} and High VRP^{beta} quintiles is 0.5819. The univariate Fama-MacBeth regressions of one-month ahead portfolio returns on $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ yields an average slope coefficient of 0.0103 for the 100 Size/BM portfolios. This positive and significant average slope coefficient implies that buying the Size/BM portfolios in highest VRP^{beta} quintile and short-selling the Size/BM portfolios in the lowest VRP^{beta} quintile generate a 0.60% return in the following month. These return magnitudes implied by the Fama-MacBeth slope coefficients (0.67% and 0.60% per month) are in line with the univariate portfolio results reported in Table 7 (0.58% and 0.49% per month, respectively).

[25] The negative values for the conditional covariances of equity portfolios with the VRP factor are consistent with the negative value for conditional covariance of the market return with the VRP factor reported earlier in Section 6.2.

9 Conclusion

Although uncertainty is more common in decision-making process than risk, relatively little attention is paid to the phenomenon of uncertainty in empirical asset pricing literature. This paper focuses on economic uncertainty and augments the original Merton's (1973) ICAPM to introduce a conditional ICAPM model with time-varying market risk and uncertainty. According to the augmented asset pricing model, the premium on equity is composed of two separate terms; the first term compensates for the market risk and the second term representing a true premium for economic uncertainty. We use the conditional ICAPM to test whether the time-varying conditional covariances of equity returns with market and uncertainty factors predict their future returns.

Since information about economic uncertainty is too imprecise to measure with available data, we have to come up with a proxy for uncertainty that should be consistent with the investment opportunity set of risk-averse investors. Following Zhou (2010), we measure economic uncertainty with the variance risk premium (VRP) of the aggregate stock market portfolio. Different from earlier studies, we provide empirical evidence that VRP is indeed closely related to economic and financial market uncertainty. Specifically, we generate several proxies for uncertainty based on the macroeconomic variables, return distributions of financial firms, credit default swap market, and investors' disagreement about individual stocks. We show that VRP is highly correlated with all measures of uncertainty.

Based on the two-factor asset pricing model, we investigate whether the market prices of risk and uncertainty are economically and statistically significant in the U.S. equity market. Using the dynamic conditional correlation (DCC) model of Engle (2002), we estimate equity portfolios' conditional covariances with the market portfolio and VRP factors and then test whether these dynamic conditional covariances predict future returns on equity portfolios. The empirical results from the size, book-to-market, and industry portfolios indicate that the DCC-based conditional covariances of equity portfolios with the market and VRP factors predict the time-series and cross-sectional variation in stock returns. We find the risk-return coefficients to be positive and highly significant, implying a strongly positive link between expected return and market risk. Similarly, the results indicate a significantly positive market price of uncertainty. That is, equity portfolios that are highly correlated with uncertainty (proxied by VRP) carry a significant premium relative to portfolios that are uncorrelated or minimally correlated with VRP. In addition to the size, book-to-market, and industry portfolios, we investigate the significance of risk, uncertainty, and return tradeoffs using the largest 500 stocks trading at NYSE, AMEX, and NASDAQ as well as stocks in the S&P 500 index. Consistent with our findings from equity portfolios, we find significantly positive market prices of risk and uncertainty for large stocks trading in the U.S. equity market.

We also examine whether the conditional covariances with VRP could be picking up the covariances with market illiquidity and/or default risk. We find that the significantly positive link between market covariance risk, uncertainty and future returns remain intact after controlling for liquidity and credit risk.

Finally, we investigate the cross-sectional asset pricing performance of our model using the long-short equity portfolios and the Fama-MacBeth regressions. The results indicate that the annual average raw and risk-adjusted returns of the equity portfolios in the highest VRP-beta quintile are 6 to 8 percent higher than the annual average returns of the equity portfolios in the lowest VRP-beta quintile. After controlling for the market, size, and book-to-market factors of Fama and French (1993), the positive relation between VRP-beta and the cross-section of portfolio returns remains economically and statistically significant. Overall, we conclude that the time-varying exposures of equity portfolios to the variance risk premia predict the time-series and cross-sectional variation in stock returns.

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Table 1 Results from Ten Decile Size, Book-to-Market, and Industry Portfolios

This table reports the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$\begin{aligned} R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) + \varepsilon_{i,t+1} \\ R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) + \varepsilon_{m,t+1} \end{aligned}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the variance risk premia (VRP_{t+1}), $Cov_t\left(R_{m,t+1}, VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and the variance risk premia (VRP_{t+1}), and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten decile size, book-to-market, and industry portfolios for the sample period from January 1990 to December 2010. The alphas (α_i) are reported for each equity portfolio and the t-statistics are presented in parentheses. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last four rows, respectively, show the common slope coefficients (A and B), the Wald$_1$ statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald$_2$ statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald$_1$ and Wald$_2$ statistics are given in square brackets.

Size	α_i, α_m	*BM*	α_i, α_m	*Industry*	α_i, α_m
Small	0.0041	Growth	0.0036	NoDur	0.0043
	(0.94)		(0.90)		(1.53)
2	0.0022	2	0.0048	Durbl	0.0019
	(0.48)		(1.38)		(0.37)
3	0.0025	3	0.0053	Manuf	0.0046
	(0.58)		(1.58)		(1.26)
4	0.0015	4	0.0065	Enrgy	0.0059
	(0.35)		(1.88)		(1.88)
5	0.0023	5	0.0057	HiTec	0.0026
	(0.54)		(1.74)		(0.45)
6	0.0023	6	0.0051	Telcm	-0.0005
	(0.61)		(1.51)		(-0.13)
7	0.0028	7	0.0058	Shops	0.0028
	(0.76)		(1.78)		(0.80)
8	0.0020	8	0.0059	Hlth	0.0036
	(0.53)		(1.76)		(1.13)
9	0.0023	9	0.0067	Utils	0.0038
	(0.67)		(1.94)		(1.39)
Big	0.0001	Value	0.0082	Other	0.0018
	(0.01)		(1.90)		(0.47)
Market	0.0008	Market	0.0032	Market	0.0019
	(0.17)		(1.23)		(0.55)
A	3.9562	*A*	2.5101	*A*	3.4055
	(3.12)		(2.53)		(2.35)
B	0.0058	*B*	0.0050	*B*	0.0060
	(2.97)		(2.27)		(2.78)
Wald$_1$	16.74	Wald$_1$	8.88	Wald$_1$	14.35
	[0.12]		[0.63]		[0.21]
Wald$_2$	1.56	Wald$_2$	1.79	Wald$_2$	0.40
	[0.21]		[0.18]		[0.53]

Table 2 Relative Performance of Conditional ICAPM with Risk and Uncertainty

This table presents the realized monthly average excess returns on the size, book-to-market, and industry portfolios and the cross-section of expected excess returns generated by the Conditional CAPM with the market factor and the Conditional ICAPM with the market and VRP factors. The last row reports the Mean Absolute Percentage Errors (MAPE) for the two competing models.

	Realized Return Benchmark	**Conditional ICAPM with VRP**	**Conditional CAPM**
Size	Average Excess Returns	Expected Excess Returns	Expected Excess Returns
Small	0.8464%	0.8461%	0.8742%
2	0.7737%	0.7677%	0.8110%
3	0.7690%	0.7647%	0.8093%
4	0.6632%	0.6637%	0.7032%
5	0.7525%	0.7550%	0.7943%
6	0.7055%	0.7025%	0.7406%
7	0.7409%	0.7379%	0.7749%
8	0.6837%	0.6810%	0.7221%
9	0.6670%	0.6643%	0.7000%
Big	0.4479%	0.4598%	0.4789%
MAPE		**0.61%**	**5.20%**
	Realized Return Benchmark	**Conditional ICAPM with VRP**	**Conditional CAPM**
Book-to-Market	Average Excess Returns	Expected Excess Returns	Expected Excess Returns
Growth	0.5286%	0.5327%	0.5645%
2	0.5614%	0.5658%	0.5961%
3	0.6140%	0.6039%	0.6488%
4	0.6752%	0.6559%	0.6960%
5	0.6119%	0.6017%	0.6423%
6	0.5439%	0.5547%	0.5803%
7	0.6014%	0.5979%	0.6360%
8	0.5885%	0.5956%	0.6233%
9	0.6827%	0.6666%	0.7133%
Value	0.8221%	0.7994%	0.8564%
MAPE		**1.66%**	**5.37%**
	Realized Return Benchmark	**Conditional ICAPM with VRP**	**Conditional CAPM**
Industry	Average Excess Returns	Expected Excess Returns	Expected Excess Returns
Telcm	0.2727%	0.2747%	0.3280%
Utils	0.4712%	0.4727%	0.4965%
Other	0.4965%	0.4910%	0.5366%
Durbl	0.5313%	0.5315%	0.5513%
Shops	0.5954%	0.5912%	0.6247%
Hlth	0.6138%	0.6088%	0.6478%
NoDur	0.6110%	0.6152%	0.6534%
Manuf	0.7172%	0.7206%	0.7474%
Enrgy	0.7606%	0.7643%	0.7824%
HiTec	0.8358%	0.8350%	0.8466%
MAPE		**0.55%**	**6.32%**

Table 3 Results from Larger Cross-Section of Industry Portfolios

This table presents the common slope estimates (A, B) from the following panel regression:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}
$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the variance risk premia (VRP_{t+1}), $Cov_t\left(R_{m,t+1}, VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and the variance risk premia (VRP_{t+1}), and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the 17, 30, 38, 48, and 49 industry portfolios for the sample period from January 1990 to December 2010. The alphas (α_i) are reported for each equity portfolio and the t-statistics are presented in parentheses. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last four rows, respectively, show the common slope coefficients (A and B), the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

17-industry portfolios		30-industry portfolios		38-industry portfolios		48-industry portfolios		49-industry portfolios	
A	2.6399	A	2.1975	A	2.2988	A	2.3271	A	2.7840
	(2.31)		(2.52)		(2.47)		(2.97)		(3.34)
B	0.0041	B	0.0036	B	0.0035	B	0.0041	B	0.0041
	(2.44)		(2.98)		(2.45)		(3.47)		(4.21)
Wald_1	16.41	Wald_1	35.11	Wald_1	30.89	Wald_1	57.20	Wald_1	52.04
	[0.56]		[0.28]		[0.75]		[0.20]		[0.39]
Wald_2	0.58	Wald_2	0.06	Wald_2	0.32	Wald_2	0.53	Wald_2	0.13
	[0.44]		[0.80]		[0.57]		[0.47]		[0.72]

Table 4 Controlling for Macroeconomic Variables

This table presents the common slope estimates from the following panel regression:

$$\begin{aligned} R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{i,t+1} \\ R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) + \lambda \cdot X_t + \varepsilon_{m,t+1} \end{aligned}$$

where X_t denotes a vector of lagged control variables; default spread (DEF), term spread (TERM), relative T-bill rate (RREL), aggregate dividend yield (DIV), inflation rate (INF), growth rate of industrial production (IP), and unemployment rate (UNEMP). The common slope coefficients (A, B, and λ) and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten size, book-to-market, and industry portfolios for the sample period January 1990 to December 2010. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last two rows the Wald_1 statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald_2 statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald_1 and Wald_2 statistics are given in square brackets.

	Size	Book-to-Market	Industry
A	4.2630	2.5763	4.0421
	(3.32)	(2.40)	(2.74)
B	0.0057	0.0051	0.0066
	(2.85)	(2.25)	(2.96)
λ_{DEF}	-0.3804	-0.0739	0.6243
	(-0.50)	(-0.09)	(1.02)
λ_{TERM}	-0.1964	-0.5366	-0.5405
	(-0.64)	(-1.69)	(-2.17)
λ_{RREL}	0.2330	0.1834	0.0104
	(0.68)	(0.52)	(0.04)
λ_{DIV}	0.0489	0.0228	0.0314
	(1.33)	(0.60)	(1.05)
λ_{INF}	0.0270	0.7158	-0.1862
	(0.04)	(0.93)	(-0.31)
λ_{IP}	0.7433	0.8689	1.1941
	(1.77)	(2.01)	(3.51)
λ_{UNEMP}	0.0031	0.0047	0.0026
	(1.13)	(1.61)	(1.15)
Wald_1	16.96	7.97	14.78
	[0.11]	[0.72]	[0.19]
Wald_2	1.46	1.63	0.67
	[0.23]	[0.20]	[0.41]

Table 5 Results from Individual Stocks

This table presents the common slope estimates (A, B) from the following panel regression:

$$\begin{aligned} R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) + \varepsilon_{i,t+1} \\ R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) + \varepsilon_{m,t+1} \end{aligned}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the variance risk premia (VRP_{t+1}), $Cov_t\left(R_{m,t+1}, VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on the market portfolio m and the variance risk premia (VRP_{t+1}), and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the largest 500 stocks trading at NYSE, AMEX, and NASDAQ, and 318 stocks in the S&P 500 index for the sample period from January 1990 to December 2010. First, the largest 500 firms is determined based on their end-of-month market cap as of the end of each month from January 1990 to December 2010. Due to the fact that the list of 500 firms changes over time as a result of changes in firms' market capitalizations, there are 738 unique firms in our first dataset. In our second dataset, the largest 500 firms is determined based on their market cap at the end of December 2010. Our last dataset contains stocks in the S&P 500 index. Since the stock composition of the S&P 500 index changes through time, we rely on the most recent sample. We also restrict our S&P 500 sample to 318 stocks with non-missing monthly return observations for the period January 1990 – December 2010. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios.

Largest 500 Stocks end-of-month		*Largest 500 Stocks as of December 2010*		*Largest 500 Stocks S&P 500 Index*	
A	6.4237	A	6.8014	A	6.0243
	(8.04)		(8.70)		(6.79)
B	0.0043	B	0.0044	B	0.0046
	(3.61)		(3.67)		(3.52)

Table 6 Controlling for Market Illiquidity and Default Risk

This table presents the common slope estimates (A, B_1, B_2, B_3) from the following panel regression:

$$\begin{aligned} R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) \\ &\quad + B_2 \cdot Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{i,t+1} \\ R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B_1 \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) \\ &\quad + B_2 \cdot Cov_t\left(R_{m,t+1}, \Delta ILLIQ_{t+1}\right) + B_3 \cdot Cov_t\left(R_{m,t+1}, \Delta TED_{t+1}\right) + \varepsilon_{m,t+1} \end{aligned}$$

where $Cov_t\left(R_{i,t+1}, R_{m,t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i ($R_{i,t+1}$) and the excess return on the market portfolio ($R_{m,t+1}$), $Cov_t\left(R_{i,t+1}, VRP_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the variance risk premia (VRP_{t+1}), $Cov_t\left(R_{i,t+1}, \Delta ILLIQ_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the change in market illiquidity ($\Delta ILLIQ_{t+1}$), $Cov_t\left(R_{i,t+1}, \Delta TED_{t+1}\right)$ is the time-t expected conditional covariance between the excess return on portfolio i and the change in TED spread (ΔTED_{t+1}), and $Var_t\left(R_{m,t+1}\right)$ is the time-t expected conditional variance of excess returns on the market portfolio. In Panel A, the parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the 10 decile size, book-to-market, and industry portfolios for the sample period from January 1990 to December 2010. In Panel B, the results are generated using a joint estimation with all test assets simultaneously (total of 30 portfolios). The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and the cross-correlations among the portfolios.

Panel A. Results from 10 Equity Portfolios

10 Equity Portfolios	A	B_1	B_2	B_3
Size	6.2227	0.0069	1.2423	
	(2.47)	(3.07)	(1.29)	
Size	3.6465	0.0052		0.6372
	(2.84)	(2.09)		(0.91)
Size	5.7826	0.0057	0.4347	1.1582
	(2.48)	(2.12)	(0.69)	(1.17)
Book-to-Market	5.3065	0.0062	2.2003	
	(2.66)	(2.65)	(1.34)	
Book-to-Market	2.5695	0.0056		0.3148
	(2.24)	(2.37)		(0.54)
Book-to-Market	6.4767	0.0079	2.8237	0.3247
	(2.13)	(2.90)	(1.69)	(0.61)
Industry	7.8266	0.0080	2.5677	
	(2.35)	(3.16)	(1.52)	
Industry	3.1868	0.0071		-0.7625
	(2.17)	(2.88)		(-1.11)
Industry	9.2805	0.0102	3.5064	-1.0014
	(2.69)	(3.49)	(1.99)	(-1.43)

Table 6 (continued)

Panel B. Results from 30 Equity Portfolios

A	B_1	B_2	B_3
2.3110	0.0053		
(2.64)	(3.72)		
3.2552	0.0060	0.6796	
(2.82)	(4.03)	(1.94)	
2.1153	0.0055		-0.0477
(2.41)	(3.49)		(-0.11)
3.0967	0.0062	0.6497	-0.0844
(2.72)	(3.78)	(1.95)	(-0.20)

Table 7 Long-Short Equity Portfolios Sorted by VRP-beta

Quintile portfolios are formed every month from January 1990 to December 2010 by sorting the 25 and 100 Size/BM portfolios based on their VRP-beta (VRP^{beta}) over the past one month. Quintile 1 (Q1) is the portfolio of Size/BM portfolios with the lowest VRP^{beta} over the past one month. Quintile 5 (Q5) is the portfolio of Size/BM portfolios with the highest VRP^{beta} over the past one month. The table reports the average excess monthly returns and the 3-factor Fama-French alphas (FF3 alpha) on the VRP-beta sorted portfolios. The last row presents the differences in monthly returns and the differences in alphas with respect to the 3-factor Fama-French model between Quintiles 5 and 1 and the corresponding t-statistics. Average excess return and risk-adjusted returns are given in percentage terms. Newey-West (1987) t-statistics are reported in parentheses.

	25 Size/BM Portfolios			**100 Size/BM Portfolios**	
	Average Excess Return	FF3 Alpha		Average Excess Return	FF3 Alpha
Q1	0.0038	-0.0042	Q1	0.0048	-0.0037
	(1.08)	(-2.66)		(1.33)	(-2.61)
Q2	0.0074	-0.0005	Q2	0.0072	-0.0008
	(2.49)	(-0.60)		(2.40)	(-0.94)
Q3	0.0078	0.0002	Q3	0.0080	0.0004
	(2.72)	(0.24)		(2.75)	(0.62)
Q4	0.0097	0.0022	Q4	0.0094	0.0019
	(3.26)	(1.80)		(3.25)	(2.15)
Q5	0.0096	0.0027	Q5	0.0097	0.0028
	(3.47)	(2.46)		(3.30)	(2.10)
Q5-Q1	0.0058	0.0069	Q5-Q1	0.0049	0.0065
	(2.51)	(3.33)		(2.14)	(2.70)

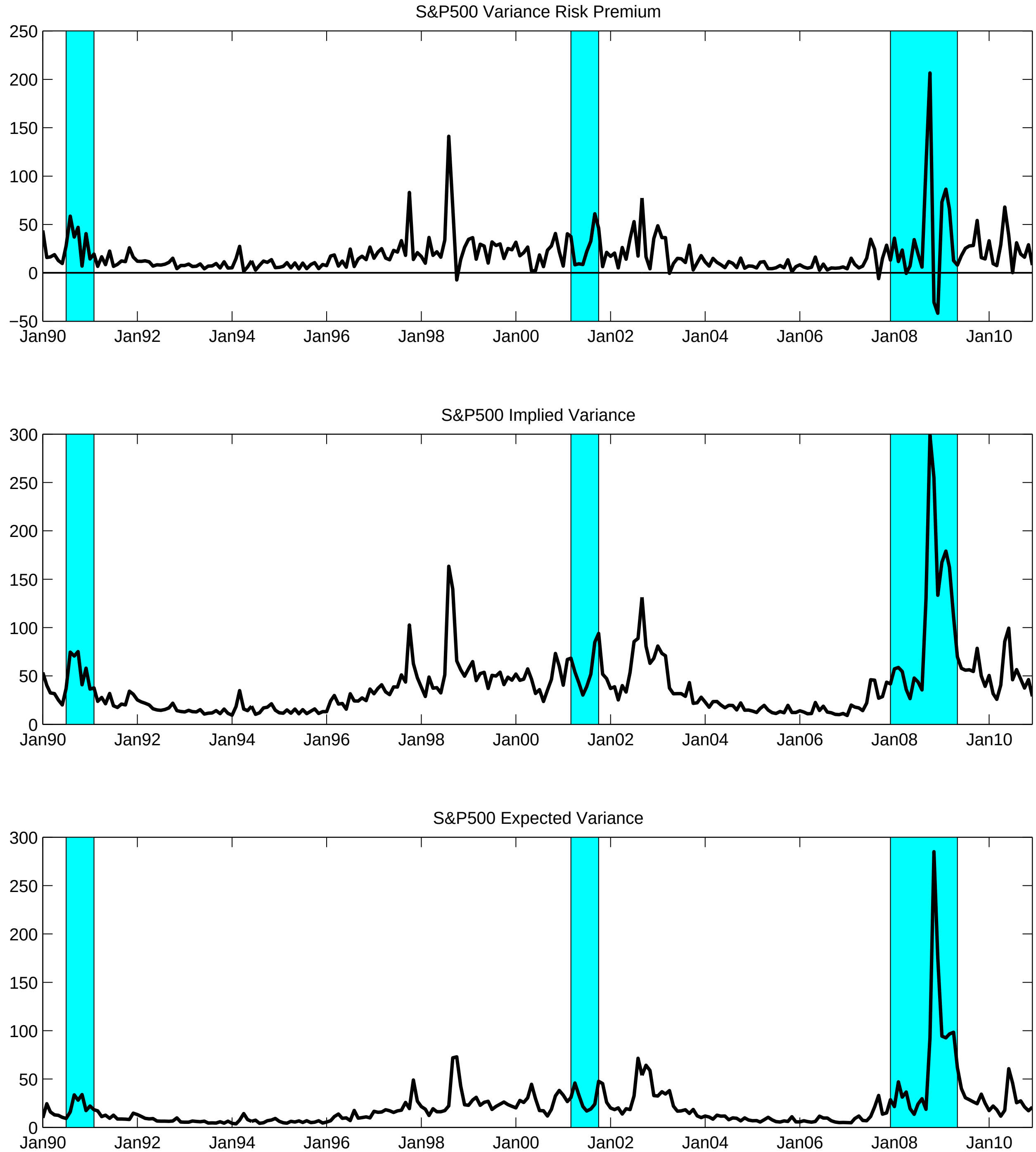

Figure 1 Variance Risk Premium, Implied and Realized Variances

This figure plots variance risk premium or the implied-expected variance difference (top panel), implied variance (middle panel), and forecasted realized variance (bottom panel) for the S&P500 market index from January 1990 to December 2010. The variance risk premium is based on the realized variance forecast from lagged implied and realized variances. The shaded areas represent NBER recessions.

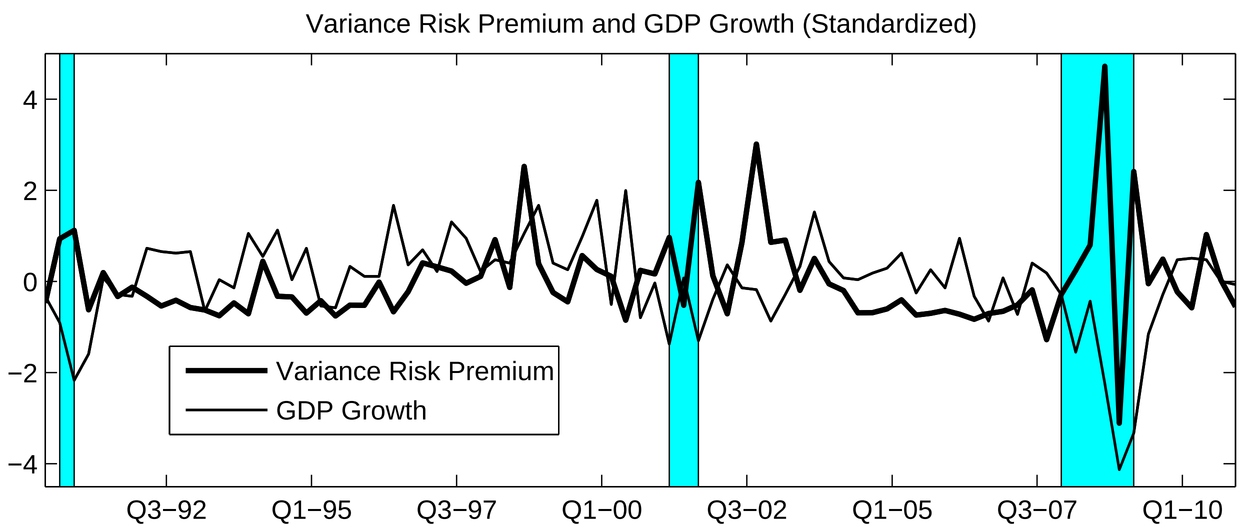

Figure 2 Variance Risk Premium and GDP Growth

The figure plots the GDP growth rates (thin line) together with the variance risk premium (thick line) from 1990Q1 to 2010Q4. Both of the series are standardized to have mean zero and variance one. The shaded areas represent NBER recessions.

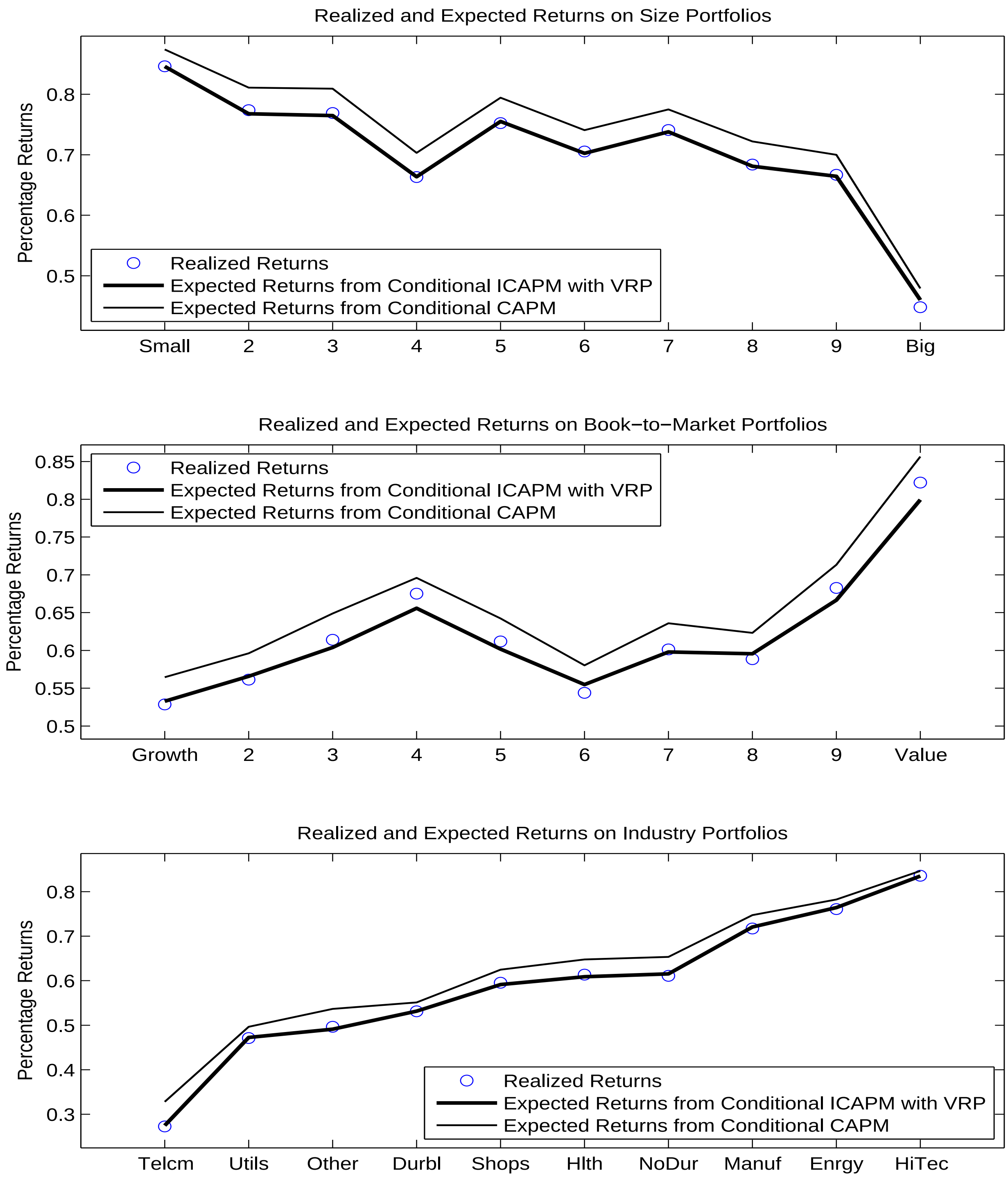

Figure 3 Relative Performance of the Conditional ICAPM with Uncertainty

This figure plots the realized monthly average excess returns on the size (top panel), book-to-market (middle panel), and industry portfolios (bottom panel) and the cross-section of expected excess returns generated by the Conditional CAPM with the market factor and the Conditional ICAPM with the market and VRP factors. The results indicate superior performance of the conditional asset pricing model introduced in the paper.

Risk, Uncertainty, and Expected Returns—Internet Appendix

A A Consumption-based Asset Pricing Model with Economic Uncertainty

To guide our economic interpretation of the empirical finding in the main paper, we follow the strategy of Campbell (1993, 1996) to substitute unobservable consumption-based measures with observable market-based measures. Under a structural model with recursive preference and consumption uncertainty (Bollerslev, Tauchen, and Zhou, 2009), one can show that the model-implied market compensations for risk and uncertainty are both positive, under reasonable parameter settings that agents are more risk averse than the log utility and that agents prefer an early resolution of economic uncertainty. In essence, the two risk factors—market return and variance risk premium—span all systematic variations in any risky assets.

A.1 An Economic Model of Return-Uncertainty Tradeoff

The representative agent in the economy is endowed with Epstein-Zin-Weil recursive preferences, and has the value function V_t of her life-time utility as

$$V_t = \left[(1-\delta) C_t^{\frac{1-\gamma}{\theta}} + \delta \left(E_t \left[V_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{A1}$$

where C_t is consumption at time t, δ denotes the subjective discount factor, γ refers to the coefficient of risk aversion, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ equals the intertemporal elasticity of substitution (IES). The key assumptions are that $\gamma > 1$, implying that the agents are more risk averse than the log utility investors; and $\psi > 1$ hence $\theta < 0$, implying that agents prefer an earlier resolution of economic uncertainty.

Suppose that log consumption growth and its volatility follow the joint dynamics

$$g_{t+1} = \mu_g + \sigma_{g,t} z_{g,t+1}, \tag{A2}$$

$$\sigma^2_{g,t+1} = a_\sigma + \rho_\sigma \sigma^2_{g,t} + \sqrt{q_t} z_{\sigma,t+1}, \tag{A3}$$

$$q_{t+1} = a_q + \rho_q q_t + \varphi_q \sqrt{q_t} z_{q,t+1}, \tag{A4}$$

where $\mu_g > 0$ denotes the constant mean growth rate, $\sigma^2_{g,t+1}$ represents time-varying volatility in consumption growth, and q_t introduces the volatility uncertainty process in the consumption growth process.[1]

Let w_t denote the logarithm of the price-dividend or wealth-consumption ratio, of the asset that pays the consumption endowment, $\{C_{t+i}\}_{i=1}^{\infty}$; and conjecture a solution for w_t as an affine function of the state variables, $\sigma^2_{g,t}$ and q_t,

$$w_t = A_0 + A_\sigma \sigma^2_{g,t} + A_q q_t. \tag{A5}$$

One can solve for the coefficients A_0, A_σ and A_q using the standard Campbell and Shiller (1988) approximation $r_{t+1} = \kappa_0 + \kappa_1 w_{t+1} - w_t + g_{t+1}$, where r_{t+1} is the return on the asset that pays the consumption endowment flow. The restrictions that $\gamma > 1$ and $\psi > 1$, hence $\theta < 0$, imply that the impact coefficients associated with both volatility and uncertainty state variables are negative; i.e., $A_\sigma < 0$ and $A_q < 0$. So if consumption risk and economic uncertainty are high, the price-dividend ratio is low, hence risk premia are high.

Given the solution of price-dividend ratio, and assume that dividend equals consumption, the model-implied premium of the market portfolio can be shown as

$$E\left[R_{m,t+1}|\Omega_t\right] \;=\; \gamma\sigma^2_{g,t} \;+\; (1-\theta)\kappa_1^2(A_q^2\varphi_q^2 + A_\sigma^2)q_t. \tag{A6}$$

The premium is composed of two separate terms. The first term, $\gamma\sigma^2_{g,t}$, is compensating for the classic consumption risk as in a standard consumption-based CAPM model. The second term, $(1-\theta)\kappa_1^2(A_q^2\varphi_q^2 + A_\sigma^2)q_t$, represents a true premium for variance risk or economic uncertainty. The restrictions that $\gamma > 1$ and $\psi > 1$ implies that the uncertainty or variance risk premium is always positive by construction.

The conditional variance of the time t to $t+1$ market return, $\sigma^2_{m,t} \equiv \mathrm{Var}_t(r_{t+1})$, can be shown as $\sigma^2_{m,t} = \sigma^2_{g,t} + \kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right)q_t$. The variance risk premium can be defined as the difference between risk-neutral and objective expectations of the return variance,[2]

$$VRP_t \equiv \mathrm{E}^Q\left[\sigma^2_{m,t+1}|\Omega_t\right] - \mathrm{E}^P\left[\sigma^2_{m,t+1}|\Omega_t\right] \approx (\theta-1)\kappa_1\left[A_\sigma + A_q\kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right)\varphi_q^2\right]q_t.$$

[1]The parameters satisfy $a_\sigma > 0, a_q > 0$, $|\rho_\sigma| < 1$, $|\rho_q| < 1$, $\varphi_q > 0$; and $\{z_{g,t}\}$, $\{z_{\sigma,t}\}$ and $\{z_{q,t}\}$ are iid Normal$(0,1)$ processes jointly independent with each other.

[2]The approximation comes from the fact that the model-implied risk-neutral conditional expectation cannot be computed in closed form, and a log-linear approximation is applied.

Moreover, provided that $\theta < 0$, $A_\sigma < 0$, and $A_q < 0$, as would be implied by the agents' preference of an earlier resolution of economic uncertainty, this difference between the risk-neutral and objective expectations of return variances is guaranteed to be positive.

However, due to the measurement difficulty in consumption data and its volatility, we will use market return volatility and variance risk premium to substitute fundamental risk and uncertainty that are harder to pin down accurately (Campbell, 1993),

$$E\left[R_{m,t+1}|\Omega_t\right] \;=\; \gamma\sigma_{m,t}^2 + \frac{(1-\theta-\gamma)\kappa_1^2(A_\sigma^2 + A_q^2\varphi_q^2)}{(\theta-1)\kappa_1\left[A_\sigma + A_q\kappa_1^2\left(A_\sigma^2 + A_q^2\varphi_q^2\right)\varphi_q^2\right]} VRP_t\,. \tag{A7}$$

Therefore the risk-return trade-off identified by γ is always positive. However, the uncertainty-return trade-off depends on the sign of $(1-\theta-\gamma)$. Under typical preference parameter setting, as in Bansal and Yaron (2004) and Bollerslev, Tauchen, and Zhou (2009), θ tends to be a large negative number, and one always has $(1-\theta-\gamma) > 0$. In other words, the model implied uncertainty-return tradeoffs should always be positive.

Campbell (1993) shows that, in an intertemporal CAPM setting (Merton, 1973), the appropriate choices for factors relevant in cross-sectional asset pricing tests should be the current market return and any variables that have information about the future market returns. Given the recent evidence that variance risk premium (VRP) possesses a significant forecasting power for short-term market returns (see, e.g., Bollerslev, Tauchen, and Zhou, 2009, among others), it is natural to postulate the following cross-sectional asset pricing implication along the lines of Campbell, Giglio, Polk, and Turley (2012):

$$E\left[R_{i,t+1}|\Omega_t\right] = A\cdot \operatorname{cov}\left[R_{i,t+1}, R_{m,t+1}|\Omega_t\right] + B\cdot \operatorname{cov}\left[R_{i,t+1}, h_{t+1}|\Omega_t\right], \tag{A8}$$

where the model implied coefficients $A = \gamma > 0$ and $B = -\theta/\psi > 0$, and we approximate the intertemporal hedging component h_t with variance risk premium VRP_t. The intuition for the positive slope coefficient B, is that investors dislike the reduced ability to hedge against a deterioration in the investment opportunity captured by VRP_t—which *positively* predicts future market returns. Therefore investors require a higher return premium to hold the assets or stocks that *positively* covary with VRP_t (Campbell, 1996).

A.2 Calibrating Uncertainty-Return Tradeoff

To give some empirical guidance on how such a modeling framework with two risk drivers—consumption risk and volatility uncertainty—can play out in empirically testing the time-series and cross-sectional stock returns, we provide some calibration evidence based on the model parameter settings used by Bollerslev, Tauchen, and Zhou (2009, or BTZ2009 for short) focusing on equity return predictability and Zhou (2010) also considering bond return and credit spread predictability. As shown in Table I, consistent with the analytical characterization above, the risk-return trade-off coefficient or A should be equal to the risk-aversion coefficient, which is 10 or 2 under the two model parameter choices. On the other hand, the uncertainty-return coefficient or B should be equal to 10.24 or 0.08, which is a highly non-linear function of both the underlying preference and structural parameters. The model implied uncertainty-return trade-off is positive.

More importantly, the positive relationship between variance risk premium and excess market return is fairly robust. There are two key preference parameters—intertemporal elasticity of substitution (IES) and risk aversion coefficient that may materially affect the sign and magnitude of the return-uncertainty trade-off. However, as shown in the top two panels of Figure 1, as long as IES—ψ is larger than one and risk aversion—γ is larger than one, the model-implied linkage between return and uncertainty should remain positive.

In contrast, when agents prefer a late resolution of uncertainty or $0 < \psi < 1$ (bottom left panel), the model implied return-uncertainty trade-off swings between positive and negative values with a bifurcation towards infinities near $\psi = 0.5$. Similarly, if agents are less risk averse than log investor or $0 < \gamma < 1$ (bottom right panel), the uncertainty-return trade-off also swings between large positive and negative values near $\gamma = 0.67$. The empirical implication is rather sharp—if we find that the exposures to variance risk premium are positively priced in stock returns, it would be consistent with our assumptions that both IES and risk aversion are larger than one—as sufficient conditions.

There is a long debate about whether the intertemporal elasticity of substitution or IES is larger than one. As emphasized by Beeler and Campbell (2009), a high IES—around 1.5—is key to the success of long-run risks model (Bansal and Yaron, 2004). Although earlier time series evidences (Hall, 1988; Campbell, 1999) suggest a small IES close to zero, the regression estimates can be downward biased if consumption volatility is time-varying (Bansal, Kiku, and Yaron, 2007). On the other hand, financial market implications on IES being less than one are found by Kandel and Stambaugh (1991) and Liu, Zhang, and Fan (2011).

Our empirical approach on estimating the risk-return and uncertainty-return trade-off from time-series and cross-section of stock returns provides an alternative reduced-form angle to judge whether IES is bigger than one. Our empirical finding of a positive uncertainty-return trade-off is consistent with an IES larger than one without imposing parametric restrictions, nor do we rely on the Euler equations or GMM estimation as in Bansal, Kiku, and Yaron (2009) and Chen, Favilukis, and Ludvigson (2011).

B Equity Portfolios

In addition to the 1990-2010 period, Table II presents the monthly raw return and CAPM Alpha differences between high-return (long) and low-return (short) equity portfolios (size, book-to-market, and industry) for the sample periods 1926-2010 and 1963-2010. For the sample period July 1926 – December 2010, the average return difference between the Small and Big portfolios is 0.60% per month with the OLS t-statistic of 2.49 and the Newey-West (1987) t-statistic of 2.36, implying that small stocks on average generate higher returns than big stocks. The CAPM Alpha (or abnormal return) for the long-short size portfolio is 0.27% per month with the OLS t-statistic of 1.22 and the Newey-West t-statistic of 1.38. This economically and statistically insignificant Alpha indicates that the static CAPM does explain the size effect for the 1926-2010 period.

For the ten book-to-market portfolios, the average return difference between the Value and Growth portfolios is 0.53% per month with the OLS t-statistic of 2.52 and the Newey-West t-statistic of 2.46, implying that value stocks on average generate higher returns than growth stocks (the so-called value premium). Similar to our findings for the size portfolios, the unconditional CAPM can explain the value premium for the 1926-2010 period; the CAPM

Alpha (or abnormal return) for the long-short book-to-market portfolio is only 0.24% per month with the OLS t-statistic of 1.25 and the Newey-West t-statistic of 1.26.

The last six rows in Table II report average return differences and CAPM Alphas for the industry portfolios (10-, 17-, 30-, 38-, 48-, and 49-industry portfolios). For the long sample period of 1926-2010, only the extreme portfolios of 48 and 49 industries generate significant return differences, whereas the average return differences for the high-return and low-return portfolios of 10, 17, 30, and 38 industries are either statistically insignificant or marginally significant. For 48- and 49-industry portfolios of Kenneth French, "Aero" industry has the highest average monthly return, whereas "Other" industry has the lowest return, yielding an average monthly return difference of 66 basis points with the Newey-West t-statistic of 2.55.[3] More importantly, the static CAPM cannot explain the industry effect; the CAPM alpha (or abnormal return) for the "Aero-Other" arbitrage portfolio is 0.50% per month and statistically significant with the t-statistic of 2.04. Although the average return differences between high-return and low-return portfolios of 30 and 38 industries are marginally significant, the CAPM Alphas are found to be significant. For 30-industry portfolios, the average return difference between "Coal" and "Other" industries is 0.51% per month and marginally significant with the t-statistic of 1.71.[4] However, the CAPM Alpha for the "Coal-Other" arbitrage portfolio is 0.65% per month with the t-statistic of 2.28. For 38-industry portfolios, the average return difference between "Oil" and "Whlsl" industries is 0.42% per month and marginally significant with the t-statistic of 1.85.[5] However, the CAPM Alpha for the "Oil-Whlsl" arbitrage portfolio is 0.49% per month with the t-statistic of 2.06.

[3]According to the 48- and 49-industry definitions and four-digit SIC codes reported at Kenneth French's online data library, "Aero" industry includes Aircraft & parts (3720-3720), Aircraft (3721-3721), Aircraft engines, engine parts (3723-3724), Aircraft parts (3725-3725), and Aircraft parts (3728-3729). "Other" industry includes Sanitary services (4950-4959), Steam, air conditioning supplies (4960-4961), Irrigation systems (4970-4971), and Cogeneration - SM power producer (4990-4991).

[4]According to the 30-industry definitions and four-digit SIC codes reported at Kenneth French's online data library, "Coal" industry includes Bituminous coal (1200-1299). "Other" industry includes Sanitary services (4950-4959), Steam, air conditioning supplies (4960-4961), Irrigation systems (4970-4971), and Cogeneration - SM power producer (4990-4991).

[5]According to the 38-industry definitions and four-digit SIC codes reported at Kenneth French's online data library, "Oil" industry includes Oil and Gas Extraction (1300-1399) and "Whlsl" industry includes Wholesale (5000-5199).

Fama and French (1992) identify economically and statistically significant value premium for the post-1963 period. Moreover, Fama and French (1992) find that the post-1963 value premium is not explained by the CAPM. However, Ang and Chen (2007) provide evidence that the value premium is captured by the CAPM for the sample period of 1926-1963. They also show that the conditional CAPM with stochastic betas can explain the return differences between value and growth portfolios even for the post-1963 period. Fama and French (2006) indicate that the performance of the CAPM with regard to the book-to-market effect varies across subperiods. We investigate the significance of size, book-to-market, and industry effects for the sample that generated heated debate on value premium. We compute the average return differences and Alphas for the subsample period of July 1963 – December 2010.

As presented in Table II, the average return difference between the Small and Big portfolios as well as the CAPM Alpha for "Small-Big" arbitrage portfolio are positive, but they are economically and statistically insignificant, indicating that the size effect disappears for the post-1963 period. Similar to the findings of Ang and Chen (2007) and Fama and French (2006), value premium remains economically and statistically significant for the sample period July 1963 – December 2010; the average raw and risk-adjusted return differences between the Value and Growth portfolios is 0.55% per month and statistically significant, implying that value stocks on average generate higher returns than growth stocks and this value premium cannot be explained by the static CAPM.

The results for the industry portfolios are similar for the post-1963 period. The high-return and low-return portfolios of 30 and 38 industries generate marginally significant, 48 and 49 industries generate significant return differences, whereas the average return differences for the high-return and low-return portfolios of 10 and 17 industries are insignificant. Specifically, for 30-, 48- and 49-industry portfolios of Kenneth French, "Coal" industry has the highest average monthly return, whereas "Other" industry has the lowest return, yielding an average raw and risk-adjusted return differences of 79 to 92 basis points per month and statistically significant. The unconditional CAPM cannot explain these industry effects either. For 38-industry portfolios, the average return and Alpha differences between

"Smoke" and "Govt" industries are about 1.06% and 1.07% per month and significant with the Newey-West t-statistics of 2.61 and 2.69, respectively.[6]

C DCC Model of Engle (2002)

We estimate the conditional covariances of each equity portfolio with the market portfolio and VRP ($\sigma_{im,t+1}$, $\sigma_{i,VRP,t+1}$) based on the mean-reverting DCC model of Engle (2002). Engle defines the conditional correlation between two random variables r_1 and r_2 that each has zero mean as

$$\rho_{12,t} = \frac{E_{t-1}\left(r_{1,t} \cdot r_{2,t}\right)}{\sqrt{E_{t-1}\left(r_{1,t}^2\right) \cdot E_{t-1}\left(r_{1,t}^2\right)}}, \tag{A9}$$

where the returns are defined as the conditional standard deviation times the standardized disturbance:

$$\sigma_{i,t}^2 = E_{t-1}\left(r_{i,t}^2\right), \quad r_{i,t} = \sigma_{i,t} \cdot u_{i,t}, \quad i = 1, 2 \tag{A10}$$

where $u_{i,t}$ is a standardized disturbance that has zero mean and variance one for each series. Equations (A9) and (A10) indicate that the conditional correlation is also the conditional covariance between the standardized disturbances:

$$\rho_{12,t} = \frac{E_{t-1}\left(u_{1,t} \cdot u_{2,t}\right)}{\sqrt{E_{t-1}\left(u_{1,t}^2\right) \cdot E_{t-1}\left(u_{1,t}^2\right)}} = E_{t-1}\left(u_{1,t} \cdot u_{2,t}\right). \tag{A11}$$

The conditional covariance matrix of returns is defined as

$$H_t = D_t \cdot \rho_t \cdot D_t, \text{ where } D_t = \text{diag}\left\{\sqrt{\sigma_{i,t}^2}\right\}, \tag{A12}$$

where ρ_t is the time-varying conditional correlation matrix

$$E_{t-1}\left(u_t \cdot u_t'\right) = D_t^{-1} \cdot H_t \cdot D_t^{-1} = \rho_t, \quad \text{where } u_t = D_t^{-1} \cdot r_t. \tag{A13}$$

Engle (2002) introduces a mean-reverting DCC model:

$$\rho_{ij,t} = \frac{q_{ij,t}}{\sqrt{q_{ii,t} \cdot q_{jj,t}}}, \tag{A14}$$

[6]According to the 38-industry definitions and four-digit SIC codes reported at Kenneth French's online data library, "Smoke" industry includes Tobacco Products (2100-2199) and "Govt" industry includes Public Administration (9000-9999).

$$q_{ij,t} = \bar{\rho}_{ij} + a_1 \cdot (u_{i,t-1} \cdot u_{j,t-1} - \bar{\rho}_{ij}) + a_2 \cdot (q_{ij,t-1} - \bar{\rho}_{ij}) \tag{A15}$$

where $\bar{\rho}_{ij}$ is the unconditional correlation between $u_{i,t}$ and $u_{j,t}$. Equation (A15) indicates that the conditional correlation is mean reverting towards $\bar{\rho}_{ij}$ as long as $a_1 + a_2 < 1$.

Engle (2002) assumes that each asset follows a univariate GARCH process and writes the log likelihood function as:

$$\begin{aligned} L &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log|H_t| + r_t' H_t^{-1} r_t\right) \\ &= -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + 2\log|D_t| + r_t' D_t^{-1} D_t^{-1} r_t - u_t' u_t + \log|\rho_t| + u_t' \rho_t^{-1} u_t\right). \end{aligned} \tag{A16}$$

As shown in Engle (2002), letting the parameters in D_t be denoted by θ and the additional parameters in ρ_t be denoted by φ, equation (A16) can be written as the sum of a volatility part and a correlation part:

$$L(\theta, \varphi) = L_V(\theta) + L_C(\theta, \varphi). \tag{A17}$$

The volatility term is

$$L_V(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\left(n\log(2\pi) + \log|D_t|^2 + r_t' D_t^{-2} r_t\right), \tag{A18}$$

and the correlation component is

$$L_C(\theta, \varphi) = -\frac{1}{2}\sum_{t=1}^{T}\left(\log|\rho_t| + u_t' \rho_t^{-1} u_t - u_t' u_t\right). \tag{A19}$$

The volatility part of the likelihood is the sum of individual GARCH likelihoods:

$$L_V(\theta) = -\frac{1}{2}\sum_{t=1}^{T}\sum_{i=1}^{n}\left(\log(2\pi) + \log\left(\sigma_{i,t}^2\right) + \frac{r_{i,t}^2}{\sigma_{i,t}^2}\right), \tag{A20}$$

which is jointly maximized by separately maximizing each term. The second part of the likelihood is used to estimate the correlation parameters. The two-step approach to maximizing the likelihood is to find

$$\hat{\theta} = \arg\max\{L_V(\theta)\}, \tag{A21}$$

and then take this value as given in the second stage:

$$\hat{\varphi} = \arg\max\{L_C(\hat{\theta}, \varphi)\}. \tag{A22}$$

D System of Regression Equations

Consider a system of n equations, of which the typical ith equation is

$$y_i = X_i \beta_i + u_i, \tag{A23}$$

where y_i is a $N \times 1$ vector of time-series observations on the ith dependent variable, X_i is a $N \times k_i$ matrix of observations of k_i independent variables, β_i is a $k_i \times 1$ vector of unknown coefficients to be estimated, and u_i is a $N \times 1$ vector of random disturbance terms with mean zero. Parks (1967) proposes an estimation procedure that allows the error term to be both serially and cross-sectionally correlated. In particular, he assumes that the elements of the disturbance vector u follow an AR(1) process:

$$u_{it} = \rho u_{it-1} + \varepsilon_{it};\ \ \rho_i < 1, \tag{A24}$$

where ε_{it} is serially independently but contemporaneously correlated:

$$Cov\left(\varepsilon_{it}\varepsilon_{jt}\right) = \sigma_{ij},\ \text{for any } i,j,\ \text{and } Cov\left(\varepsilon_{it}\varepsilon_{js}\right) = 0,\ \text{for } s \neq t \tag{A25}$$

Equation (A23) can then be written as

$$y_i = X_i \beta_i + P_i u_i, \tag{A26}$$

with

$$P_i = \begin{bmatrix} \left(1-\rho_i^2\right)^{-1/2} & 0 & 0 & ... & 0 \\ \rho_i \left(1-\rho_i^2\right)^{-1/2} & 1 & 0 & ... & 0 \\ \rho_i^2 \left(1-\rho_i^2\right)^{-1/2} & \rho & 1 & ... & 0 \\ . & & & & \\ . & & & & \\ . & & & & \\ \rho_i^{N-1} \left(1-\rho_i^2\right)^{-1/2} & \rho^{N-2} & \rho^{N-3} & ... & 1 \end{bmatrix}. \tag{A27}$$

Under this setup, Parks (1967) presents a consistent and asymptotically efficient three-step estimation technique for the regression coefficients. The first step uses single equation regressions to estimate the parameters of autoregressive model. The second step uses single equation regressions on transformed equations to estimate the contemporaneous covariances. Finally, the Aitken estimator is formed using the estimated covariance,

$$\hat{\beta} = \left(X^T \Omega^{-1} X\right)^{-1} X^T \Omega^{-1} y, \tag{A28}$$

where $\Omega \equiv E[uu^T]$ denotes the general covariance matrix of the innovation. In our application, we use the aforementioned methodology with the slope coefficients restricted to be the same for all equity portfolios and individual stocks. In particular, we use the same three-step procedure and the same covariance assumptions as in equations (A24) to (A27) to estimate the covariances and to generate the t-statistics for the parameter estimates.

E DCC with Asymmetric GARCH

Because the conditional variance and covariance of stock market returns are not observable, different approaches and specifications used in estimating the conditional variance and covariance could lead to different conclusions. We have so far used the bivariate GARCH(1,1) model of Bollerslev (1986) in equations (18)-(19) and (24)-(25) to obtain conditional variance and covariance estimates. In this section, we investigate whether changing these specifications influences our main findings.

The current volatility in the GARCH(1,1) model is defined as a symmetric, linear function of the last period's unexpected news and the last period's volatility. Since, in a symmetric GARCH process, positive and negative information shocks of the same magnitude produce the same amount of volatility, the symmetric GARCH model cannot cope with the skewness of stock return distribution. If a negative return shock causes more volatility than a positive return shock of the same size, the symmetric GARCH model underpredicts the amount of volatility following negative shocks and overpredicts the amount of volatility following positive shocks. Furthermore, if large return shocks cause more volatility than a quadratic function allows, then the symmetric GARCH model underpredicts volatility after a large return shock and overpredicts volatility after a small return shock.

In this section we use an asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993) that explicitly takes account of skewed distributions and allows good news and bad news to have different impacts on the conditional volatility forecasts. To test whether such variations in the variance forecasting specification alter our conclusion, we re-estimate the DCC-based conditional covariances using the following alternative specification:

$$\begin{aligned}
&R_{i,t+1} = \alpha_0^i + \alpha_1^i R_{i,t} + \varepsilon_{i,t+1}\\
&R_{m,t+1} = \alpha_0^m + \alpha_1^m R_{m,t} + \varepsilon_{m,t+1}\\
&VRP_{t+1} = \alpha_0^{VRP} + \alpha_1^{VRP} VRP_t + \varepsilon_{VRP,t+1}\\
&E_t\left[\varepsilon_{i,t+1}^2\right] \equiv \sigma_{i,t+1}^2 = \beta_0^i + \beta_1^i \varepsilon_{i,t}^2 + \beta_2^i \sigma_{i,t}^2 + \beta_3^i \varepsilon_{i,t}^2 D_{i,t}^-\\
&E_t\left[\varepsilon_{m,t+1}^2\right] \equiv \sigma_{m,t+1}^2 = \beta_0^m + \beta_1^m \varepsilon_{m,t}^2 + \beta_2^m \sigma_{m,t}^2 + \beta_3^m \varepsilon_{m,t}^2 D_{m,t}^-\\
&E_t\left[\varepsilon_{VRP,t+1}^2\right] \equiv \sigma_{VRP,t+1}^2 = \beta_0^{VRP} + \beta_1^{VRP} \varepsilon_{VRP,t}^2 + \beta_2^{VRP} \sigma_{VRP,t}^2 + \beta_3^{VRP} \varepsilon_{VRP,t}^2 D_{VRP,t}^-\\
&E_t\left[\varepsilon_{i,t+1}\varepsilon_{m,t+1}\right] \equiv \sigma_{im,t+1} = \rho_{im,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{m,t+1}\\
&E_t\left[\varepsilon_{i,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{i,VRP,t+1} = \rho_{i,VRP,t+1} \cdot \sigma_{i,t+1} \cdot \sigma_{VRP,t+1}\\
&E_t\left[\varepsilon_{m,t+1}\varepsilon_{VRP,t+1}\right] \equiv \sigma_{m,VRP,t+1} = \rho_{m,VRP,t+1} \cdot \sigma_{m,t+1} \cdot \sigma_{VRP,t+1}
\end{aligned} \tag{A29}$$

where $D_{i,t}^-$, $D_{m,t}^-$, and $D_{VRP,t}^-$ are indicator functions that equals one when $\varepsilon_{i,t+1}$, $\varepsilon_{m,t+1}$, and $\varepsilon_{VRP,t+1}$ are negative and zero otherwise. The indicator function generates an asymmetric GARCH effect between positive and negative shocks. $\rho_{im,t+1}$, $\rho_{i,VRP,t+1}$, and $\rho_{m,VRP,t+1}$ are the time-t expected conditional correlations estimated using the mean-reverting DCC model of Engle (2002).

A notable point in Table III is that the main findings from an asymmetric GARCH specification of the conditional covariances are very similar to those reported in Table 2. Specifically, the risk aversion coefficients are estimated to be positive and highly significant for all equity portfolios; A is in the range of 2.53 to 3.54 with the t-statistics ranging from 2.58 to 3.11, implying a significantly positive link between expected return and risk. Similar to our results from GARCH(1,1) specification, asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993) yields positive and significant coefficient estimates on the covariance between equity portfolios and the variance risk premia. Specifically, the uncertainty aversion coefficients (B) are in the range of 0.0054 to 0.0075 with the t-statistics between 2.68 and 3.30. These results show that equity portfolios that are highly correlated with uncertainty (proxied by VRP) carry a significant premium relative to portfolios that are uncorrelated or lowly correlated with VRP.

With this alternative covariance specification, we also examine the empirical validity of the conditional asset pricing model by testing the joint hypothesis. As shown in Table III, the Wald_1 statistics for the size, book-to-market, and industry portfolios are, respectively, 16.91, 7.89, and 14.41 with the corresponding p-values of 0.11, 0.72, and 0.21. The significantly positive risk and uncertainty aversion coefficients and the insignificant Wald_1 statistics indicate that the two-factor model explains the time-series and cross-sectional variation in equity portfolios. Finally, we investigate whether the model with asymmetric GARCH specification explains the return spreads between Small and Big; Value and Growth; and HiTec and Telcm portfolios. The last row in Table III reports Wald_2 statistics from testing the equality of conditional alphas for high-return and low-return portfolios ($H_0 : \alpha_1 = \alpha_{10}$). For the size, book-to-market, and industry portfolios, the Wald_2 statistics provide no evidence for a significant conditional alpha for "Small-Big", "Value-Growth", and "HiTec-Telcm" arbitrage portfolios. Overall, the DCC-based conditional covariances from the asymmetric GARCH model captures the time-series and cross-sectional variation in returns on size, book-to-market, and industry portfolios and generates significantly positive risk-return and uncertainty-return tradeoffs.

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Table I Model Calibration Parameter Setting

This table reports the calibration parameter values for the stochastic volatility-of-volatility model used in this paper. BTZ2009 refers to the calibration setting of Bollerslev, Tauchen, and Zhou (2009), with an emphasis on equity risk premium and its short-run predictability, while the setting of Zhou (2010) also considers bond risk premium and credit spread and their forecastability from variance risk premium. The Campbell-Shiller linearization constants are $\kappa_1 = 0.9$ and $\kappa_0 = 0.3251$.

Calibration Parameters	BTZ2009	Zhou (2010)
Preference Parameters:	$\delta = 0.997$	$\delta = 0.997$
	$\gamma = 10$	$\gamma = 2$
	$\psi = 1.5$	$\psi = 1.5$
Endowment Parameters:	$\mu_g = 0.0015$	$\mu_g = 0.0015$
	$a_\sigma = 1.34 \times 10^{-6}$	$a_\sigma = 0.002$
	$\rho_\sigma = 0.978$	$\rho_\sigma = 0.1$
	$a_q = 2 \times 10^{-7}$	$a_q = 1.4 \times 10^{-5}$
	$\rho_q = 0.8$	$\rho_q = 0.98$
	$\varphi_q = 0.001$	$\varphi_q = 0.008$
Risk-Return Trade-off (A)	10	2
Uncertainty-Return Trade-off (B)	10.24	0.08

Table II Monthly Raw Returns and CAPM Alphas for the Long-Short Equity Portfolios

This table presents the monthly raw return and CAPM Alpha differences between high-return (long) and low-return (short) equity portfolios. The results are reported for the size, book-to-market (BM), and industry portfolios for the sample periods July 1926 – December 2010, July 1963 – December 2010, and January 1990 – December 2010. The OLS t-statistics are reported in parentheses. The Newey-West t-statistics are given in square brackets.

	July 1926 – December 2010				July 1963 – December 2010				January 1990 – December 2010		
Portfolio	Long-Short	Return Diff.	Alpha	Portfolio	Long-Short	Return Diff.	Alpha	Portfolio	Long-Short	Return Diff.	Alpha
10 Size	Small-Big	0.60%	0.27%	10 Size	Small-Big	0.38%	0.29%	10 Size	Small-Big	0.40%	0.35%
		(2.49)	(1.22)			(1.58)	(1.41)			(1.22)	(1.06)
		[2.36]	[1.38]			[1.81]	[1.26]			[1.13]	[0.98]
10 BM	Value-Growth	0.53%	0.24%	10 BM	Value-Growth	0.55%	0.55%	10 BM	Value-Growth	0.29%	0.28%
		(2.52)	(1.25)			(2.77)	(2.76)			(0.92)	(0.86)
		[2.46]	[1.26]			[2.48]	[2.40]			[0.79]	[0.71]
10 Industry	Durbl-Telcm	0.27%	-0.09%	10 Industry	Utils-Enrgy	0.28%	0.17%	10 Industry	HiTech-Telcm	0.56%	0.31%
		(1.44)	(-0.56)			(1.50)	(0.93)			(1.55)	(0.92)
		[1.31]	[-0.54]			[1.50]	[0.91]			[1.64]	[0.95]
17 Industry	Cars-Other	0.28%	0.07%	17 Industry	Mines-Durbl	0.43%	0.45%	17 Industry	Mines-Durbl	0.56%	0.61%
		(1.65)	(0.44)			(1.54)	(1.58)			(1.15)	(1.23)
		[1.55]	[0.44]			[1.48]	[1.51]			[1.16]	[1.22]
30 Industry	Coal-Other	0.51%	0.65%	30 Industry	Coal-Other	0.82%	0.79%	30 Industry	Coal-Other	1.66%	1.54%
		(1.79)	(2.26)			(2.14)	(2.05)			(2.25)	(2.08)
		[1.71]	[2.28]			[1.95]	[1.84]			[2.12]	[1.90]
38 Industry	Oil-Whlsl	0.42%	0.49%	38 Industry	Smoke-Govt	1.06%	1.07%	38 Industry	Mines-Govt	1.34%	1.45%
		(1.90)	(2.18)			(2.38)	(2.40)			(1.48)	(1.59)
		[1.85]	[2.06]			[2.61]	[2.69]			[1.75]	[1.90]
48 Industry	Aero-Other	0.66%	0.50%	48 Industry	Coal-Other	0.92%	0.92%	48 Industry	Coal-Other	1.79%	1.71%
		(2.66)	(2.02)			(2.24)	(2.25)			(2.38)	(2.26)
		[2.55]	[2.04]			[2.06]	[2.04]			[2.33]	[2.13]
49 Industry	Aero-Other	0.66%	0.50%	49 Industry	Coal-Other	0.92%	0.92%	49 Industry	Coal-Other	1.79%	1.71%
		(2.66)	(2.02)			(2.24)	(2.25)			(2.38)	(2.26)
		[2.55]	[2.04]			[2.06]	[2.04]			[2.33]	[2.13]

Table III Results from Asymmetric GARCH Model

This table reports the portfolio-specific intercepts and the common slope estimates from the following panel regression:

$$
\begin{aligned}
R_{i,t+1} &= \alpha_i + A \cdot Cov_t\left(R_{i,t+1}, R_{m,t+1}\right) + B \cdot Cov_t\left(R_{i,t+1}, VRP_{t+1}\right) + \varepsilon_{i,t+1} \\
R_{m,t+1} &= \alpha_m + A \cdot Var_t\left(R_{m,t+1}\right) + B \cdot Cov_t\left(R_{m,t+1}, VRP_{t+1}\right) + \varepsilon_{m,t+1}
\end{aligned}
$$

where the conditional variance and covariances are estimated using the asymmetric GARCH model of Glosten, Jagannathan, and Runkle (1993). The parameters and their t-statistics are estimated using the monthly excess returns on the market portfolio and the ten decile size, book-to-market, and industry portfolios for the sample period from January 1990 to December 2010. The alphas (α_i) are reported for each equity portfolio and the t-statistics are presented in parentheses. The t-statistics are adjusted for heteroskedasticity and autocorrelation for each series and cross-correlations among the portfolios. The last four rows, respectively, show the common slope coefficients (A and B), the Wald$_1$ statistics from testing the joint hypothesis $H_0 : \alpha_1 = \alpha_2 = ...\alpha_m = 0$, and the Wald$_2$ statistics from testing the equality of Alphas for high-return and low-return portfolios (Small vs. Big; Value vs. Growth; and HiTec vs. Telcm). The p-values of Wald$_1$ and Wald$_2$ statistics are given in square brackets.

Size	α_i, α_m	*BM*	α_i, α_m	*Industry*	α_i, α_m
Small	0.0052	Growth	0.0035	NoDur	0.0051
	(1.23)		(0.87)		(1.94)
2	0.0037	2	0.0047	Durbl	0.0028
	(0.85)		(1.35)		(0.57)
3	0.0040	3	0.0052	Manuf	0.0055
	(0.99)		(1.55)		(1.61)
4	0.0030	4	0.0064	Enrgy	0.0064
	(0.75)		(1.85)		(1.85)
5	0.0038	5	0.0056	HiTec	0.0029
	(0.97)		(1.71)		(0.52)
6	0.0037	6	0.0050	Telcm	0.0004
	(1.05)		(1.48)		(0.11)
7	0.0041	7	0.0057	Shops	0.0036
	(1.19)		(1.76)		(1.04)
8	0.0034	8	0.0058	Hlth	0.0043
	(0.97)		(1.74)		(1.37)
9	0.0036	9	0.0066	Utils	0.0042
	(1.11)		(1.92)		(1.58)
Big	0.0012	Value	0.0081	Other	0.0030
	(0.38)		(1.88)		(0.81)
Market	0.0018	Market	0.0033	Market	0.0028
	(0.57)		(1.20)		(0.82)
A	3.2927	A	2.5303	A	3.5369
	(3.11)		(2.62)		(2.58)
B	0.0054	B	0.0060	B	0.0075
	(3.12)		(2.68)		(3.30)
Wald$_1$	16.91	Wald$_1$	7.89	Wald$_1$	14.41
	[0.11]		[0.72]		[0.21]
Wald$_2$	1.48	Wald$_2$	1.99	Wald$_2$	0.46
	[0.22]		[0.16]		[0.50]

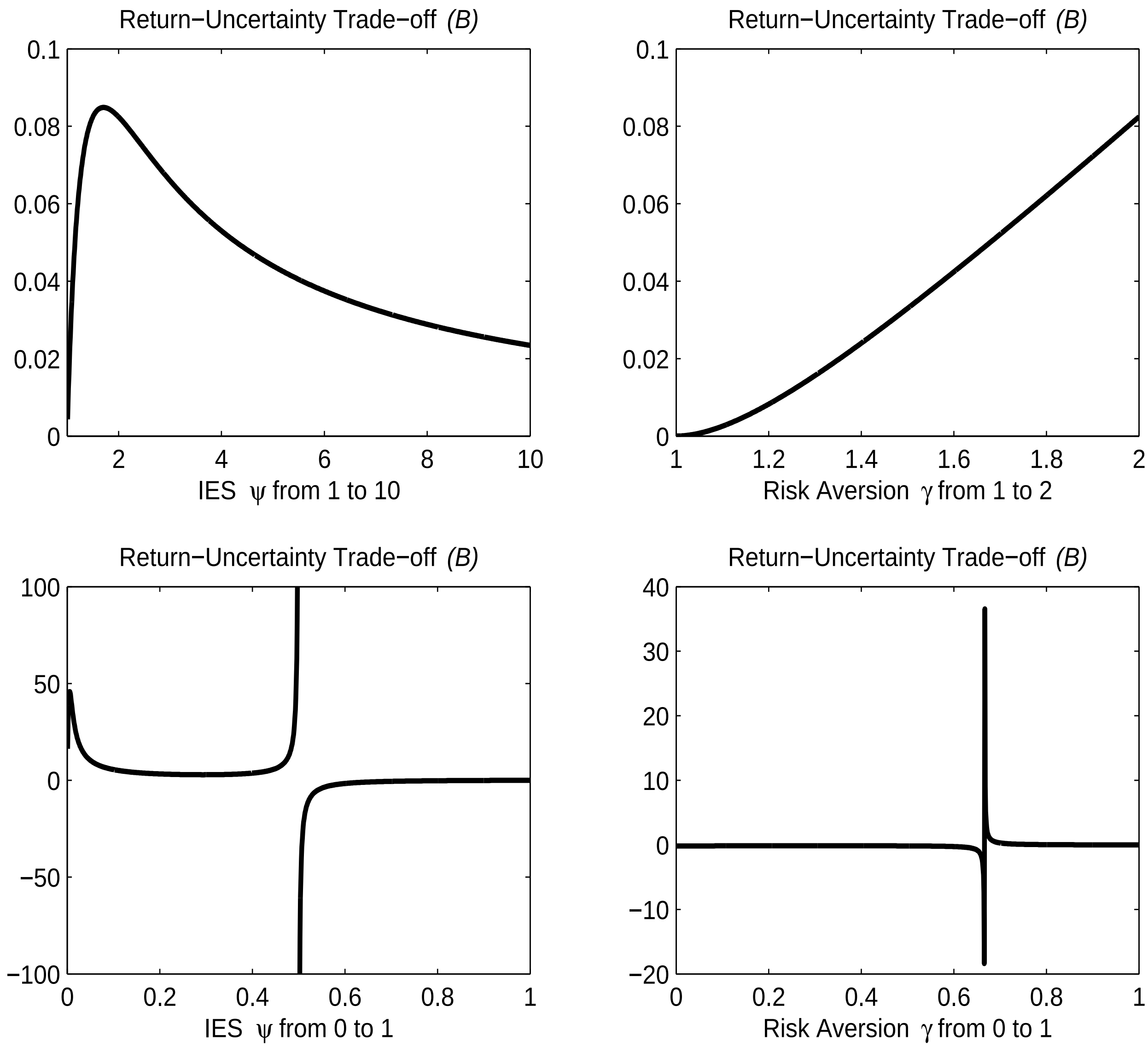

Figure 1 Return-Uncertainty Trade-off Coefficient

The figure shows the model-implied relationship between market excess return and variance risk premium (VRP), or the return-uncertainty trade-off coefficient (B) as implied by the model. The top panels show how the value of B changes with respect to the intertemporal elasticity of substitution (IES) $\psi = [1, 10]$ (left) and the risk aversion coefficient $\gamma = [1, 2]$ (right), and the lower two panels with respect to $\psi = [0, 1]$ (left) and $\gamma = [0, 1]$ (right). The benchmark calibration setting is based on Zhou (2010) and specified in Table I.