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Stock Market Tournaments

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Abstract

We propose a new theory of suboptimal risk-taking based on contractual externalities. We examine an industry with a continuum of firms. Each firm's manager exerts costly hidden effort. The productivity of effort is subject to systematic shocks. Firms' stock prices reflect their performance relative to the industry average. In this setting, stock-based incentives cause complementarities in managerial effort choices. Externalities arise because shareholders do not internalize the impact of their incentive provision on the average effort. During booms, they over-incentivise managers, triggering a rat-race in effort exertion, resulting in excessive risk relative to the second-best. The opposite occurs during busts.

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1 Introduction

The financial crisis of 2008 has led to renewed attention to the excess risk-taking by managers. Studies have shown that during this period managers (especially of financial firms) chose to expose their firms to large systematic risks, e.g., by having investments with large market betas (Cheng et al. (2011)). Many pundits and scholars have suggested that the design of corporate managers' compensation schemes is a major factor underlying such behavior.[1] It remains unclear, however, why compensation contracts would encourage managers to take on excessive risks, and more importantly systematic risks as opposed to idiosyncratic risks.[2] In this paper, we propose an explanation based on contractual externalities.

Specifically, we examine an industry with a continuum of firms. Each firm has a manager who exerts costly and hidden effort. The productivity of the effort is random and correlated among all firms in the industry. The firm's share is traded in a stock market that is informationally efficient. Speculators who trade in this market have access to information about each firm's fundamental value *relative to the industry average.* As a result, a firm's stock price reflects its performance relative to the industry average and the stock market becomes a tournament punishing losers and rewarding winners. In this setup, we analyze the impact of stock price based optimal linear contracts on the strategic interaction among managers' effort and risk choices.[3]

In this economy, risk-averse shareholders of each firm in the industry contract with the firm's risk-averse manager to elicit effort and share risk. We show that contracts based on stock prices generate complementarities in effort-taking among managers of all firms in the industry. This is because when the average effort level in the industry is high, a manager has to increase her effort in order to avoid a drop in her firm's stock price, which in turn influences her compensation. Complementarities in managerial efforts create an externality in stock-based incentive provision among shareholders of all firms in the industry. This is because shareholders of a given firm do not take into consideration that their stock-based

[1] Rajan (2005) is among the first to warn that compensation package could cause excessive risk taking in the financial institutions. The Citigroup CEO Chuck Prince proposed a now infamous musical chair theory of corporate investment/risk-taking, implying that the stock market exerts pressure for the Citi to over-invest in the subprime: "dance while the music is playing."

[2] There are several features of compensation contracts that might encourage excessive risk taking by managers, such as limited liability or convex payoff structure. However, these features alone do not distinguish between systematic versus idiosyncratic risk taking.

[3] Because the productivity of effort is random, increasing the effort exposes the firm to more systematic risk. Hence, a manager's effort choice and the firm's systematic risk exposure are tightly linked.

incentive provision affects the effort choices by managers of other firms, hence the industry average effort level. This externality in stock-based incentive provision among shareholders causes either excessive or insufficient effort provision and risk-taking in equilibrium relative to the second-best where a planner sets contracts internalizing the externality.

For example, when the expected industry productivity is high, e.g., during a boom, a firm's shareholders would like to elicit high effort from their manager by increasing stock-based incentives. Since these shareholders do not internalize the impact of their incentive provision on the effort choice by the managers of other firms, their increased incentives trigger a rat race in effort- and risk-taking in the whole industry. The equilibrium managerial effort and risk exposure is excessive relative to the second best. By contrast, when the expected industry productivity is low, e.g., during a recession, the logic is reversed and the race is to the bottom: Relative to the second-best, stock-based incentive provision in equilibrium is too low, leading to under-provision of effort and insufficient risk exposure. The planner can improve the total welfare by making stock-based incentives counter-cyclical: enforcing lower (higher) stock-based incentives among managers during booms (busts).

An important point is that the excessive risk-taking in our model is systematic rather than idiosyncratic in nature.[4] As mentioned above in a stock market tournament, a firm's stock price reflects its performance relative to the industry average. Consider the case when the risk in the productivity shock is purely systematic and is common across firms. In this case, if a firm's manager matches the industry average effort level, the firm's performance relative to the industry average, and consequently its stock price, do not contain any exposure to the common productivity shock. Therefore, by matching their peer's effort choice level, managers can remove the systematic risk embedded in their stock-based compensations.[5] This incentive to hedge the systematic risk from their compensations by managers results in complementarities in their effort choices and is the underlying cause for the risk-taking rat race during booms (and race to the bottom during busts.) By contrast, as we show in an extension of the model, when productivity shocks are idiosyncratic across firms within the industry, managerial effort choices are not complements. In this case, since productivity shocks are uncorrelated across firms, if a firm's manager matches the industry average effort

[4] In this paper we treat the industry shock and the systematic shock as the same and use the terms interchangeably.

[5] In the less extreme case, when the risk in the productivity shock is correlated across firms a similar result holds. In this case, managers can reduce the systematic risk embedded in the stock-based compensation by matching the average effort level in the industry. We focus on the case where the productivity shock is common for ease of exposition.

level, the firm's performance relative to the industry average still contains the exposure to its own productivity shock. Consequently, managers cannot remove the exposure to the productivity shock in their own compensations by doing what their peers are doing and hence have no reason to be influenced by their peers' effort choices. This result on excessive systematic risk-taking is consistent with the recent empirical findings on bank CEO incentives and risk-taking behavior (Cheng et al. (2011)).

Comparative-static analysis on the equilibrium properties provides a rich set of new testable empirical hypotheses. In conducting this analysis, we first delineate how the two purposes of contracting – risk-sharing and incentive provision – shape the equilibrium and the second-best contracts. We define the risk sharing effect as the level of stock-based compensation when the sole purpose of contracting is risk sharing (e.g., when idiosyncratic risks are very high) and the incentive effect as the level of stock based compensation when the sole purpose of contracting is incentive provision (e.g., when the expected industry productivity is very high). Our comparative static results are based on how variations in model parameters affect the power of stock-based compensations through these two effects. For example, suppose that the incentive effect requires a more powerful stock price based contract than the risk-sharing effect. In this case, if the expected productivity increases, then shareholders care more about incentivising than risk-sharing in setting up contracts, and the relative importance of the incentive effect becomes larger as the expected productivity gets higher. As a result, stock-based compensation and risk-taking behavior are pro-cyclical. This indicates that the empirical findings in Cheng et al. (2011) could be reversed during market downturns: Financial firms with large stock-based compensation may choose investment with lower market-betas during market downturns. Empirical implications can also be generated based on the industry-specific information environment. In Appendix B, we show that when stock prices reflect both the relative performance signal and the absolute performance signal, the complementarities in effort-taking among managers are more strengthened when the absolute performance signal is relatively noisy. Hence a testable hypothesis is that the boom-bust cycle of risk-taking is more pronounced in industries with new innovations. This is because information about the productivity of a new technology is likely to be noisy and investors might have to rely more on the relative performance information in pricing stocks and setting up contracts.

Besides empirical implications, the theoretical findings in the paper also have policy relevance. For example, we find that to control the socially suboptimal risk-taking behavior of the firms, the government should impose restrictions on stock-based pay during booms and do the opposite in recessions. In an extension of the main model, we study the case when shareholders can use a noisy version of the final firm value as an additional contractual instrument. Relative to stock-based compensations, instruments based on the (noisy) final value are more costly for shareholders since they force managers to bear systematic risks. However, by using both stock- and (noisy) final value-based instruments, the impact of the externality of the stock market tournament on risk-taking is reduced. In fact, when the noise in this additional instrument approaches zero, the second best is achieved. The contractual instrument based on the noisy final value of the firm bear some resemblance to clawbacks in compensation contracts. Clawback arrangements typically refer to deferred bonuses (punishment), which are held by firms away from the employees for years and are tied specifically to the future (as compared to short-term) performance of the firm. Our finding on the noisy final value instrument lends support to using claw-back types of instruments in compensation contracts to control managerial incentives to undertake excessive systematic risks.

It has been argued theoretically that there is no reason to tie managers' compensation to variables that are independent of their efforts but may affect profits such as general industry conditions (Holmström 1979; 1982). Still, most compensation contracts observed in reality are tied to firms' profits or stock prices. These are traditionally viewed as absolute performance measures and they depend to a large extent on factors outside the control of managers. This has puzzled economists since it is usually rather easy to remove some of these variables from compensation contracts by, for example, rewarding managers only to the extent that they outperform industry benchmarks. A corollary of our finding is that the scope for explicit relative performance in optimal contracts is limited when the stock market generates a tournament providing an implicit relative performance measure. In fact, the results in this paper are not restricted to stock market tournaments. A broader interpretation of our results is that using relative performance instruments in managerial contracts may induce tournaments and cause excessive or insufficient levels of effort and risk-taking.

Relation to the literature. Our paper is built heavily on the contract theory literature that studies the stock price as an incentive instrument such as Diamond and Verrecchia (1982); Holmström and Tirole (1993); and Bolton et al. (2006). Our paper is also related to papers that study the tradeoffs between short- and long-term incentives such as Axelson and Baliga (2009) and Peng and Röell (2011). We differ from these papers by studying the tournament aspect of stock prices and the associated consequences.

By studying stock market tournaments, our paper is related to the contract theory literature on (rank order) tournaments (Akerlof (1976); Lazear and Rosen (1981); Green and

Stokey (1983); Nalebuff and Stiglitz (1983); and Bhattacharya and Mookherjee (1986)). The most closely related paper is Nalebuff and Stiglitz (1983). They examine a setting similar to ours where the agents can affect the exposure to the common risk through their effort choice. They argue that a rank order tournament offers flexibility by adjusting to the common risk and study the optimal size of prize, gap of winning, and penalty. In their setup the common risk is observable to the agents and the second best effort does not exceed the first best optimum. We differ from this literature by studying the externality aspect of the tournament.

Through the results on excessive systematic risk-taking, our paper bears resemblance to the literature that studies the incentive for banks or bank managers to take on excessive risk collectively that causes the financial crisis due to bailouts (Acharya and Yorulmazer (2007); Acharya and Yorulmazer (2008); Farhi and Tirole (2011); and Acharya et al. (2011)).[6] Our focus on the tournament aspect of the stock market in generating these incentives is unique. Our paper is also related to the contracting literature where agents can choose both effort and risk level (Diamond (1998); Biais and Casamatta (1999); Palomino and Prat (2003); and Makarov and Plantin (2010)) when the contract yields convex payoff for the agent, although the risk in question is not systematic risk per se.

The structure of the paper is as follows. In Section 2, we lay out the model. In Section 3, we solve for the optimal linear contract under the equilibrium and the second best, compare the two and present some comparative static results. In Section 4, we discuss some extensions of the main model. Section 5 concludes.

2 Model

Our model is based on Holmström and Tirole (1993) and Bolton et al. (2006). In this section, we describe our setup, information environment, and equilibrium definition.

2.1 The Set-up

There is a continuum of firms. One manager is assigned to each firm and both are indexed by $[0, 1]$. For each firm, there are two types of shareholders, inside shareholders and

[6] Acharya et al. (2011) is closely related to the career concern literature (Holmström (1999) and Holmström and Ricart I Costa (1986)). Holmström (1999) and Holmström and Ricart I Costa (1986) have shown that when they are drawn from different types of abilities, risk-averse managers exert less effort than the first best.

outside shareholders. Inside shareholders of a firm (henceforth shareholders) set the firm's manager's contract and maximize their expected utility based on the expected final value of their holding of the firm. All (inside) shareholders are potentially risk averse, and their utility is given by $u_s(w) = -\exp(-r_s w)$. We view these shareholders as the major stakeholders of the firm who derive certain private benefits and cannot easily (or will not) diversify their risk exposure in the firm away. Hence we assume that they are potentially averse to the firm risk.[7] Outside shareholders (henceforth speculators) speculate in the stock market. We view speculators as investors who hold diversified portfolios and hence we assume that they are risk-neutral.[8]

There are three dates $t = 0, 1, 2$. At $t = 0$, shareholders of firm i offer manager $i \in [0, 1]$ a contract. We assume that contracts are offered simultaneously. Manager i observes her contract and decides whether to accept or reject it, and if she accepts the contract, she chooses hidden effort denoted by e_i on a project of the firm. We assume that each firm owns one project. Manager i's effort is costly and the cost is specified as $C(e_i) = e_i^2/2$. We assume that all managers have identical CARA preferences so that $u(w) = -\exp(-rw)$. Shareholders of firm i hold fraction δ of the firm's shares until liquidation at time $t = 2$. The rest of the shares are publicly traded by speculators in a competitive market. At $t = 1$, speculators receive a payoff-relevant public signal about firm i and set price P_i. At $t = 2$, the final values of all firms' projects are realized and all agents receive their payoffs.

2.2 Production Technology

We assume that manager i generates output V_i, which is a random function of her unobservable effort,

$$V_i = V(e_i, \tilde{h}, \tilde{\epsilon}_i). \tag{1}$$

The randomness arises from a common random variable $\tilde{h}$, and a firm-specific random variable $\tilde{\epsilon}_i$. We interpret $\tilde{h}$ as the industry-wide productivity shock and $\tilde{\epsilon}_i$ as an output shock specific to the individual firm. The important assumption is that $\partial^2 V_i/(\partial h \partial e_i) \neq 0$, i.e., the state of nature that is common across firms affects the productivity of effort.

[7]The literature typically focuses on the case where shareholders are risk-neutral. In our model risk-neutrality is not an innocuous assumption. Later in the paper we show that varying risk-aversion of shareholders leads to different implications about the managerial contractual arrangement.

[8]It is not essential that speculators are risk neutral. All results remain the same if speculators have the same utility function as inside shareholders. This assumption is purely for expositional clarity.

Our main results are based on a linear specification where $V_i = \tilde{h}e_i + \tilde{\epsilon}_i$. In our model, the random variable $\tilde{h}$ is normally distributed with mean $\bar{h} > 0$ and variance σ_h^2 (i.e., precision $\tau_h = 1/\sigma_h^2$). The random variable $\tilde{\epsilon}_i$ is normally distributed with mean zero and variance σ_ϵ^2 (i.e., precision $\tau_\epsilon = 1/\sigma_\epsilon^2$). To show that the tournament aspect of the model is crucially dependent on the productivity shock being common across firms, we analyze an alternative specification where the productivity function is $V_i = \tilde{k}_i e_i + \tilde{\epsilon}_i$. Here $\tilde{k}_i$ is a firm-specific random term.

Note that in our specifications, the productivity shock enters multiplicatively with effort. A similar specification appears in Nalebuff and Stiglitz (1983) who use it to analyze tournament design. This type of specification has also been used increasingly in corporate finance models to analyze questions related to the return to managerial talent (See Aghion and Stein (2008) and Edmans et al. (2009)). When $\sigma_h = 0$, the specification for output in our model is essentially same as the one in Holmström and Tirole (1993). In the more general case where $\sigma_h > 0$, higher average effort generates a higher return, but since the productivity of effort is random it also leads to higher volatility. Here, we have in mind a broad interpretation of managerial effort as choosing the scale of the project, e.g. by devoting more resources (time, personnel, etc) to it.[9]

2.3 Information Structure

A key aspect of our model is that stock prices reflect firms' *relative* valuation to some extent. Empirical evidence indeed provides support for the relative nature of payoff-relevant information in the stock market. For example, Cohen et al. (2003) show that roughly eighty percent of return variation in portfolios created by sorting on book-to-market ratios can be attributed to the relative performance of firms to their industry, i.e., intra-industry variations, and only about twenty percent can be explained by inter-industry variations. They also find the dominance of intra-industry information over inter-industry information on the other dimensions such as forecasting one-year to fifteen-year ahead returns and profits.[10]

Further, there is empirical evidence that speculators are better at gathering information about relative valuations than absolute valuations. For example, Da and Schaumburg

[9] In this way, our setup is different from those models where manager can choose effort and level of risk separately as in Diamond (1998); Biais and Casamatta (1999); Palomino and Prat (2003); and Makarov and Plantin (2010).

[10] Specifically, Cohen et al. (2003) show that the intra-industry effect is nine (four) times larger than the inter-industry effect at one-year horizon and 19 (four) times larger at the 15-year horizon in terms of profit (return).

(2011) show that when forming a portfolio using relative valuation information produced by analysts' forecasts, investors can generate abnormal returns in the future, although it is impossible to do so using absolute valuation information produced by analysts' forecasts. This suggests that stock analysts are better at ranking stocks than setting the price level for them. Anecdotally, the popularity of relative-value/pairwise trading where investors buy one and short another stock also lends support to this relative information structure.

Motivated by empirical evidence, we endow speculators in the model with information about a firm's fundamental value relative to its industry peers. Specifically, we specify the public signal received by speculators about firm i at $t = 1$ as: $s_i = V_i - \bar{V}$, where $\bar{V} = \tilde{h}\bar{e}$ is the average value of a firm, and $\bar{e}$ is the average effort exerted by the managers in the industry. Therefore, this signal can also be rewritten as $s_i = \tilde{h}\left(e_i - \bar{e}\right) + \tilde{\epsilon}_i$. In an extension, we show that the essence of the model does not change if speculators also observe a noisy signal t_i which reflects the absolute valuation of firm i. In the main specification we choose to focus on the relative information signal for expositional clarity. The more general analysis is in Appendix B.

The key element of our model is the existence of relative information in the stock market and our information structure is a parsimonious way, but not the only way, to capture it. Alternatively, the fact that the stock price punishes losers and rewards winners can be captured using a reduced-form model where speculators have a limited amount of risk capital due to regulatory or institutional constraints. In this case, by engaging in a long-short strategy, that is, buying a relatively better-performing stock and shorting a stock with a relatively poor performance in the same industry, speculators can achieve a higher return with limited exposure to systematic risk (which requires more regulatory capital relative to idiosyncratic risk).

2.4 Equilibrium Definition

As is standard in the theoretical literature on executive compensation, we restrict attention to linear compensation contracts. We assume that manager i's compensation contract has two components. The first component is a fixed wage W_i. The second component is stock appreciation right for which the firm pays the manager $a_i P_i$ where P_i is the stock price of firm i at $t = 1$ (more on the determination of this price below).[11] Therefore, manager i's

[11]We assume that the time required to observe the realization of the final value $\tilde{V}$ exceeds the time horizon of a contract that can feasibly be written between shareholders and the manager. Therefore, $\tilde{V}$ is not a viable contractual instrument. In Section 4, we discuss the implications when instruments related to

total compensation I_i is given by

$$I_i(a_i, W_i) = a_i P_i + W_i.$$

Manager i's utility is given by $u(I_i - C(e_i))$ where I_i is her income and $C(e_i)$ is the cost of exerting effort e_i.

Now we are ready to specify manager i's optimization problem. We assume managers' reservation utility is $u(\bar{I})$. Manager i accepts contract (a_i, W_i) if her expected utility from accepting the contract exceeds her reservation utility

$$E[u(I_i(a_i, W_i) - C(e_i(a_i, W_i)))] = E[u(a_i P_i + W_i - C(e_i(a_i, W_i)))] \geq u(\bar{I}) \quad (2)$$

where $e_i(a_i, W_i)$ is the optimal effort choice conditional on accepting the contract. That is,

$$e_i(a_i, W_i) = \arg\max_{e_i \geq 0} E[u(a_i P_i + W_i - C(e_i))]. \quad (3)$$

We look for a rational expectations equilibrium of the model as follows.

Definition 1: A rational expectations equilibrium consists of a stock price P_i, a contract (a_i^*, W_i^*) and effort choice $e_i^* = e_i(a_i^*, W_i^*)$ for each $i \in [0, 1]$ such that:

(i) The stock market is informationally efficient in that sense that

$$P_i(a_i^*, W_i^*, s_i) = E[V_i^* - I_i(a_i^*, W_i^*) | s_i] = E[V_i^* - a_i^* P_i - W_i^* | s_i] \quad (4)$$

where $V_i^* = \tilde{h} e_i^* + \tilde{\epsilon}_i$;

(ii) The contract (a_i^*, W_i^*) solves shareholders' problem. Shareholders choose (a_i, W_i) to maximize $E[u_s(\delta(V_i - I_i))]$ subject to $E[u(a_i P_i + W_i - C(e_i))] \geq u(\bar{I})$, where $e_i = e_i(a_i, W_i)$ (given in (3)) and $V_i = \tilde{h} e_i(a_i, W_i) + \tilde{\epsilon}_i$.

In the above definition of equilibrium we assume implicitly that speculators do not observe the actual contracts or effort levels but they correctly anticipate them in equilibrium. We solve for a symmetric equilibrium where firms' contract choices and managers' equilibrium effort choices are the same. In particular, manager i's equilibrium effort e_i^* is equal to the average effort $\bar{e}$.[12]

We begin our analysis in Section 3 by first solving for the equilibrium in the contractual environment discussed above. In Section 4, we discuss how the equilibrium effort level compares with the first-best optimum and extend the model to a contractual environment where a signal of the final value can be used as a contractual device.

$\tilde{V}$ can be contracted upon.

[12] We will follow the convention that superscript $*$ denotes equilibrium values and upper-bar denotes averages across all firms.

3 Stock Market Tournaments

In this section we first solve for the equilibrium price and illustrate the tournament aspect of the stock market. We then solve for managers' equilibrium effort choices for a given contract. Finally, we solve for the choice of optimal contract, compare it with the second-best contract, and conduct comparative static analysis.

3.1 Prices

In a symmetric equilibrium the effort choices are all identical. Since speculators take equilibrium efforts as given, the public signal from their perspective is $s_i = \tilde{h}\left(e_i^* - \bar{e}\right) + \tilde{\epsilon}_i = \tilde{\epsilon}_i$. That is, in a symmetric equilibrium the public signal is informative only about the idiosyncratic component of the firm value and not about the common uncertainty $\tilde{h}$.[13] The equilibrium value of firm i given public information is

$$E\left[V_i^* \left| s_i\right.\right] = E\left[\tilde{h} e_i^* + \tilde{\epsilon}_i \left| s_i = \tilde{\epsilon}_i\right.\right] = \bar{h}\bar{e} + s_i.$$

Thus, by (4), the stock price of firm i at time 1 is given by:

$$P_i \;=\; E\left[V_i^* - a_i^* P_i - W_i^* \left| s_i\right.\right] = \left(\bar{h}\bar{e} + s_i\right) - a_i^* P_i - W_i^*,$$

or

$$P_i = \frac{1}{1+a_i^*}\left(\bar{h}\bar{e} + s_i - W_i^*\right). \tag{5}$$

Note that when manager i increases her effort, holding the average effort $\bar{e}$ constant, s_i and P_i will be higher on average. More interestingly, P_i depends on manager i's effort relative to the average effort in the industry. It arises because of the relative nature of the information structure. This price function reflects the tournament aspect of the stock market. In a tournament when others increase their efforts, a player on average will have a lower rank and consequently a lower payoff. The same is true in our model of the stock market. Although firms' stocks are on average correctly priced at $\bar{h}\bar{e}$, an (unexpected) increase in the average industry effort lowers both the ranking of firm i within the industry and its stock price.

[13] The empirical finance literature has found that a significant portion of stock returns is due to firm-specific idiosyncratic shocks. This portion has been shown to be about 70 percent average across 40 countries and is significantly higher in countries with more developed markets such as the U.S. (Li and Myers (2006)). To explain relatively low co-movements in stock returns in the developed markets than in the developing markets, Morck et al. (2000) and Li and Myers (2006) point out that there is more information discovery about firm-specific idiosyncratic shocks in the developed markets hence stock returns in these countries are less driven by macro or systematic news. Our results on equilibrium prices point out an alternative explanation for these empirical facts that is based on relative information.

3.2 The Manager's Problem

Given the equilibrium price in (5), we can now write manager i's compensation as:

$$I_i = a_i P_i + W_i = a_i \left(\frac{1}{1+a_i^*} \left(\bar{h}\bar{e} + s_i - W_i^* \right) \right) + W_i = x_i (\underbrace{\bar{h}\bar{e} + s_i}_{E[V_i|s_i]} - W_i^*) + W_i, \tag{6}$$

where $x_i = a_i / (1 + a_i^*)$. Thus, x_i denotes the pay sensitivity to the expected final value conditional on public signal s_i, which can be interpreted as the stock-pay sensitivity or the magnitude of stock-based incentive in the contract. We restrict attention to contracts where $x_i \in [0,1]$, that is, the manager's stock-based compensations do not exceed the entire equity of the firm. For expositional clarity, from now on, we state the contract terms as (x_i, W_i) instead of (a_i, W_i).

Using (6), given a contract (x_i, W_i) and average effort $\bar{e}$, manager i chooses e_i to maximize:

$$E\left[u\left(x_i \left(\bar{h}\bar{e} + s_i - W_i^* \right) + W_i - C\left(e_i\right) \right) \right]. \tag{7}$$

Plugging $s_i = \tilde{h}\left(e_i - \bar{e}\right) + \tilde{\epsilon}_i$, and computing the expectation in the above expression, manager i's problem in (7) can be restated as choosing e_i to maximize:[14]

$$x_i \bar{h} e_i - x_i W_i^* + W_i - C\left(e_i\right) - \frac{1}{2} r \left(\left(x_i \left(e_i - \bar{e} \right) \right)^2 \frac{1}{\tau_h} + x_i^2 \frac{1}{\tau_\epsilon} \right). \tag{8}$$

From (8) we see how a given incentive package shapes manager i's exposure to industry and idiosyncratic risks. The amount of industry risk she faces depends on two things: the power of the stock-based pay x_i and the difference between her effort and the industry average effort $(e_i - \bar{e})$. Her exposure to the idiosyncratic risk, on the other hand, depends solely on the power of stock-based pay x_i. From this we can see that to lower the systematic component of her compensation risk, manager i has an incentive to match the industry average effort $\bar{e}$. Put differently, including the stock price in the compensation contract allows manager i to hedge her exposure to industry risk. At the same time, this hedging possibility creates a complementarity among effort choices of managers. Taking the first-order condition and solving for e_i, we obtain manager i's effort choice as

$$e_i = \frac{x_i \bar{h} + \frac{r}{\tau_h} x_i^2 \bar{e}}{1 + \frac{r}{\tau_h} x_i^2}. \tag{9}$$

[14] Recall that from the speculators' perspective, the signal reflects only idiosyncratic variations in the firm value. However, from the manager's perspective, her effort is a choice variable and the signal reflects the difference between her effort and the average effort.

Note that manager i's effort is increasing in $\bar{e}$, the average effort exerted by all the other managers. Thus, when the average effort increases, manager i's best response is to increase her effort.

Typically, the more risk-averse a manager is (i.e., the higher r is) and/or the more volatile the industry shock becomes (i.e., the lower τ_h is), the lower effort level she will choose. This is because by lowering her effort, a manager reduces her exposure to the industry risk. This effect is captured by the term r/τ_h in the denominator of (9). In the presence of effort complementarities, the term r/τ_h is also in the numerator capturing the fact that when r is higher or τ_h is lower, a manager has a stronger incentive to match the average effort to hedge the industry risk. Through this second effect, for a given contract x_i, the manager's effort may increase when her risk aversion is higher or the industry productivity shock becomes more volatile.

3.3 Shareholders

Now we turn to the shareholders' problem and the characterization of the optimal contract. Shareholders in firm i choose the contract terms $(x_i,\, W_i)$ to maximize their expected utility:

$$E\left[u_s\left(\delta\left(V_i - I_i\right)\right)\right] \tag{10}$$

subject to manager i's participation constraint: $E\left[u\left(a_i P_i + W_i - C\left(e_i\right)\right)\right] \geq u\left(\bar{I}\right)$ where e_i is given by (9).

We proceed to solve for the equilibrium contract terms $(x_i,\, W_i)$. Using (6) we obtain firm i's shareholders' final payoff as

$$\delta(V_i - I_i) = \delta\left(-x_i\left(\bar{h}\bar{e} + s_i - W_i^*\right) + \left(\tilde{h}e_i + \tilde{\epsilon}_i\right) - W_i\right).$$

Plugging $s_i = \tilde{h}\left(e_i - \bar{e}\right) + \tilde{\epsilon}_i$ and computing expectations, the shareholders' problem in (10) can be stated as

$$\max_{x_i, W_i} \delta\left(\left(1 - x_i\right)\bar{h}e_i + x_i W_i^* - W_i\right) - \frac{1}{2} r_s \delta^2 \left(\left(e_i - x_i\left(e_i - \bar{e}\right)\right)^2 \frac{1}{\tau_h} + \left(1 - x_i\right)^2 \frac{1}{\tau_\epsilon}\right) \tag{11}$$

where e_i is given by (9). Using (8) and manager i's individual rationality constraint we obtain

$$-\left(x_i\bar{h}e_i - x_i W_i^* + W_i\right) = -C\left(e_i\right) - \frac{1}{2} r \left(\left(x_i\left(e_i - \bar{e}\right)\right)^2 \frac{1}{\tau_h} + x_i^2 \frac{1}{\tau_\epsilon}\right) - \bar{I}.$$

We substitute the above equation into (11) and simplify it further as

$$\begin{aligned}\max_{x_i} \quad & \delta\left(\bar{h}e_i - C(e_i) - \frac{1}{2}r\left((x_i(e_i-\bar{e}))^2\frac{1}{\tau_h} + x_i^2\frac{1}{\tau_\epsilon}\right) - \bar{I}\right) \\ & -\frac{1}{2}r_s\delta^2\left(((1-x_i)e_i + x_i\bar{e})^2\frac{1}{\tau_h} + (1-x_i)^2\frac{1}{\tau_\epsilon}\right). \end{aligned} \tag{12}$$

The above expression has an intuitive interpretation as it is firm i's shareholders' and its manager's combined surplus. The first term takes into account the cost of its manager's effort, her disutility from her risk exposure, and the certainty equivalent of the payment that she receives. The second term captures firm i's shareholders' disutility from their risk exposures. These risk exposures come from the volatility of the residual value of the firm net the stock compensation to its manager.

Firm i's shareholders take $\bar{e}$ as given when choosing the optimal linear contract which we denote by x_i^{PO}. The following proposition characterizes the equilibrium contract.

Proposition 1: For τ_h large enough, an equilibrium contract exists and it is unique. The equilibrium contract term x_i^{PO} satisfies

$$\frac{\bar{h}^2}{\frac{r}{\tau_h}\left(x_i^{PO}\right)^2+1}\left(1-x_i^{PO}\right)\left(1-\frac{r_s\delta}{\tau_h}x_i^{PO}\right) - \frac{1}{\tau_\epsilon}\left(rx_i^{PO} - r_s\delta\left(1-x_i^{PO}\right)\right) = 0 \tag{13}$$

and $x_i^{PO} \in (0,1)$. The equilibrium effort level is $e_i^{PO} = \bar{e} = x_i^{PO}\bar{h}$.

Proposition 1 guarantees the existence of a unique equilibrium as long as the systematic risk is not too large. In fact, there is a unique equilibrium contract for most reasonable parameter values. In the rest of the paper we will restrict attention to situations where the equilibrium is unique and use (13) to characterize the unique equilibrium.

In principle, shareholders may also use the average of the stock prices, i.e., a stock index, to incentivise the manager. The price for such an index in our model is

$$\bar{P} = \int P_i di = \frac{1}{1+a^*}\left(\bar{h}\bar{e} + \tilde{h}\left(\int_0^1 (e_i - \bar{e})\, d_i\right) - W^*\right) = \frac{1}{1+a^*}\left(\bar{h}\bar{e} - W^*\right),$$

where a^* and W^* are the equilibrium contract terms. Note that the price for the stock index is constant. Therefore it is uninformative about effort and including it cannot improve the optimal contract. This finding is the result of the relative nature of the information embedded in the stock price and hence, leads immediately to the following corollary.

Corollary 1: Shareholders cannot improve the optimal linear contract by including a stock index.

3.4 Equilibrium Properties

There are several notable features of the equilibrium contract. From the solution in Proposition 1, we observe the incentive and risk-sharing effects that shape the contract between shareholders and managers. We define the *incentive effect* as the level of stock-based compensation when the sole purpose of the contract is to incentivise managers to exert effort, and the *risk-sharing effect* as the level of stock-based compensation when the purpose of the contract is to allow risk-sharing between shareholders and managers. To illustrate, we label the terms corresponding to these two effects in the equilibrium condition (13) as follows

$$\frac{\bar{h}^2}{\frac{r}{\tau_h}\left(x_i^{PO}\right)^2+1}\underbrace{\left(1-x_i^{PO}\right)\left(1-\frac{r_s\delta}{\tau_h}x_i^{PO}\right)}_{\text{The Incentive Effect}}-\frac{1}{\tau_\epsilon}\underbrace{\left(rx_i^{PO}-r_s\delta\left(1-x_i^{PO}\right)\right)}_{\text{The Risk-Sharing Effect}}=0. \tag{14}$$

The relative importance of these two effects in the contract is captured by the coefficients of these two terms in (14). To understand this decomposition, note that systematic risk is entirely borne by a firm's shareholders in equilibrium. A firm's shareholders and its manager share only the idiosyncratic risk. To see why this is this case, recall in equilibrium $e_i^{PO}=\bar{e}$. This implies that a firm's manager's systematic risk exposure in her compensation contract is zero in equilibrium (from (8)). Consequently, the risk-sharing effect (i.e., the second term) dominates the solution if the idiosyncratic risk $1/\tau_\epsilon$ is large. By comparison, when the expected productivity of effort $\bar{h}$ grows, shareholders are keen to get managers to work harder and therefore the incentive effect (i.e., the first term) dominates in the solution.

The magnitude of the incentive effect also depends on shareholders' effective risk aversion $r_s\delta$ and the level of systematic risk (τ_h) but not on managers' risk aversion since systematic risk is not borne by managers. The magnitude of the risk-sharing effect, by contrast, depends on the relative risk-aversions of both shareholders and managers. The following corollary characterizes the optimal contract in the limiting cases when only one of the two effects dominates.

Corollary 2: When $\bar{h}$ or τ_ϵ goes to infinity, the optimal linear contract reflects only the incentive effect and is given by $x_i^{PO}=\min\{1,\tau_h/(r_s\delta)\}$. When $\bar{h}$ or τ_ϵ goes to zero, the optimal linear contract reflects only the risk-sharing effect and is given by $x_i^{PO}=r_s\delta/(r_s\delta+r)$.

3.5 Comparison with the Second Best

In this section, we compare the equilibrium effort and risk-taking with the second-best level. We define the second best as the solution to the planner's problem where the planner maximizes the sum of the utilities of all shareholders conditional on the incentive and individual rationality constraints for managers. Formally,

DEFINITION 2: A second-best solution consists of stock prices P_i for each $i \in [0,1]$, contract $\left(a^{SB}, W^{SB}\right)$ and effort choice e^{SB} such that:

(i) The stock market is informationally efficient in the sense that

$$P_i\left(a^{SB}, W^{SB}, s_i\right) = E\left[V^{SB} - I_i\left(a^{SB}, W^{SB}\right)|s_i\right]$$

where $V^{SB} = \tilde{h}e^{SB} + \tilde{\epsilon}_i$ and $e^{SB} = e_i\left(a^{SB}, W^{SB}\right)$;

(ii) The contract $\left(a^{SB}, W^{SB}\right)$ solves the planner's problem. The planner chooses (a, W) to maximize $\int_0^1 E\left[u_s\left(\delta\left(V_i - I_i\right)\right)\right] di$, subject to $E\left[u\left(aP_i + W - C\left(e_i\left(a, W\right)\right)\right)\right] \geq u\left(\bar{I}\right)$, where $V_i = \tilde{h}e_i\left(a, W\right) + \tilde{\epsilon}_i$.

This definition calls for several comments. The planner's role is limited to coordinating the contracts written by shareholders of different firms. In particular, the planner cannot set prices and must give managers incentives to exert effort. Also, our definition requires the planner to give managers their reservation utility, which implies that including managers' utility in the planner's objective would not change the solution. Similarly, including speculators' payoffs in the planner's objective would not change the second-best solution since the first part of the definition implies that speculators make zero profit given their information.

From Definition 2 we see that the planner chooses the contract term x to maximize the sum of shareholders' utility subject to incentive and participation constraints. Since shareholders' optimization problems are identical, the planner's problem can be seen equivalently as maximizing the utility of shareholders of an arbitrary firm taking into account that $\bar{e} = x\bar{h}$. That is, the planner internalizes the impact of the contract term x on the industry average effort level $\bar{e}$ when choosing x. Thus, the planner's problem is

$$\max_x -\frac{1}{2}\bar{h}^2 x\delta\left(x\left(1+\delta\left(\frac{r_s}{\tau_h}\right)\right) - 2\right) - \frac{1}{2}\delta x^2\frac{r}{\tau_\epsilon} - \frac{1}{2}\delta^2\left(1-x\right)^2\frac{r_s}{\tau_\epsilon}. \tag{15}$$

The first-order condition of the problem is

$$\bar{h}^2\underbrace{\left(1 - x_i^{SB}\left(\frac{r_s\delta}{\tau_h} + 1\right)\right)}_{\text{The Incentive Effect}} - \frac{1}{\tau_\epsilon}\underbrace{\left(rx_i^{SB} - r_s\delta\left(1 - x_i^{SB}\right)\right)}_{\text{The Risk-Sharing Effect}} = 0. \tag{16}$$

and the solution to the planner's problem is :

$$x^{SB} = \frac{\frac{r_s}{\tau_\epsilon}\delta + \bar{h}^2}{\frac{r}{\tau_\epsilon} + \frac{r_s}{\tau_\epsilon}\delta + \bar{h}^2\left(\frac{r_s}{\tau_h}\delta + 1\right)}. \tag{17}$$

Like the optimal equilibrium contract, the second-best solution also reflects the incentive and risk-sharing effects as shown in (16). Specifically, when the expected productivity of effort grows ($\bar{h}$ increases) or the idiosyncratic risk diminishes (τ_ϵ increases), we observe that the incentive effect dominates, and in the limit the second-best contract becomes $x^{SB} = 1/(r_s\delta/\tau_h + 1)$. Note that this limit is strictly lower than the corresponding limit of the equilibrium contract characterized in Corollary 2. As the expected productivity of effort declines ($\bar{h}$ decreases) or the idiosyncratic risk increases (τ_ϵ decreases), the risk-sharing effect dominates. Notice that in this limit the second-best contract is the same as the corresponding limit of the equilibrium contract. This is because externality vanishes when the sole propose of contracting is risk-sharing. The following corollary summarizes the limiting results of the second-best contract.

Corollary 3: When $\bar{h}$ or τ_ϵ goes to infinity, the second-best contract with the stock price as the only incentive instrument reflects only the incentive effect and is given by $x_i^{SB} = 1/(r_s\delta/\tau_h + 1)$. When $\bar{h}$ or τ_ϵ goes to zero, the second-best contract with stock price as the only incentive instrument reflects only risk-sharing effect and is given by $x_i^{PO} = r_s\delta/(r_s\delta + r)$.

Comparing (14) and (16), we see that based on the incentive effect alone, the stock-pay sensitivity is larger in the equilibrium contract than in the second best contract. Moreover, based on the risk-sharing effect alone, the stock-pay sensitivity is the same in both the equilibrium and the second best contracts. However, this does not mean that the stock-pay sensitivity in the equilibrium always exceeds that in the second best. This is because the incentive effect gets a lower weight (relative to the risk-sharing effect) in equilibrium (see (14)) than in the second-best (see (16)). Loosely speaking, this is because in equilibrium shareholders do not internalize the complementarities among managers' effort choices and "freeride" on shareholders of other firms to provide incentives. This leads them to choose a relatively smaller weight on the incentive effect in equilibrium compared to the second best. In cases where the risk-sharing effect dominates (e.g., when the idiosyncratic risk $1/\tau_\epsilon$ is high) and requires a smaller stock-pay sensitivity (e.g., when shareholders are substantially less risk averse than managers), shareholders, who over-weight this effect in equilibrium, may end up giving smaller stock-pay sensitivities than in the second best. The next proposition gives the exact condition that characterizes when the stock-pay sensitivities in equilibrium exceed those under the second-best contract.

Proposition 2: The second best contract requires smaller stock-pay sensitivities than the equilibrium contract, i.e, $x^{SB} < x^{PO}$, and $e^{SB} < e^{PO}$ if and only if

$$\bar{h}^2 \frac{r_s}{\tau_h}\delta \left(\frac{r_s}{\tau_h}\delta + 1\right) + \frac{r_s}{\tau_h}\frac{r_s}{\tau_\epsilon}\delta^2 \left(1 + \frac{r}{\tau_h}\right) + \frac{r}{\tau_\epsilon}\left(\frac{r_s}{\tau_h}\delta - \frac{r}{\tau_h}\right) > 0. \tag{18}$$

Thus, if this inequality is reversed second-best contract requires larger stock-pay sensitivities.

The above condition shows that the level of stock-pay sensitivities in managers' contracts, and consequently excess risk-taking behavior, is pro-cyclical. When $\bar{h}$ is high, e.g., during booms, the expected productivity of effort is very high and shareholders would like to offer the manager of their own firm a high powered stock-based incentive. Since shareholders do not internalize the impact of their own incentive-provision on increasing the industry average effort, they over-incentivise their own manager using stock prices, and trigger a rat race in managerial effort- and risk-taking. Consequently, in equilibrium there is excessive effort- and risk-taking by managers. The planner, in this case, can improve the total welfare by enforcing lower stock-based pay-sensitivities in managers' compensation contracts.

By contrast, when $\bar{h}$ is low, e.g., during downturns, the expected productivity of effort is low and shareholders would like to offer the manager of their own firm low stock-pay sensitivities. Since shareholders do not internalize the impact of their own incentive-provision on increasing the industry average effort, they under-incentivise their own manager using stock prices. This again triggers a race but this time causes a race to the bottom: There is insufficient effort- and risk-taking. In this case, the planner can improve the total welfare by enforcing higher stock-based pay-sensitivities.

3.6 Comparative Statics

Next, we examine how the equilibrium and second-best levels of effort and risk-taking vary with the parameters of the model. Specifically, we look at how the contract term x, that is, the stock-based incentive or pay sensitivity, changes with the characteristics of the firm's project (mean productivity of effort $\bar{h}$, systematic risk τ_h, and idiosyncratic risk τ_ϵ) as well as the risk aversion parameters of shareholders (r_s) and the manager (r) in equilibrium and in second-best. This set of comparative statics generates some hypotheses testable in cross-section. For example, one can test the equilibrium relationship between the power of the contract, the risk-taking behavior with shareholders' or managers' risk-aversions, industry productivity and risk characteristics for cross-sections of firms.

The next proposition characterizes the comparative statics of the equilibrium and second-best contracts with respect to parameters $\bar{h}$ and τ_ϵ.

Proposition 3:

i. If $\tau_h/(r_s\delta) > r_s\delta/(r_s\delta + r)$, the equilibrium contract x^{PO} increases in $\bar{h}$ and τ_ϵ. If $\tau_h/(r_s\delta) < r_s\delta/(r_s\delta + r)$, x^{PO} decreases in $\bar{h}$ and τ_ϵ.

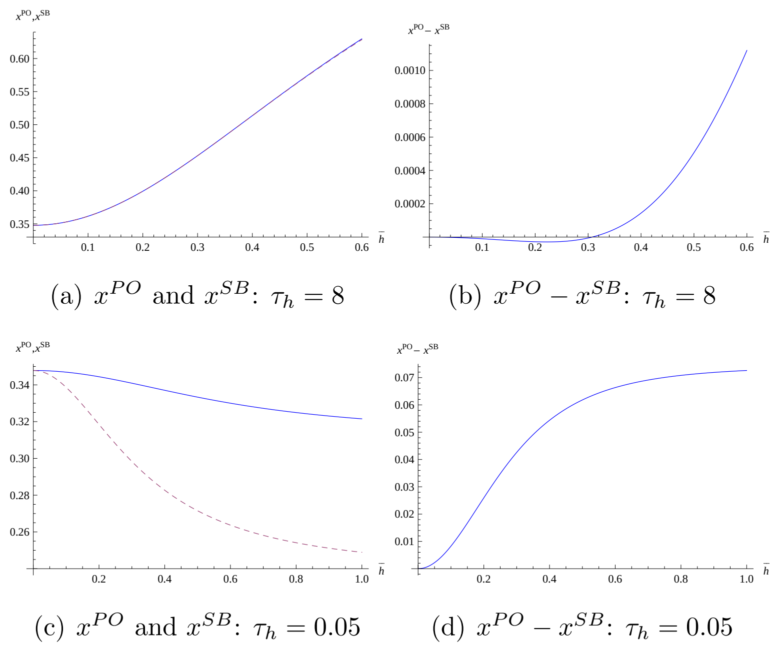
ii. If $1/(r_s\delta/\tau_h + 1) > r_s\delta/(r_s\delta + r)$ then the second-best contract x^{SB} increases in $\bar{h}$ and τ_ϵ. If $1/(r_s\delta/\tau_h + 1) < r_s\delta/(r_s\delta + r)$ then x^{SB} decreases in $\bar{h}$ and τ_ϵ.

To understand this proposition, recall that in Corollaries 2 and 3 we find that when either $\bar{h}$ or τ_ϵ is zero, contracting in equilibrium and in second-best reflects only the risk-sharing effect, requiring the stock-pay sensitivity to be $r_s\delta/(r_s\delta + r)$. When $\bar{h}$ or τ_ϵ approaches infinity, the contracting reflects only the incentive effect, requiring the stock-pay sensitivity to be $\min\{1, \tau_h/(r_s\delta)\}$ in equilibrium and $\min\{1, 1/(r_s\delta/\tau_h + 1)\}$ in second-best. Thus, the above proposition says that when the incentive effect requires a larger (smaller) stock-pay sensitivity than the risk-sharing effect, the power of the equilibrium contract x^{PO} and the power of the second best contract x^{SB} is monotone increasing (decreasing) in $\bar{h}$ and τ_ϵ. Intuitively, when $\bar{h}$ and τ_ϵ are increasing, that is, the average productivity of effort is higher and the idiosyncratic risk of the firm's project is lower, shareholders as well as the planner are more concerned about the incentivising than risk-sharing, that is, the purpose of the contracting is weighted more towards the incentive effect rather than the risk sharing effect. Since the former requires a larger (smaller) pay sensitivity, both x^{PO} and x^{SB} are increasing (decreasing) in $\bar{h}$ and τ_ϵ. The monotonicity result established in Proposition 3 leads to the following corollary.

Corollary 4: The equilibrium contract x^{PO} always takes a value between $\min\{1, \tau_h/(r_s\delta)\}$ and $r_s\delta/(r_s\delta + r)$. The second best contract x^{SB} always takes a value between $1/(r_s\delta/\tau_h + 1)$ and $r_s\delta/(r_s\delta + r)$.

In Figures 1 and 2, we illustrate these findings regarding $\bar{h}$ and τ_ϵ using numerical examples. We consider two cases where the ratio $r_s\delta/(r_s\delta + r)$ is at the same value, 0.35. Therefore, the power of the contract for the two cases should be the same if the sole purpose of the contracting is for risk-sharing between shareholders and managers. In panels (a) and (b) of both figures, we consider a case with a less volatile project ($\tau_h = 8$). In this case the incentive effect requires a larger stock pay sensitivity than the risk sharing effect (both in equilibrium and second-best).[15] In panels (c) and (d) of both figures, we consider a case with

[15] Since $\min\{1, \tau_h/(r_s\delta)\} = 1$ and $\min\{1, 1/(r_s\delta/\tau_h + 1)\} = 0.98$ both exceed $r_s\delta/(r_s\delta + r) = 0.35$.



(a) x^{PO} and x^{SB}: $\tau_h = 8$ (b) $x^{PO} - x^{SB}$: $\tau_h = 8$

(c) x^{PO} and x^{SB}: $\tau_h = 0.05$ (d) $x^{PO} - x^{SB}$: $\tau_h = 0.05$

Figure 1: Mean Productivity of Effort ($\bar{h}$): The solid and the dashed lines in panels (a) and (c) represent how the stock-based incentive (x) changes with respect to the mean productivity of effort ($\bar{h}$) in equilibrium and in the planner's optimum. In panels (b) and (d), the solid line represents the difference between the equilibrium and the planner's optimum. The parameters are fixed at $\tau_\epsilon = 1$, $r = 0.3$, $r_s = 0.2$, and $\delta = 0.8$.

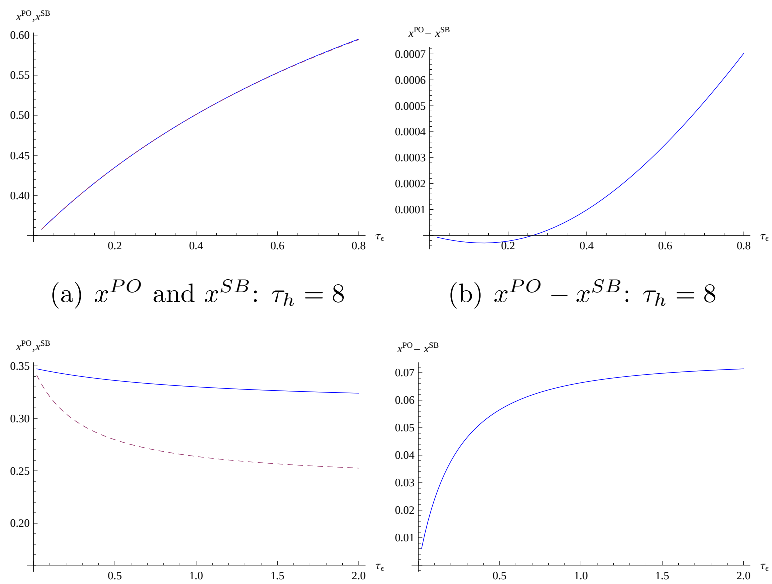
a relatively more volatile project ($\tau_h = 0.05$). In this case the incentive effect requires a lower stock pay sensitivity than the risk sharing effect (both in equilibrium and second-best).[16]

Panels (a) and (c) in Figure 1 show how equilibrium and second-best contracts vary with $\bar{h}$. The power of the contract starts at 0.35 when $\bar{h} = 0$ and the sole purpose of contracting is risk-sharing. As predicted by Proposition 3, it is monotonically increasing in $\bar{h}$ when the incentive effect requires a larger stock pay sensitivity which is the case in panel (a), but is monotonically decreasing in $\bar{h}$ when the incentive effect requires a lower stock pay sensitivity which is the case in panel (b).[17]

Panels (b) and (d) in Figure 1 show how the difference between the two varies with $\bar{h}$. We observe that in the region where equilibrium incentive is stronger than the second best (i.e., $x^{PO} - x^{FB}$ is positive), higher $\bar{h}$ leads to larger $x^{PO} - x^{FB}$. This shows that the externality becomes stronger as $\bar{h}$ increases. However, as we discussed earlier, the gap between the equilibrium and the second best converges to a constant when $\bar{h}$ approaches

[16]Since $\min\{1, \tau_h/(r_s\delta)\} = 0.31$ and $\min\{1, 1/(r_s\delta/\tau_h + 1)\} = 0.238$ are both below $r_s\delta/(r_s\delta + r) = 0.35$.

[17]From Corollary 2 and (17) we know that when $\bar{h}$ goes to infinity, the incentive effect becomes the only factor determining the equilibrium contract and the difference between x^{PO} and x^{SB} converges to a constant. This convergence can be observed in panels (a) and (c).



(a) x^{PO} and x^{SB}: $\tau_h = 8$ (b) $x^{PO} - x^{SB}$: $\tau_h = 8$

(c) x^{PO} and x^{SB}: $\tau_h = 0.05$ (d) $x^{PO} - x^{SB}$: $\tau_h = 0.05$

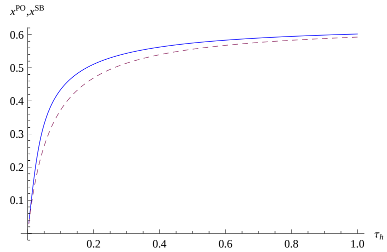
Figure 2: Idiosyncratic Risk (τ_ϵ): The solid and the dashed lines in panels (a) and (c) represent how the stock-based incentive (x) changes with respect to the inverse of the firm's idiosyncratic risk (τ_ϵ) in equilibrium and in the planner's optimum. In panels (b) and (d), the solid line represents the difference between the equilibrium and the planner's optimum. The parameters are fixed at $\bar{h} = 0.6$, $r = 0.3$, $r_s = 0.2$, and $\delta = 0.8$.

infinity. In addition, in panel (b), we observe a region where $x^{PO} - x^{FB}$ is negative.[18] This is the region where the average productivity of effort ($\bar{h}$) is low and shareholders give lower powered incentives in equilibrium than in the second best. In this region, the negative gap has a non-monotonic relationship with $\bar{h}$. It grows, i.e., the negative externality becomes stronger, when $\bar{h}$ becomes smaller, but narrows when $\bar{h}$ approaches zero as the externality vanishes and risk-sharing becomes the sole purpose of contracting.

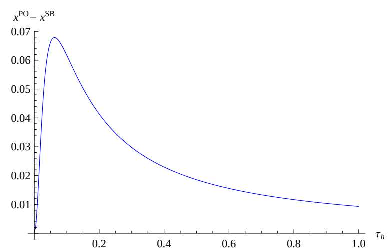
The graphs in Figure 2 show how the power of the contract varies with the idiosyncratic risk of the project τ_ϵ. Similar to $\bar{h}$, the effect of the idiosyncratic risk depends on the incentive and risk-sharing effects. As τ_ϵ increases, the incentive effect starts to dominate. Panels (a) and (c) illustrate the results in Proposition 3 and show the monotonic relationship between the equilibrium and second-best contracts with τ_ϵ. The intuition behind Figure 2 is similar to that behind Figure 1 and hence is omitted.

Now we turn to comparative statics on τ_h. Algebraically, we observe that in both the equilibrium and the second-best (see (14) and (16)) when τ_h is larger, that is, the systematic risk is lower, the incentive effect requires a larger stock-pay sensitivity. Further, when τ_h

[18] In panel (d) of Figure (1), we do not observe a negative $x^{PO} - x^{FB}$ because condition in (18) is always satisfied in this case.



(a) x^{PO} and x^{SB}



(b) $x^{PO} - x^{SB}$

Figure 3: Systematic Risk (τ_h): The solid and the dashed lines in panel (a) represent how the stock-based incentive (x) changes with respect to the inverse of project systematic risk (τ_h) in equilibrium and in the planner's optimum. In panel (b), the solid line represents the difference between the equilibrium and the planner's optimum. The parameters are fixed at $r = 0.3$, $r_s = 0.2$, $\tau_\epsilon = 1$, $\bar{h} = 2$, and $\delta = 0.8$.

is larger, coefficients on the incentive effect in (14) and (16) become larger, indicating that shareholders are more concerned about incentivising. Intuitively, stock-based compensations allow managers to remove their exposure to the industry risk and, as a result, shareholders have to bear it entirely. Because of this, when the systematic risk is smaller (higher τ_h), shareholders are more willing to give larger stock-pay compensations (larger x_i). The following proposition states this result formally.

Proposition 4: Both x^{PO} and x^{SB} are strictly increasing in τ_h.

The graphs in Figure 3 illustrate the above proposition and how $x^{PO} - x^{SB}$ varies with τ_h. In panel (a), we observe indeed that the power of the contract is increasing as the systematic risk decreases in both the equilibrium and the second best. In the case shown in panel (b), there is a gap between the planner's optimum and the equilibrium outcome and the planner prefers to have lower stock-pay sensitivities. This gap grows when the systematic risk is higher (τ_h is smaller) and complementarities in effort-taking become larger; but quickly converges to zero when the systematic risk approaches infinity (τ_h is close to zero) and both the equilibrium effort and the second-best effort approach zero.

Comparative statics of the equilibrium contract x^{PO} with respect to r is quite complex because there are two potentially conflicting effects. First, since contracts impose risk on managers, as managers get more risk averse (i.e., r becomes larger), both the equilibrium and second-best require reducing stock-pay sensitivities to improve risk sharing. This is the direct effect and is shown in (14) and (16) where r affects the magnitude of the risk-sharing effect: As r increases, the risk-sharing effect requires lower stock-pay sensitivities. Second, increasing r also increases the relative importance of the risk-sharing effect on the equilibrium contract (See (14)) which is an indirect effect. (This indirect effect does not exist for the second-best contract.) The relationship between r and stock-pay sensitivities depends on the magnitude of these two effects. If the risk-sharing effect requires a larger stock-pay sensitivity than the incentive effect (i.e., $\delta r_s/(r+\delta r_s) > \tau_h/(r_s\delta)$), it is possible to have a region where the equilibrium incentive is increasing in r. This is because as r increases, the equilibrium contract reflects the risk-sharing purpose more than the incentive-provision purpose, but the risk-sharing effect requires a larger stock-pay sensitivity than the incentive effect.[19] As the next proposition shows, however, when $\tau_h/(r_s\delta) > r_s\delta/(r_s\delta+r)$, the indirect effect reinforces the direct effect and x^{PO} is decreasing in r.

Proposition 5: The second best contract x^{SB} is decreasing in r. If $\tau_h/(r_s\delta) > r_s\delta/(r_s\delta+r)$ then the equilibrium contract x^{PO} decreases in r.

The risk aversion of shareholders, r_s, on the other hand, affects the magnitude of both incentive and risk-sharing effects as shown in (14) and (16) and hence has an even more complicated effect.[20] The risk-sharing effect dictates a larger stock-pay sensitivity to managers as shareholders become more risk-averse; while the incentive effect requires shareholders to reduce the stock-pay sensitivity due to the concern that managers might be incentivised to take on too much systematic risk. In the equilibrium case, there is an indirect effect which is that the coefficient on the incentive effect varies with r_s, indicating that the relative importance of the incentive effect changes with r_s. When τ_h is sufficiently large the risk-sharing effect dominates, and we get the following result.

Proposition 6: For τ_h sufficiently large, the second best contract x^{SB} and the equilibrium contract x^{PO} are increasing in r_s.

4 Discussions

4.1 Systematic or Idiosyncratic Risk-Taking?

In this section, we show that the excessive (insufficient) risk-taking is related to the common/systematic rather than firm-specific/idiosyncratic risk. To highlight the source of externality we consider the case where the productivity shock is idiosyncratic rather than

[19]It is straightforward to create numerical examples of this but they require extreme values for risk aversion parameters.

[20]The parameter δ is similar to r_s and hence its comparative statics is omitted.

common to the industry. Specifically, we let $V_i = \tilde{k}_i e_i + \tilde{\epsilon}_i$ where $\tilde{k}_i$ is a firm-specific random term which is independently and normally distributed across managers with mean $\overline{k}$ and variance $1/\tau_k$.

In this case, the public signal about firm i that speculators receive at $t = 1$ is $s_i = V_i - \bar{V}$, where $\bar{V} = \overline{k}\bar{e}$ is the average value of a firm, and $\bar{e}$ is the average effort exerted by the managers in the industry. Therefore, this signal can also be rewritten as $s_i = \tilde{k}_i e_i - \overline{k}\bar{e} + \tilde{\epsilon}_i$.

In a symmetric equilibrium, $s_i = (\tilde{k}_i - \overline{k})\bar{e} + \tilde{\epsilon}_i$. The equilibrium value of firm i given public information is

$$E\left(V_i^* \left| s_i\right.\right) = E\left(\tilde{k}_i e_i^* + \tilde{\epsilon}_i \middle| s_i = \left(\tilde{k}_i - \overline{k}\right)\bar{e} + \tilde{\epsilon}_i\right) = \overline{k}\bar{e} + s_i.$$

Thus, by equation (4), firm i's stock price at time 1 is given by

$$P_i = E\left(V_i^* - a_i^* P_i - W_i^* \left| s_i\right.\right) = \left(\overline{k}\bar{e} + s_i\right) - a_i^* P_i - W_i^*,$$

which is

$$P_i = \frac{1}{1 + a_i^*}\left(\overline{k}\bar{e} + s_i - W_i^*\right).$$

Given this equilibrium price, we can now write manager i's compensation as

$$I_i \;=\; a_i P_i + W_i = a_i\left(\frac{1}{1 + a_i^*}\left(\overline{k}\bar{e} + s_i - W_i^*\right)\right) + W_i = x_i(\underbrace{\overline{k}\bar{e} + s_i}_{E(V_i|s_i)} - W_i^*) + W_i \qquad (19)$$

with x_i again given by $a_i/(1 + a_i^*)$. Using (19), given a contract (x_i, W_i) and average effort $\bar{e}$, manager i chooses e_i to maximize

$$E\left(u\left(x_i\left(\overline{k}\bar{e} + s_i - W_i^*\right) + W_i - C\left(e_i\right)\right)\right).$$

Plugging in $s_i = \tilde{k}_i e_i - \overline{k}\bar{e} + \tilde{\epsilon}_i$ and computing the expectation in the above equation, the manager's problem can be restated as choosing e_i to maximize

$$x_i\overline{k}e_i - x_i W_i^* + W_i - C\left(e_i\right) - \frac{1}{2}r\left(\left(x_i e_i\right)^2\frac{1}{\tau_k} + x_i^2\frac{1}{\tau_\epsilon}\right).$$

Taking the first-order condition and solving for e_i, we obtain manager i's effort choice as

$$e_i = \frac{x_i\bar{k}}{1 + \left(\frac{r}{\tau_k}\right)x_i^2}. \qquad (20)$$

The above equation shows that, as one would expect, a volatile firm-specific risk $(1/\tau_k)$ lowers the effort level. More importantly, it shows that when the productivity shock is idiosyncratic, there are no complementarities between the industry average effort and an individual manager's effort. Therefore, the results we obtained earlier on excessive (insufficient) risk-taking can only arise in an environment where the productivity shock has a systematic component across firms in the industry.

4.2 Can Effort with Stock Market Tournaments Exceed the First-Best?

In this section we compare the effort choice in equilibrium with another benchmark: the first-best case where efforts are observable (and enforceable). In this benchmark case, there is no need to incentivise managers. The contracting between a firm's shareholders and its manager is purely for risk-sharing. Further, it is possible to contract on the final value of the firm in this benchmark. Managers will be given, in addition to a fixed wage, an ownership share of the final value for this purpose. Intuitively, one might conjecture that without the agency problem, the first-best effort should exceed the equilibrium effort. However, when contracts are based only on stock prices, we will show that it is possible for the equilibrium effort to exceed the first-best effort.

Let's denote the proportion of the final value given to manager i by b_i^{FB} and the fixed wage by W_i^{FB}. The value of the firm at $t = 2$ is the final liquidation claim of the project, V_i, less the payment to the manager, $b_i^{FB} V_i + W_i^{FB}$. Since they hold a proportion δ of the shares of the firm, shareholders of firm i choose its manager's effort level, e_i^{FB} and the contract terms (b_i^{FB}, W_i^{FB}) to maximize

$$E\left[u_s\left(\delta\left(\left(1-b_i^{FB}\right)V_i - W_i^{FB}\right)\right)\right], \tag{21}$$

subject to manager i's participation constraint

$$u\left(b_i^{FB} V_i + W_i^{FB} - C\left(e_i^{FB}\right)\right) \geq u\left(\bar{I}\right), \tag{22}$$

where e_i^{FB} is the first-best effort level. Substituting for $V_i = \tilde{h} e_i^{FB} + \tilde{\epsilon}_i$ in (22) and computing expectation we obtain

$$b_i^{FB} \bar{h} e_i^{FB} + W_i^{FB} - C\left(e_i\right) - \frac{1}{2} r\left(\left(b_i^{FB}\right)^2\left(e_i^{FB}\right)^2 \frac{1}{\tau_h} + \left(b_i^{FB}\right)^2 \frac{1}{\tau_\epsilon}\right) = \bar{I}. \tag{23}$$

Solving for W_i^{FB} in (23), substituting it into (21), and calculating expectations, the shareholders' objective function (21) can be rewritten as

$$\begin{aligned} &\max_{b_i^{FB}, e_i^{FB}} \delta\left(\bar{h} e_i^{FB} - C\left(e_i^{FB}\right) - \frac{1}{2} r\left(\left(b_i^{FB}\right)^2\left(e_i^{FB}\right)^2 \frac{1}{\tau_h} + \left(b_i^{FB}\right)^2 \frac{1}{\tau_\epsilon}\right)\right) \\ &-\frac{1}{2} r_s \delta^2\left(\left(1-b_i^{FB}\right)^2\left(e_i^{FB}\right)^2 \frac{1}{\tau_h} + \left(1-b_i^{FB}\right)^2 \frac{1}{\tau_\epsilon}\right) - \bar{I}. \end{aligned} \tag{24}$$

Therefore, for a given b_i^{FB}, the optimal effort level is

$$e_i^{FB} = \frac{\bar{h}}{1 + \frac{r}{\tau_h}\left(b_i^{FB}\right)^2 + \delta \frac{r_s}{\tau_h}\left(1-b_i^{FB}\right)^2}. \tag{25}$$

Substituting this back into the shareholders' objective function (24), we see that the shareholders choose b_i^{FB} to maximize

$$\frac{1}{2}\frac{\bar{h}^2}{\delta\frac{r_s}{\tau_h}\left(1-b_i^{FB}\right)^2+\frac{r}{\tau_h}\left(b_i^{FB}\right)^2+1}-\frac{1}{2}\left(\delta\frac{r_s}{\tau_\epsilon}\left(1-b_i^{FB}\right)^2+\frac{r}{\tau_\epsilon}\left(b_i^{FB}\right)^2\right).$$

Therefore, b_i^{FB} is given by,[21]

$$b_i^{FB}=\frac{r_s\delta}{r+r_s\delta}. \tag{26}$$

That is, firm i's shareholders give its manager an ownership share that depends on the ratio of their coefficients of absolute risk aversion. This result echoes our early finding when the sole purpose of contracting is risk-sharing in equilibrium. Substituting this into (25) we obtain the first-best effort level as

$$e_i^{FB}=\frac{\bar{h}}{1+\frac{r_s\delta}{\tau_h}\left(\frac{r}{r+r_s\delta}\right)}. \tag{27}$$

The above analysis shows that in the first best optimum, managers exert higher effort when the average productivity of effort ($\bar{h}$) and a lower variance ($1/\tau_h$), and exert lower effort when either shareholders or managers become more risk averse (i.e. r_s or r increases). Further, it shows that when shareholders have to hold a larger share of the final value, i.e., a larger $r/\left(r+\delta r_s\right)$, meaning that they have to take on more firm risk, they prefer managers exert a lower level of effort.

Now we compare the first-best outcome with the equilibrium outcome when the contract is based on stock prices. In this contractual environment, as we discussed earlier, shareholders of individual firms do not internalize the effect of their own incentive provision on the effort level of the mangers of the other firms in the same industry. They may over-incentivise and the externality may escalate and exceed the first-best effort level, e_i^{FB}. This is the next Proposition.

Proposition 7: The effort level under stock price-based contracts, e_i^{PO}, exceeds e_i^{FB} if $\bar{h}$ is large enough and shareholders are not too risk averse.

That is, when the average productivity of effort is high and the shareholders are not too risk averse, externalities can be so strong that the shareholders provide too much stock price based incentives causing the equilibrium effort to exceed the first-best effort benchmark.

[21] To see this note that $\delta\frac{r_s}{\tau_h}\left(1-b_i^{FB}\right)^2+\frac{r}{\tau_h}\left(b_i^{FB}\right)^2$ and $\delta\frac{r_s}{\tau_\epsilon}\left(1-b_i^{FB}\right)^2+\frac{r}{\tau_\epsilon}\left(b_i^{FB}\right)^2$ are both minimized at $b_i^{FB}=r_s\delta/\left(r+r_s\delta\right)$.

4.3 Contracting with Clawbacks

In this section, we consider the case where shareholders can use a noisy version of the final value of the firm as an additional contractual instrument with managers, which we denote by SV. This contractual instrument is similar to the clawback type of clauses in the compensation contract. Typically these are deferred bonuses (or punishment) which are tied to the firm's future (long-term) performance. Specifically, we assume that

$$SV_i = \tilde{V}_i + \tilde{\eta}_i.$$

In this setup, manager i's total compensation has three components, a fixed wage W_i, stock appreciation rights $a_i P_i$ and a long term component $b_i SV_i$. Thus manager i's compensation is given by

$$I_i\left(a_i, b_i, W_i\right) = a_i P_i + b_i SV_i + W_i.$$

Firm i's stock price is

$$\begin{aligned} P_i\left(a_i^*, b_i^*, W_i^*, s_i\right) &= E\left[V_i^* - I_i\left(a_i^*, b_i^*, W_i^*\right)|s_i\right] = E\left[\left(1-b_i^*\right)V_i^* - a_i^* P_i - b\eta_i - W_i^*\,|s_i\right] \\ &= \left(1-b_i^*\right)\left(\bar{h}\bar{e}+s_i\right) - a_i^* P_i - W_i^* \\ &= \frac{1-b_i^*}{1+a_i^*}\left(\bar{h}\bar{e}+s_i-W_i^*\right). \end{aligned} \tag{28}$$

Given the equilibrium price from (28), manager i's compensation is

$$\begin{aligned} I_i &= a_i P_i + b_i SV_i + W_i = a_i\left(\frac{1-b_i^*}{1+a_i^*}\left(\bar{h}\bar{e}+s_i-W_i^*\right)\right) + b_i SV_i + W_i \\ &= x_i(\underbrace{\bar{h}\bar{e}+s_i}_{E[V_i|s_i]} - W_i^*) + (z_i - x_i)\underbrace{\left(\tilde{h}e_i + \tilde{\epsilon}_i + \tilde{\eta}\right)}_{V_i+\tilde{\eta}} + W_i, \end{aligned} \tag{29}$$

with x_i and z_i given by

$$x_i = \frac{a_i\left(1-b_i^*\right)}{1+a_i^*} \text{ and } z_i = x_i + b_i.$$

As before, x_i denotes the pay sensitivity to the stock price. Since b_i denotes pay sensitivity to the clawback, z_i denotes the overall pay sensitivity of the contract. We restrict attention to contracts where $z_i \in [0,1]$, that is, the total pay sensitivity does not exceed the entire equity of the firm. For expositional clarity, from now on, we state the contract terms as (x_i, z_i, W_i) instead of (a_i, b_i, W_i).

Using (29), given a contract (x_i, z_i, W_i) and average effort $\bar{e}$, manager i chooses e_i to maximize

$$E\left[u\left(x_i\left(\bar{h}\bar{e}+s_i-W_i^*\right) + (z_i-x_i)\left(\tilde{h}e_i+\tilde{\epsilon}_i+\tilde{\eta}\right) + W_i - C\left(e_i\right)\right)\right]. \tag{30}$$

Plugging $s_i = \tilde{h}(e_i - \bar{e}) + \tilde{\epsilon}_i$, and computing expectation in the above equation, manager i's problem in (30) can be restated as choosing e_i to maximize

$$z_i \bar{h} e_i - x_i W_i^* + W_i - C(e_i) - \frac{1}{2} r \left((x_i (e_i - \bar{e}) + (z_i - x_i) e_i)^2 \frac{1}{\tau_h} + (z_i - x_i)^2 \frac{1}{\tau_\eta} + z_i^2 \frac{1}{\tau_\epsilon} \right). \quad (31)$$

Taking the first-order condition, and solving for e_i, we obtain manager i's effort choice as

$$e_i = \frac{z_i \bar{h} + \frac{r}{\tau_h} z_i x_i \bar{e}}{1 + \frac{r}{\tau_h} z_i^2}. \quad (32)$$

Firm i's shareholders choose the contract terms $(x_i,\ z_i,\ W_i)$ to maximize their expected utility

$$E\left[u_s\left(\delta\left(V_i - I_i\right)\right)\right] \quad (33)$$

subject to its manager's participation constraint: $E\left[u\left(a_i P_i + b_i S V_i + W_i - C(e_i)\right)\right] \geq u\left(\bar{I}\right)$ where e_i is given by (32).

Using (29) we obtain:

$$\delta(V_i - I_i) = \delta\left(-x_i\left(\bar{h}\bar{e} + s_i - W_i^*\right) + (1 - z_i + x_i)\left(\tilde{h} e_i + \tilde{\epsilon}_i\right) - (z_i - x_i)\tilde{\eta} - W_i\right).$$

Plugging in $s_i = \tilde{h}(e_i - \bar{e}) + \tilde{\epsilon}_i$, and computing expectations, firm i's shareholders' problem in (33) can be stated as

$$\begin{aligned} \max_{x_i, z_i, W_i} \quad & \delta\left((1 - z_i)\bar{h} e_i + x_i W_i^* - W_i\right) \\ & -\frac{1}{2} r_s \delta^2 \left(((1 - z_i + x_i) e_i - x_i (e_i - \bar{e}))^2 \frac{1}{\tau_h} + (z_i - x_i)^2 \frac{1}{\tau_\eta} + (1 - z_i)^2 \frac{1}{\tau_\epsilon} \right) \end{aligned}$$

where e_i is given by (32). Using (31) and firm i's manager's individual rationality constraint we obtain:

$$-\left(z_i \bar{h} e_i - x_i W_i^* + W_i\right) = -C(e_i) - \frac{1}{2} r \left((x_i (e_i - \bar{e}) + (z_i - x_i) e_i)^2 \frac{1}{\tau_h} + (z_i - x_i)^2 \frac{1}{\tau_\eta} + z_i^2 \frac{1}{\tau_\epsilon} \right) - \bar{I}.$$

We substitute the above equation in (33) and simplify firm i's shareholders' problem further as

$$\begin{aligned} \max_{x_i, z_i} \quad & \delta\left(\bar{h} e_i - C(e_i) - \frac{1}{2} r \left((x_i (e_i - \bar{e}) + (z_i - x_i) e_i)^2 \frac{1}{\tau_h} + (z_i - x_i)^2 \frac{1}{\tau_\eta} + z_i^2 \frac{1}{\tau_\epsilon} \right) - \bar{I} \right) \\ & -\frac{1}{2} r_s \delta^2 \left(((1 - z_i + x_i) e_i - x_i (e_i - \bar{e}))^2 \frac{1}{\tau_h} + (z_i - x_i)^2 \frac{1}{\tau_\eta} + (1 - z_i)^2 \frac{1}{\tau_\epsilon} \right). \quad (34) \end{aligned}$$

A general closed form solution to the above maximization problem is difficult to provide. We provide a complete characterization for the case where τ_η approaches infinity and illustrate the solution numerically for intermediate values of τ_η. This analysis shows that when shareholders have access to a more precise final value contractual instrument, SV, the impact of the externality weakens and the gap between the equilibrium and the second best narrows.

The following proposition characterizes the equilibrium solution when the noise in the final value instrument approaches zero, that is, when τ_η approaches infinity.

PROPOSITION 8: When τ_η approaches infinity, the optimal linear contract with stock price and the final value instrument is unique. The contract term z_i^* is the unique positive root to the following equation:

$$\begin{aligned} H(z) &= -\bar{h}^2\delta\left(\frac{r}{\tau_h}z+1\right)\left(\frac{r}{\tau_h}+\frac{r_s}{\tau_h}\delta\right)^2(z-1) \\ &\quad -\left(\frac{r}{\tau_h}+\frac{r_s}{\tau_h}\delta+\left(\frac{r}{\tau_h}\right)^2 z^2\left(\frac{r_s}{\tau_h}\delta+1\right)+2\frac{r_s}{\tau_h}\frac{r}{\tau_h}z\delta\right)^2\left(\delta z\frac{r}{\tau_\epsilon}+\delta^2(z-1)\frac{r_s}{\tau_\epsilon}\right)=0 \end{aligned} \tag{35}$$

and $z_i^* \in (0,1)$.

The contract term x_i^* is :

$$x_i^* = \frac{\frac{r}{\tau_h}\left(1+\frac{r}{\tau_h}(z_i^*)^2\right)+\frac{r_s}{\tau_h}\delta\left(z_i^*-1\right)\left(\frac{r}{\tau_h}z_i^*+1\right)}{\left(\frac{r}{\tau_h}z_i^*+1\right)\left(\frac{r}{\tau_h}+\frac{r_s}{\tau_h}\delta\right)} > 0. \tag{36}$$

The equilibrium effort level is

$$e_i^* = \bar{e} = \frac{\bar{h}z_i^*\left(\frac{r}{\tau_h}z_i^*+1\right)\left(\frac{r}{\tau_h}+\frac{r_s}{\tau_h}\delta\right)}{\frac{r}{\tau_h}\left(\frac{r}{\tau_h}(z_i^*)^2+1\right)+\frac{r_s}{\tau_h}\delta\left(\frac{r}{\tau_h}z_i^*+1\right)^2}. \tag{37}$$

Finally, managers are given a strictly positive share of the final value instrument, i.e., $b_i^* \geq 0$ if

$$\bar{h}^2\frac{r_s}{\tau_h}\delta\left(\frac{r_s}{\tau_h}\delta+1\right)+\frac{r_s}{\tau_h}\frac{r_s}{\tau_\epsilon}\delta^2\left(1+\frac{r}{\tau_h}\right)+\frac{r}{\tau_\epsilon}\left(\frac{r_s}{\tau_h}\delta-\frac{r}{\tau_h}\right) > 0. \tag{38}$$

Otherwise, managers are given a negative share of the final value instrument.[22]

Furthermore, the next proposition shows that the equilibrium and the second-best result coincide when the noise in the final value instrument approaches zero.

[22]Note that we do not put any restrictions on the amount of the final-value based pay sensitivity b_i. If b_i is restricted to be non-negative (e.g., because managers might privately destroy some of the value of the firm at time 2), we might have a corner solution. Specifically, when (38) is negative, we set $b_i = 0$.

Proposition 9: When τ_η approaches infinity, the planner's optimum coincides with the equilibrium when shareholders use both the stock price and the final value instrument as contractual instruments.

In other words, this proposition states that if the final value instrument is perfectly correlated with the final value realization, shareholders are able to completely counteract the impact of complementarities among managers' effort-taking through optimal contracting. To see this algebraically, we rewrite shareholders' objective function (34) as

$$\begin{aligned}\max_{x_i^{'},z_i} \quad & \delta\left(\bar{h}e_i - C\left(e_i\right) - \frac{1}{2}r\left(\left(z_ie_i - x_i^{'}\right)^2\frac{1}{\tau_h} + z_i^2\frac{1}{\tau_\epsilon}\right) - \bar{I}\right) \\ & -\frac{1}{2}r_s\delta^2\left(\left(\left(1-z_i\right)e_i + x_i^{'}\right)^2\frac{1}{\tau_h} + \left(1-z_i\right)^2\frac{1}{\tau_\epsilon}\right), \end{aligned} \tag{39}$$

where $x_i^{'} = x_i\bar{e}$. That is, x_i and $\bar{e}$ always enter together in shareholders' objective function. Shareholders can completely eliminate the impact of the industry average effort $\bar{e}$ through choosing x_i: Shareholders could choose a lower x_i when $\bar{e}$ is high and vice versa. By redefining shareholders' optimization problem this way, we see that it coincides with the planner's problem and Proposition 9 is obvious.

In the setting that τ_η approaches infinity, shareholders have two incentive instruments: the firm's stock price and the final value, and face two types of risk: one is related to the source of complementarities and the other is idiosyncratic. Using the two instruments a firm's shareholders can choose optimally its manager's exposure to each type of risks. Since they can choose the firm's exposure to systematic risk regardless of the industry average effort, they are able to undo the complementarities among managers' effort choices. The planner, therefore, has no role to play in this environment.

Now we compare the results in this setting with those in the earlier setting where the final value instrument is not available. Note that when (38) is satisfied, shareholders would like to offer managers a positive share of the final value instrument. If the final value instrument is not available, when the same condition is satisfied, the equilibrium stock price-based incentive would exceed the second-best as stated in (18) of Proposition 2. Intuitively, this means that when shareholders are constrained from giving a positive share of the final value instrument to elicit effort, they increase the use of stock price-based instrument instead, triggering complementarities in effort, causing excessive effort and risk level in equilibrium relative to the second best. Similarly, when shareholders are constrained from giving a negative share of the final value instrument to control effort, the oppositive would happen. They lower the use of stock price-based instrument instead, resulting in insufficient equilibrium effort and risk-taking relative to the second best.

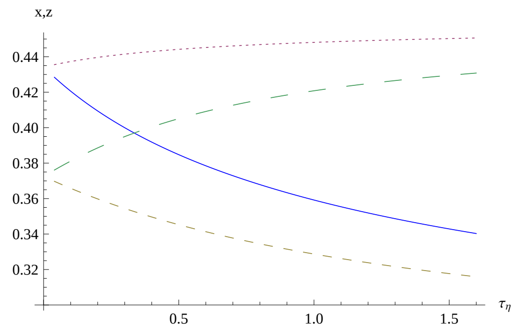


Figure 4: Noisy Final Value (τ_η): The dotted and the long-dashed lines represent how the total incentive changes with respect to the noise of the final value instrument (τ_η) in equilibrium and in the planner's optimum, respectively. The solid and short-dashed lines represent how the stock-based incentive (x) changes with respect to the noise of the final value instrument (τ_η) in equilibrium and in the planner's optimum, respectively. The parameters are fixed at $r = 0.3$, $r_s = 0.2$, $\tau_\epsilon = 1$, $\bar{h} = 0.6$, $\tau_h = 0.1$, and $\delta = 0.8$.

However, in reality, the time required to observe the realization of the final value tends to exceed the time horizon of the contracts between shareholders and managers. Feasible instruments are often correlated with the final value with some noise. Next we provide a numerical example, showing how the equilibrium and the second best incentives change with the noise in the final value instrument. When the noise in the final value instrument goes down, i.e., τ_η increases, the impact of the externality generated by stock market tournaments becomes smaller as shareholders' ability to span the risk space of the manager strengthens.

The graph in Figure 4 illustrates this intuition. It shows that as the final value instrument becomes more precise (i.e., τ_η becomes larger), the total pay sensitivities increase and the stock price-based pay sensitivities decrease in both equilibrium and in the second-best. Further, as expected, the gap between the equilibrium and the second best narrows as the impact of the externality weakens when τ_η gets larger.

5 Conclusion

In this paper, we propose a new explanation for excessive (insufficient) managerial risk taking. The explanation is based on the idea that the stock-based compensation contracts generate a tournament among managers within an industry. This tournament introduces complementarities among managers to exert efforts. In the presence of such complementarities, shareholders, when setting the optimal contract for their own manager, do not internalize the impact of their incentive provision on managers of other firms in the industry.

The tournament effect is double-edged. Stock-based compensation allows managers to remove the common industry risk and is therefore desirable for shareholders since it is a cost-effective incentive instrument. At the same time, it lets shareholders bear the systematic risk entirely and gives managers an incentive to take on too much systematic risk. However, since shareholders in individual firms do not internalize the impact of their own incentive provision on the average effort in the industry, effort- and risk-taking among managers may escalate. For example, shareholders are eager to provide more powerful incentives during booms, causing the industry average effort to be high, triggering a rat race among managers to exert effort and exposing shareholders to too much systematic risk, relative to the second-best or even the first-best level. During recessions, the opposite might happen: The incentive provision is too weak and the equilibrium level of effort and risk exposure is lower than the second best.

Our results may offer an explanation to the expansion of the financial industry in recent years and the reckless systematic risk-taking that followed. Our analysis indicates the regulatory reform of executive compensation is a complex issue. Tackling managerial risk-taking behavior needs to be counter-cyclical: lowering stock price-based incentives during booms; and the opposite during recessions. Clawback types of incentive instruments may also moderate the impact of externalities in the economy. Simple restrictions on contractual terms such as lowering incentive pay in general may generate socially suboptimal outcomes when shareholders are acting optimally in setting the terms of the contracts.

References

Acharya, Viral and Tanju Yorulmazer, "Too many to fail: An analysis of time-inconsistency in bank closure policies," *Journal of Financial Intermediation*, 2007, *16* (1), 1–31.

_ **and** _ , "Cash in the market pricing and optimal resolution of bank failures," *Review of Financial Studies*, 2008, *21* (6), 2705–2742.

_ **, Marco Pagano, and Paolo Volpin**, "Seeking alpha: Excess risk taking and competition for managerial talent," 2011, New York University, University of Naples Federico II, London Business School and CEPR Working Paper.

Aghion, Philippe and Jeremy Stein, "Growth vs. margins: Destabilizing consequences of giving the stock market what it wants," *Journal of Finance*, 2008, *63*, 1025–1058.

Akerlof, George, "The economics of caste and of the rat race and other woeful tales," *Quarterly Journal of Economics*, 1976, *90* (4), 599–617.

Axelson, Ulf and Sandeep Baliga, "Liquidity and manipulation of executive compensation schemes," *Review of Financial Studies*, 2009, *22*, 3907–3939.

Bhattacharya, Sudipto and Dilip Mookherjee, "Portfolio choice in research and development," *Rand Journal of Economics*, 1986, *17*, 594–605.

Biais, Bruno R. and Catherine Casamatta, "Optimal leverage and aggregate investment," *Journal of Finance*, 1999, *54*, 1291–1323.

Bolton, Patrick, José Scheinkman, and We Xiong, "Executive compensation and short-termist behavior in speculative markets," *Review of Economic Studies*, 2006, *73*, 577–610.

Cheng, Ing-Haw, Harrison Hong, and Jose A. Scheinkman, "Yesterday's heros: Compensation and creative risk-taking," 2011, Princeton University Working Paper.

Cohen, Randy, Christopher Polk, and Tuomo Vuolteenaho, "The value spread," *Journal of Finance*, 2003, *58*, 609–641.

Da, Zhi and Ernst Schaumburg, "Relative valuation and analyst target price forecasts," *Journal of Financial Markets*, 2011, *14*, 161–192.

Diamond, Douglas W. and Robert E. Verrecchia, "Optimal managerial contracts and equilibrium security prices," *Journal of Finance*, 1982, *37.*

Diamond, Peter A., "Managerial incentives: On the near linearity of optimal compensation," *Journal of Political Economy*, 1998, *34*, 931–957.

Edmans, Alex, Xavier Gabaix, and Augustin Landier, "A multiplicative model of optimal CEO incentives in market equilibrium," *Review of Financial Studies*, 2009, *22*, 4881–4917.

Farhi, Emmanuel and Jean Tirole, "Collective moral hazard, maturity mismatch and systemic bailouts," *American Economic Review*, 2011, *forthcoming.*

Green, Jerry R. and Nancy L. Stokey, "A comparison of tournaments and contracts," *The Journal of Political Economy*, 1983, *91*, 349–364.

Holmström, Bengt, "Moral hazard and observability," *Bell Journal of Economics*, 1979, *10*, 74–91.

_ , "Moral hazard in teams," *Bell Journal of Economics*, 1982, *13* (2), 324–340.

_ , "Managerial incentive problems: A dynamic perspective," *Review of Economic Studies*, 1999, *66*, 169–182.

_ **and Jean Tirole**, "Market liquidity and performance monitoring," *Journal of Political Economy*, 1993, *101*, 678–709.

_ **and Joan Ricart I Costa**, "Managerial incentives and capital management," *Quarterly Journal of Economics*, 1986, *101*, 835–860.

Lazear, Edward P. and Sherwin Rosen, "Rank-order tournaments as optimum labor contracts," *The Journal of Political Economy*, 1981, *89*, 841–864.

Li, Jin and Stewart C. Myers, "R^2 around the world: New theory and new tests," *Journal of Financial Economics*, 2006, *79*, 257–292.

Makarov, Igor and Guillaume Plantin, "Rewarding trading skills without inducing gambling," 2010, London Business School Working Paper.

Morck, Randall, Bernard Yeung, and Wayne Yu, "The information content of stock markets: Why do emerging markets have synchronous stock price movements?" *Journal of Financial Economics*, 2000, *58*, 215–260.

Nalebuff, Barry J. and Joseph E. Stiglitz, "Prizes and incentives: Towards a general theory of compensation and competition," *The Bell Journal of Economics*, 1983, *14*, 21–34.

Palomino, Federic and Andrea Prat, "Risk taking and optimal contracts for money managers," *Rand Journal of Economics*, 2003, *34* (1), 113–137.

Peng, Lin and Alisa Röell, "Managerial incentives and stock price manipulation," 2011, Baruch College and Columbia University Working Paper.

Rajan, Raghuram, "Has financial development made the world riskier?" *Proceedings, Federal Reserve Bank of Kansas City*, 2005, pp. 313–369.

Appendix A

Proof of Proposition 1

Let

$$t=\frac{r}{\tau_h},\quad v=\frac{r}{\tau_\epsilon},\ p=\frac{r_s}{\tau_h},\ q=\frac{r_s}{\tau_\epsilon}.$$

First we use (9) to plug in for e_i in the shareholders' problem (34) and take the derivative of the objective function with respect to x_i to find the first-order condition as a function of $\bar{e}$. This first-order condition is sufficient for a maximum because the shareholders' problem is concave when τ_h is large. In equilibrium, $\bar{e}=x_i\bar{h}$. Therefore any equilibrium must solve for the first-order condition and $\bar{e}=x_i\bar{h}$. To find an equilibrium we plug $\bar{e}=x_i\bar{h}$ in the first order condition. After simplifying we find the equilibrium condition:

$$-\bar{h}^2\delta\frac{(1-x_i)\left(p\delta x_i-1\right)}{tx_i^2+1}-\delta\left(vx_i-q\delta\left(1-x_i\right)\right)=0.$$

The above function takes the value $\bar{h}^2\delta+\delta^2 q>0$ at $x_i=0$ and $-\delta v<0$ at $x_i=1$. Therefore the equilibrium condition has at least one solution. Next we show that there is exactly one solution if τ_h is large enough. Let's focus on the first part of the condition:

$$\Psi\left(x\right)=-\bar{h}^2\delta\frac{(1-x)\left(px\delta-1\right)}{tx^2+1},\tag{40}$$

where $\Psi\left(0\right)=\bar{h}^2\delta$ and Ψ becomes zero only at $x=1$ and $x=1/p\delta$.

Let's consider its derivative:

$$\Psi'\left(x\right)=\bar{h}^2\delta\frac{1}{\left(tx^2+1\right)^2}\left(\left(t+pt\delta\right)x^2-\left(2t-2p\delta\right)x-\left(p\delta+1\right)\right).$$

The derivative is negative at $x=0$. At $x=1$ its value is $\bar{h}^2\delta\frac{(p\delta-1)}{(t+1)}$. Since the numerator is a quadratic, Ψ' either stays negative for all $x\in[0,1]$ (this happens if $p\delta<1$) or crosses to positive once at some $x<1$ (this happens if $p\delta>1$.)

There are two cases:

Case 1: $p\delta<1$. In this case Ψ is declining for all $x\in[0,1]$. Thus $\Psi\left(x\right)-\delta\left(vx-q\delta\left(1-x\right)\right)$ is also declining for all $x\in[0,1]$. Since $\Psi\left(0\right)+\delta^2 q>0$ and $\Psi\left(1\right)-\delta v<0$, $\Psi\left(x\right)-\delta\left(vx-q\delta\left(1-x\right)\right)$ crosses zero only once between 0 and 1.

Case 2: $p\delta>1$. In this case Ψ is declining up to some $x'\in(1/p\delta,1)$ and increases after that and crosses zero from above at $1/\delta p$ and from below at 1. Suppose τ_h is large enough so that $\frac{1}{\delta p}>\frac{q\delta}{v+q\delta}$. Note that $\Psi\left(x\right)-\delta\left(vx-q\delta\left(1-x\right)\right)>\Psi\left(x\right)$ for all $x<q\delta/\left(v+\delta q\right)$

and $\Psi(x) - \delta(vx - q\delta(1-x)) < \Psi(x)$ for all $x > q\delta/(v+\delta q)$. Moreover when $\Psi(x)$ is declining $\Psi(x) - \delta(vx - q\delta(1-x))$ is also declining. These facts together imply that $\Psi(x) - \delta(vx - q\delta(1-x))$ crosses zero only once and at some $x < 1/\delta p$.

Proof of Proposition 2

Proof follows from plugging x^{SB} in the equilibrium condition (13) and checking whether its value is positive (in which case $x^{SB} < x^{PO}$) or negative (in which case $x^{SB} > x^{PO}$.)

The next lemma is useful in proving the comparative statics results:

Lemma 1: Suppose there is a unique equilibrium. (i) $\Psi(x) - \delta(vx - q\delta(1-x))$ crosses zero from above at x_i^{PO}. (ii) If $1/\delta p > q\delta/(v+\delta q)$ then $x_i^{PO} < 1/\delta p$, otherwise $x_i^{PO} > 1/\delta p$.

Proof of Lemma 1

Part (i) follows from $\Psi(0) + q\delta^2 > 0$, $\Psi(1) - \delta vx < 0$ and uniqueness. The proof of (ii) is immediate if $1/\delta p > 1$. Otherwise, $\Psi(x)$ (defined in (40)) crosses zero at $1/\delta p < 1$. $\Psi(x) - \delta(vx - q\delta(1-x))$ is above $\Psi(x)$ for $x < q\delta/(v+\delta q)$ and is below $\Psi(x)$ for $x > q\delta/(v+\delta q)$. This and the fact that there is a unique equilibrium x_i^{PO} prove part (ii).

Proof of Proposition 3

Equilibrium x_i^{PO} solves

$$\Psi\left(x_i^{PO}\right) - \delta\left(vx_i^{PO} - q\delta\left(1 - x_i^{PO}\right)\right) = 0$$

where Ψ is given in (40). We write $\Psi\left(x_i^{PO}\left(\bar{h}\right), \bar{h}\right)$ to make the dependence of Ψ and x_i^{PO} on $\bar{h}$ explicit. (We use similar notation for other parameters, e.g. $\Psi\left(x_i^{PO}(\tau_h), \tau_h\right)$.)

Taking the total derivative of the equilibrium condition with respect to $\bar{h}$ we obtain

$$\frac{\partial x_i^{PO}\left(\bar{h}\right)}{\partial \bar{h}} = \frac{-\frac{\partial \Psi}{\partial \bar{h}}}{\frac{\partial \Psi\left(x_i^{PO}\left(\bar{h}\right), \bar{h}\right)}{\partial x} - \delta(v + q\delta)}.$$

Denominator is negative by Lemma 1 (i). By Lemma 1 (ii),

$$\frac{\partial \Psi\left(x_i^{PO}, \bar{h}\right)}{\partial \bar{h}} = -2\bar{h}\delta \frac{\left(1 - x_i^{PO}\right)\left(px_i^{PO}\delta - 1\right)}{t\left(x_i^{PO}\right)^2 + 1} \gtrless 0$$

if $1/\delta p \gtrless q\delta/(v+\delta q)$ which proves part (i) for $\bar{h}$. Proof for the result on τ_ϵ is entirely analogous. Proof of part (ii) follows directly from taking the derivative of x^{SB} with respect to $\bar{h}$ and τ_ϵ.

Proof of Proposition 4

Taking the total derivative of the equilibrium condition with respect to τ_h, we obtain

$$\frac{\partial x_i^{PO}(\tau_h)}{\partial \tau_h} = \frac{-\frac{\partial \Psi}{\partial \tau_h}}{\frac{\partial \Psi\left(x_i^{PO}(\tau_h),\tau_h\right)}{\partial x} - \delta\left(v + q\delta\right)}.$$

Denominator is negative by Lemma 1 (i). Moreover,

$$\frac{\partial \Psi\left(x_i^{PO}, \tau_h\right)}{\partial \tau_h} = \bar{h}^2 \delta\left(1 - x_i^{PO}\right)\frac{\left(\frac{r_s}{(\tau_h)^2} x_i^{PO}\delta\right) + \left(\frac{r}{(\tau_h)^2}\left(x_i^{PO}\right)^2\right)}{\left(\frac{r}{\tau_h}\left(x_i^{PO}\right)^2 + 1\right)^2} > 0.$$

Thus x_i^{PO} is increasing in τ_h. From the definition of x_i^{SB} we see immediately that x_i^{SB} is also strictly increasing in τ_h.

Proof of Proposition 5

Taking the total derivative of the equilibrium condition with respect to r, we obtain:

$$\frac{\partial x_i^{PO}(r)}{\partial r} = \frac{-\frac{\partial \Psi}{\partial r} + \delta \frac{1}{\tau_\epsilon} x_i^{PO}}{\frac{\partial \Psi\left(x_i^{PO}(r), r\right)}{\partial x} - \delta\left(v + q\delta\right)}.$$

Denominator is negative by Lemma 1 (i). By Lemma 1 (ii), if $1/\delta p > q\delta/\left(v + \delta q\right)$ then

$$\frac{\partial \Psi\left(x_i^{PO}, r\right)}{\partial r} - \delta \frac{1}{\tau_\epsilon} x_i^{PO} = 2\bar{h}^2 \delta \frac{\left(1 - x_i^{PO}\right)\left(p x_i^{PO}\delta - 1\right)}{\left(t\left(x_i^{PO}\right)^2 + 1\right)^2} t x - \delta \frac{1}{\tau_\epsilon} x_i^{PO} < 0.$$

Thus, $\frac{\partial x_i^{PO}(r)}{\partial r} < 0$ if $1/\delta p > q\delta/\left(v + \delta q\right)$.

Proof of Proposition 6

Taking the total derivative of the equilibrium condition with respect to r_s we obtain:

$$\frac{\partial x_i^{PO}(r_s)}{\partial r_s} = \frac{-\frac{\partial \Psi}{\partial r_s} - \delta^2 \frac{1}{\tau_\epsilon}\left(1 - x_i^{PO}\right)}{\frac{\partial \Psi\left(x_i^{PO}(r), r\right)}{\partial x} - \delta\left(v + q\delta\right)}.$$

Note,

$$\frac{\partial \Psi\left(x_i^{PO}, r_s\right)}{\partial r_s} + \delta^2 \frac{1}{\tau_\epsilon}\left(1 - x_i^{PO}\right) = (1 - x)\left(-\bar{h}^2 \delta \frac{\frac{1}{\tau_h} x_i^{PO}\delta}{t\left(x_i^{PO}\right)^2 + 1} + \delta^2 \frac{1}{\tau_\epsilon}\right)$$

and the above expression is strictly positive for τ_h sufficiently large.

Proof of Proposition 7

Note that $e_i^{PO} > e_i^{FB}$ if and only if

$$\frac{x^{PO}\bar{h} + t\left(x^{PO}\right)^2 \bar{e}}{1 + t\left(x^{PO}\right)^2} > \frac{\bar{h}}{1 + \delta p\left(\frac{t}{t+\delta p}\right)}.$$

Since $\bar{e} = x^{PO}\bar{h}$ we can rewrite the condition as

$$\frac{1}{t + p\delta}\left(t\left(x^{PO}\right)^2 + 1\right)\left(\left(t + p\delta + p\delta t\right)x^{PO} - p\delta - t\right) > 0.$$

Thus $e_i^{PO} > e_i^{FB}$ if and only if

$$x^{PO} > \overline{x}^{FO} = \frac{p\delta + t}{t + p\delta + pt\delta}.$$

Plugging $\overline{x}^{FO}$ in (13) we see that $x^{PO} > \overline{x}^{FO}$ if and only if

$$\bar{h}^2 pt\delta^2 \frac{-p^2\delta^2 + p\delta + t}{p^2t^2\delta^2 + 3p^2t\delta^2 + p^2\delta^2 + 4pt^2\delta + 2pt\delta + t^3 + t^2} - \delta\left(v\frac{p\delta + t}{t + p\delta + pt\delta} - q\delta\left(1 - \frac{p\delta + t}{t + p\delta + pt\delta}\right)\right).$$

A sufficient condition to satisfy the above condition is that $\bar{h}$ is large enough and

$$p\delta + t > p^2\delta^2.$$

This inequality would hold as long as r_s is small enough.

Similarly, $e_i^{PO} > e_i^{NH}$ if and only if

$$\frac{x^{PO}\bar{h} + t\left(x^{PO}\right)^2 \bar{e}}{1 + t\left(x^{PO}\right)^2} > \frac{\bar{h}}{1 + \delta p}.$$

Once again, since $\bar{e} = x^{PO}\bar{h}$ we can rewrite the condition as

$$\left(t\left(x^{PO}\right)^2 + 1\right)\left(x^{PO}\left(1 + p\delta\right) - 1\right) > 0.$$

Thus $e_i^{PO} > e_i^{NH}$ if and only if

$$x^{PO} > \overline{x}^{NH} = \frac{1}{1 + p\delta}.$$

Plugging $\overline{x}^{NH}$ in (13) we see that $x^{PO} > \overline{x}^{NH}$ if and only if

$$\bar{h}^2 p\frac{\delta^2}{p^2\delta^2 + 2p\delta + t + 1} - \delta\left(v\frac{1}{1 + p\delta} - q\delta\left(1 - \frac{1}{1 + p\delta}\right)\right) > 0.$$

A sufficient condition to satisfy the above condition is that $\bar{h}$ is large enough.

Proof of Proposition 8:

For expositional clarity, we will drop the subscript i. So shareholders' objective can be written as

$$\left(\delta\bar{h}e_i - \frac{1}{2}\delta e_i^2 - \frac{1}{2}\delta\left((ze_i - x\bar{e})^2 t + z^2 v\right)\right) - \frac{1}{2}\delta^2\left(\left(1-z\right)e_i + x\bar{e}\right)^2 p + \left(1-z\right)^2 q. \quad (41)$$

and the first order condition yields the optimal level of effort as a function of contract terms and the average effort

$$e_i = \frac{z\bar{h} + tzx\bar{e}}{1+tz^2}.$$

Next substituting for the effort level in the objective function of (41) we obtain

$$\begin{aligned}&\left(\delta\bar{h}\left(\frac{z\bar{h}+tzx\bar{e}}{1+tz^2}\right) - \frac{1}{2}\delta\left(\frac{z\bar{h}+tzx\bar{e}}{1+tz^2}\right)^2 - \frac{1}{2}\delta\left(\left(z\left(\frac{z\bar{h}+tzx\bar{e}}{1+tz^2}\right) - x\bar{e}\right)^2 t + z^2 v\right)\right)\\ &-\frac{1}{2}\delta^2\left(\left(\left(1-z\right)\left(\frac{z\bar{h}+tzx\bar{e}}{1+tz^2}\right) + x\bar{e}\right)^2 p + \left(1-z\right)^2 q\right). \quad (42)\end{aligned}$$

For a given z the above function is negative quadratic in x. Thus for a given z shareholders' objective function is maximized at x_z which is given by

$$x_z = \frac{\bar{h}z\left(t\left(tz^2+1\right) - p\delta\left(1-z\right)\left(tz+1\right)\right)}{\bar{e}\left(t\left(tz^2+1\right) + p\delta\left(tz+1\right)^2\right)}. \quad (43)$$

We need to check that $x_z \geq 0$ for the optimal z. If x_z is negative, we have a corner solution and the optimal value of x is obtained at a corner where $x_z = 0$. Substituting (43) for x_z in (42) we reduce shareholders' problem to choosing z to maximize

$$\frac{1}{2}\bar{h}^2 z\delta\left(t+p\delta\right)\frac{z\left(t-1\right)+2}{t+p\delta+t^2z^2+pt^2z^2\delta+2ptz\delta} - \frac{1}{2}\delta z^2 v - \frac{1}{2}\delta^2\left(1-z\right)^2 q.$$

Taking the derivative with respect to z, we obtain

$$\frac{-\bar{h}^2\delta\left(tz+1\right)\left(t+p\delta\right)^2\left(z-1\right)}{\left(t+p\delta+t^2z^2+pt^2z^2\delta+2ptz\delta\right)^2} - \delta zv - \delta^2\left(z-1\right)q.$$

Note that the above function starts as positive and crosses to negative once. Thus the objective function is maximized at z^* that solves

$$H\left(z\right) = -\bar{h}^2\delta\left(tz+1\right)\left(t+p\delta\right)^2\left(z-1\right) - \left(t+p\delta+t^2z^2\left(p\delta+1\right)+2ptz\delta\right)^2\left(\delta zv + \delta^2\left(z-1\right)q\right) = 0.$$

Note that $z^* \in (0,1)$. In a symmetric equilibrium

$$\bar{e} = \frac{z^*\bar{h} + tz^* x_{z^*}\bar{e}}{1+t\left(z^*\right)^2}$$

Plugging for x_{z^*}, we obtain

$$\bar{e} = \frac{\bar{h}z^*\left(tz^*+1\right)\left(t+p\delta\right)}{t\left(t\left(z^*\right)^2+1\right)+p\delta\left(tz^*+1\right)^2}.$$

Using the above to substitute for $\bar{e}$ in (43), we obtain

$$x_z = \frac{t\left(1+t\left(z^*\right)^2\right)+p\delta\left(z^*-1\right)\left(tz^*+1\right)}{\left(tz^*+1\right)\left(t+p\delta\right)}.$$

Next we show that $x_{z^*} \in (0, z^*)$. First, note that

$$x_{z^*} < z^* \Leftrightarrow p\delta\left(1+tz^*\right) > t\left(1-z^*\right),$$

or if and only if

$$z^* > \left(\frac{t-p\delta}{t\left(1+p\delta\right)}\right).$$

Note

$$H\left(\frac{t-p\delta}{t\left(1+p\delta\right)}\right) = \frac{1}{t}\frac{\delta}{\left(p\delta+1\right)^3}\left(t+p\delta\right)^2\left(t+1\right)^2\left(pv\delta-tv+h^2p^2\delta^2+h^2p\delta+pq\delta^2+pqt\delta^2\right).$$

Thus $x_{z^*} < z^*$ if and only if

$$\left(h^2\frac{r_s}{\tau_h}\delta\left(\frac{r_s}{\tau_h}\delta+1\right)+\frac{r_s}{\tau_\epsilon}\delta\frac{r_s}{\tau_h}\delta\left(\frac{r}{\tau_h}+1\right)\right) > \frac{r}{\tau_\epsilon}\left(\frac{r}{\tau_h}-\frac{r_s}{\tau_h}\delta\right).$$

Substituting for t, v, p and q we obtain:

$$h^2\frac{r_s}{\tau_h}\delta\left(\frac{r_s}{\tau_h}\delta+1\right)+\frac{r_s}{\tau_\epsilon}\delta\frac{r_s}{\tau_h}\delta\left(\frac{r}{\tau_h}+1\right)+\frac{r}{\tau_\epsilon}\frac{r_s}{\tau_h}\delta-\frac{r}{\tau_\epsilon}\frac{r}{\tau_h} > 0$$

or

$$\left(\tau_h\delta^2+r\delta^2+\tau_\epsilon h^2\delta^2\right)r_s^2+\left(\tau_h\tau_\epsilon\delta h^2+\tau_h r\delta\right)r_s-\tau_h r^2 > 0.$$

Thus $x_{z^*} < z^*$ if and only if

$$r_s > \frac{1}{2\delta\left(\tau_h+r+\tau_\epsilon h^2\right)}\left(\sqrt{4\tau_h r^3+5\tau_h^2r^2+1\tau_h^2\tau_\epsilon^2h^4+4\tau_h\tau_\epsilon h^2r^2+2\tau_h^2\tau_\epsilon h^2r}-\tau_h\left(r+\tau_\epsilon h^2\right)\right).$$

Note that the right side approaches 0 as h increases and thus the condition becomes easier to satisfy.

Next we show $x_{z^*} > 0$. Note that

$$x_{z^*} > 0 \Leftrightarrow t\left(t\left(z^*\right)^2+1\right)-\left(p\delta\right)\left(-t\left(z^*\right)^2+\left(t-1\right)z^*+1\right) > 0.$$

Using $H(z^*) = 0$, we can write the above condition as

$$t\left(t\left(z^*\right)^2+1\right)+\frac{\left(t+p\delta+t^2\left(z^*\right)^2\left(p\delta+1\right)+2ptz^*\delta\right)^2\left(\delta z^* v+\delta^2\left(z^*-1\right)q\right)}{\delta\left(t+p\delta\right)^2}>0.$$

Observe that

$$\delta z^* v+\delta^2\left(z^*-1\right)q>0 \Leftrightarrow z^*>\frac{q\delta}{v+q\delta}.$$

Since $H\left(\frac{q\delta}{v+q\delta}\right)>0$, indeed $z^*>q\delta/\left(v+q\delta\right)$, which proves that $x_{z^*}>0$. Finally given z^* and x_{z^*} we can solve for a_i^* and b_i^* by solving:

$$b^* = z^* - x_{z^*}$$

and

$$a_i^* = \frac{x_{z^*}}{1-z^*}.$$

Appendix B

Relative and Absolute Information:

In this appendix we consider the case where speculators receive a signal about the absolute valuation of the firm in addition to the signal about the relative valuation. Suppose that the two signals that speculators receive about firm i are:

$$s_i = V_i - \bar{V} = \tilde{h}\left(e_i - \bar{e}\right) + \tilde{\epsilon}_i$$

and

$$t_i = \left(\tilde{h} + \tilde{\zeta}\right) e_i$$

where $\tilde{\zeta}$ is normally distributed with mean zero and precision τ_ζ. The noise $\tilde{\zeta}$ in the signal t_i reflects the difficulty in assessing the industry-wide productivity. As before all noise terms are standard normal and payoffs, timing of the moves and the equilibrium definition are exactly as in the main model. We focus on symmetric equilibrium where all managers put the same level of effort in equilibrium. As a result from speculators' perspective, in equilibrium, the two signals are

$$s_i = \tilde{\epsilon}_i \text{ and } t_i = \left(\tilde{h} + \tilde{\zeta}\right) \bar{e}.$$

Notice that in equilibrium the absolute signal is the same for all firms (and the relative signal is completely idiosyncratic), thus speculators cannot learn about firm i's final value from signals they have about the other firms in the industry. Moreover, the absolute signal does not fully reveal the industry risk $\tilde{h}$ because of the additional noise $\tilde{\zeta}$.

From speculators' perspective, the expected value of firm i in equilibrium is given by

$$\begin{aligned} E\left(V_i^* \left| s_i, t_i\right.\right) &= E\left(\tilde{h}\bar{e} + \tilde{\epsilon}_i \middle| s_i = \tilde{\epsilon}_i, t_i = \left(\tilde{h} + \tilde{\zeta}\right)\bar{e}\right) \\ &= (1-\alpha)\,\overline{h}\bar{e} + \alpha t_i + s_i \end{aligned}$$

where

$$\alpha = \frac{\tau_\zeta}{\tau_h + \tau_\zeta}.$$

We can the compute the price as:

$$P_i = \frac{1}{1 + a_i^*}\left((1-\alpha)\,\overline{h}\bar{e} + \alpha t_i + s_i\right) - \frac{1}{1 + a_i^*} W_i^*.$$

Given the equilibrium price we can write manager i's compensation as:

$$I_i = a_i P_i + W_i = x_i \left((1-\alpha)\, \overline{h}\bar{e} + \alpha t_i + s_i \right) - x_i W_i^* + W_i \tag{44}$$

where $x_i = a_i/(1+a_i^*)$. Therefore, given a contract (x_i, W_i) and average effort $\bar{e}$, manager i chooses e_i to maximize

$$E\left(u\left(x_i \left((1-\alpha)\, \overline{h}\bar{e} + \alpha t_i + s_i \right) - x_i W_i^* + W_i - C(e_i) \right) \right).$$

Plugging $s_i = \tilde{h}\,(e_i - \bar{e}) + \tilde{\epsilon}_i$, and $t_i = (\tilde{h} + \tilde{\zeta}) e_i$, and computing expectations, manager i's problem can be restated as choosing e_i to maximize

$$-\alpha x_i \overline{h}\bar{e} + (1+\alpha) x_i \overline{h} e_i - x_i W_i^* + W_i - C(e_i) - \frac{1}{2} r \left((\alpha x_i e_i + x_i (e_i - \bar{e}))^2 \frac{1}{\tau_h} + (\alpha x_i e_i)^2 \frac{1}{\tau_\zeta} + x_i^2 \frac{1}{\tau_\epsilon} \right)$$

Taking the first order condition, we find the optimal effort as

$$e_i = \frac{(1+\alpha)(x_i \overline{h} + \frac{r}{\tau_h} x_i^2 \bar{e})}{1 + \left(\frac{r}{\tau_\zeta}\alpha^2 + \frac{r}{\tau_h}(1+\alpha)^2 \right) x_i^2}. \tag{45}$$

Note that there are complementarities in managers' efforts in this model just as in the main model. However, the strength of complementarities depends negatively on the weight speculators put on the absolute signal, α, if the noise in the absolute signal is very large, that is $1/\tau_\zeta$ is large.

Since shareholders of firm i hold δ share of the claim $(V_i - I_i)$, substituting for I_i, we obtain shareholders' payoff at $t = 2$:

$$\delta(V_i - I_i) = \delta \left(-x_i \left((1-\alpha)\, \overline{h}\bar{e} + s_i + \alpha t_i \right) + \left(\tilde{h} e_i + \tilde{\epsilon}_i \right) + x_i W_i^* - W_i \right).$$

Plugging $s_i = \tilde{h}\,(e_i - \bar{e}) + \tilde{\epsilon}_i$, and $t_i = (\tilde{h} + \tilde{\zeta}) e_i$, and computing expectations, shareholders' problem can be stated as choosing x_i to maximize

$$\begin{aligned} &\delta \left(\overline{h} e_i - C(e_i) - \bar{I} - \frac{1}{2} r \left((\alpha x_i e_i + x_i (e_i - \bar{e}))^2 \frac{1}{\tau_h} + (\alpha x_i e_i)^2 \frac{1}{\tau_\zeta} + x_i^2 \frac{1}{\tau_\epsilon} \right) \right) \\ &-\frac{1}{2} r_s \delta^2 \left((x_i \bar{e} + (1 - \alpha x_i - x_i) e_i)^2 \frac{1}{\tau_h} + (\alpha x_i e_i)^2 \frac{1}{\tau_\zeta} + (1 - x_i)^2 \frac{1}{\tau_\epsilon} \right) \end{aligned} \tag{46}$$

subject to (45).

As before, the second-best solution would take into account that $e_i = \bar{e}$. Therefore in the second-best x maximizes

$$\begin{aligned} &\delta \left(\overline{h}\bar{e} - C(\bar{e}) - \bar{I} - \frac{1}{2} r \left((\alpha x \bar{e})^2 \frac{1}{\tau_h} + (\alpha x \bar{e})^2 \frac{1}{\tau_\zeta} + x^2 \frac{1}{\tau_\epsilon} \right) \right) \\ &-\frac{1}{2} r_s \delta^2 \left(((1 - \alpha x)\, \bar{e})^2 \frac{1}{\tau_h} + (\alpha x \bar{e})^2 \frac{1}{\tau_\zeta} + (1 - x)^2 \frac{1}{\tau_\epsilon} \right) \end{aligned} \tag{47}$$

subject to

$$\bar{e} = \frac{(1+\alpha)x\bar{h}}{1 + \left(\frac{r}{\tau_\zeta}\alpha^2 + \frac{r}{\tau_h}\alpha(1+\alpha)\right)x^2}.$$

Contrasting shareholders' problem in (46) with the second-best in (47), we see that the equilibrium and the second best would induce different incentives and effort levels. The case that we studied in the main model corresponds to the limit where τ_ζ approaches zero.