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Euler Equation Estimation on Micro Data*

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Abstract

First order conditions from the dynamic optimization problems of consumers and firms are important tools in empirical macroeconomics. When estimated on micro-data these equations are typically linearized so standard IV or GMM methods can be employed to deal with the measurement error that is endemic to survey data. However, it has recently been argued that the approximation bias induced by linearization may be worse than the problems that linearization is intended to solve. This paper explores this issue in the context of consumption Euler equations. These equations form the basis of estimates of key macroeconomic parameters: the elasticity of inter-temporal substitution (EIS) and relative prudence. We numerically solve and simulate 6 different life-cycle models, and then use the simulated data as the basis for a series of Monte Carlo experiments in which we consider the validity and relevance of conventional instruments, the consequences of different data sampling schemes, and the effectiveness of alternative estimation strategies. The first-order Euler equation leads to biased estimates of the EIS, but that bias is perhaps not too large when there is a sufficient time dimension to the data, and sufficient variation in interest rates. A sufficient time dimension can only realistically be achieved with a synthetic cohort. Estimates are unlikely to be very precise. Bias will be worse the more impatient agents are. The second order Euler equation suffers from a weak instrument problem and offers no advantage over the first-order approximation.

JEL Classifications: E21, C20

Keywords: Euler Equations, Measurement Error, Instrumental Variables, GMM.

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1 Introduction

Models based on dynamic optimization problems faced by economic agents have long been the work horses of macroeconomics. Since Hall (1978), first order conditions (known as Euler Equations) from these optimization problems have been used extensively to test these models against economic data and as a basis for estimating preference and technology parameters. This tradition includes: tests and estimates of the life-cycle models of consumption (see for example, Attanasio et. al. 1999); studies of the investment behaviour of firms (for example, Bond and Meghir 1994, or Mulligan, 2004); and tests of asset pricing theories (for example, Mehra and Prescott, 1985).

The main attraction of the Euler equation approach is that it allows researchers to estimate preference parameters with limited data and without fully specifying the stochastic processes that agents face. In principle, researchers do not need to model agents' expectations or (in the case of consumers) observe their wealth when estimating preference and technology parameters. Unfortunately these advantages of the Euler Equation approach are significantly diminished by practical problems that arise from the nature of the available data.

In this paper, we evaluate the econometric problems that arise in the estimation of Euler equations on micro data, focusing in particular on the consumption Euler equation. The key parameters that researcher recover from the consumption Euler equation are the elasticity of of inter-temporal substitution (EIS) and the degree of relative prudence. Estimated values of these parameters are central to our thinking about macroeconomic policies such as the UK's temporary VAT cut during the crisis of 2008-9 (Crossley et al., 2009). We numerically solve and simulate 6 different life-cycle models, and then use the simulated data as the basis for a series of Monte Carlo experiments in which we consider the validity and relevance of conventional instruments, the consequences of different data sampling schemes, and the effectiveness of alternative estimation strategies.

Euler equations represent the behavior of an individual agent (households and firms). There is substantial evidence that estimating these models with aggregate data can lead not only to biased parameter estimates but also to false rejections of the underlying models (see for example, Attanasio and Weber, 1993). As a result, researchers have moved towards using household/firm level data instead of aggregate time series data. This solution generates a further problem because measurement error is endemic to such survey data. This measurement error interacts with the nonlinear structure of the Euler equation: in the presence of measurement error, standard non-linear GMM methods yield inconsistent estimates (Amemiya, 1985). This problem is in turn normally addressed by taking a first-

, or possibly second-order approximation to the Euler equation (to give a linearized or approximate Euler equation) and then using standard linear IV and GMM techniques to deal with the measurement error.

However, it may be that this particular cure is worse than the disease. Higher order terms that are ignored in the approximation are potentially correlated with the typical instruments (lagged variables) used in these estimation procedures, and this can lead to substantial bias. Consequently, the usefulness of approximate Euler Equations is now in question. Several papers have explored this issue in the context of the consumption Euler equation. The common approach of these studies is that they solve and simulate a life-cycle consumption model and then perform Monte-Carlo experiments with the simulated data to investigate whether a linearized Euler equation yields good estimates of the preference parameters (that is, they investigate whether it is possible to recover the preference parameters values that were assumed in solving the model and generating the simulated data). Ludvigson and Paxson (2001) investigate the estimation of the relative prudence parameter in an environment with a fixed interest rate and impatient agents. Following the empirical strategy of Dynan (1993), they employ a second-order approximation to the Euler equation as the basis for estimation. They conclude that this strategy for estimating the prudence parameter is not useful because instruments typically employed are correlated with the approximation error. Carroll (2001) reaches the same conclusion for the elasticity of inter-temporal substitution (EIS) in an environment with cross-sectional variation in interest rates. A common feature of these studies is the lack of time series variation in interest rates. Attanasio and Low (2004) argue that with sufficiently long sample periods and enough time-series variation in the inter-temporal price (interest rate), good estimates of the EIS can be obtained with linearized Euler equations.[1]

We go beyond the previous literature in a number of key respects. First, we consider a range of different economic models and a range of different data structures (for example, true panels and synthetic panels constructed at the birth cohorts level). Second, we add realistic measurement error to our simulated data. Third, unlike previous studies, we examine the validity of usual instruments directly (as well as documenting the distribution of resulting estimates). As we know the true parameters values underlying simulated data, we can construct the true residuals to the linearized Euler equation, and hence test instrument validity even in cases when the model is just identified. Fourth, we assess

[1] There are alternatives. Alan, Attanasio and Browning (2009) introduce two new nonlinear GMM estimators that deal with measurement error in particular circumstances. Alan and Browning (2010) propose an approach that is not based on Euler equations but rather on modeling expectation errors directly.

not only instrument validity but also instrument relevance.[2] The issue of instrument relevance (whether instruments are weak) has been studied for consumption Euler equations estimated on aggregate data (Yogo, 2004), but not, to our knowledge, in the case of approximate Euler equations estimated on micro data. Fifth, we propose a useful way of predicting when estimation based linearized Euler equations is likely to be effective. Finally, we consider the consequences of parameter heterogeneity for linearized Euler Equation Estimation.

We proceed as follows. We study variants of a standard life cycle model in which consumers have time-separable (CRRA) preferences and face (aggregate) interest rate uncertainty and uninsurable idiosyncratic income risk. We first solve numerically for policy functions in 6 different models (that is, 6 different choices with respect to parameter values and constraints), which cover and extend the range of different economic environments studied in the literature. We then simulate these models to generate populations of agents that are ex ante identical but ex post heterogeneous.

Next, we propose simple non-parametric statistic to summarize the key characteristic of these different environments. The average derivative of log consumption with respect to cash-on-hand weights the semi-elasticity of the policy function (consumption function) with respect to cash-on-hand by the agents' ex-post cash-on-hand (asset) distribution. While the consumption function varies with different values of preference and income process parameters, it will typically exhibit the same general shape: steep at low wealth levels and much flatter (and near linear) at high wealth levels (where agents are largely self insured). As we shall elaborate below, for a given risk aversion, the approximation bias in a linearized Euler equation depends on the variance of consumption growth. This depends primarily on the sensitivity of log consumption to shocks to cash-on-hand, which is controlled by the semi-elasticity of the consumption function. Thus the key consequence of different parameter values (especially discount rates) is not so much the shape of the resulting policy function but rather which part of the policy function is ex-post relevant for agents. The average semi-elasticity measure integrates over the ex-post distribution of the key state variable (cash-on-hand) and so captures both the semi-elasticity of the policy function per se, and the ex-post relevance of different parts of the state space (and hence different parts of the policy function). We use this measure to predict models for which linearized Euler Equations are likely to be a poor basis for estimation.

[2] Following common practice, we use "valid" to refer to an instrument that is uncorrelated with the residual in the equation of interest (and "invalid" to describe an instrument that is correlated with the residual.) We use "relevant" to describe an instrument that is strongly correlated with the regressor whose correlation with the residual motivates the use of and instrumental variable or GMM estimator. An instrument that is not strongly correlated with the endogenous regressor is "weak" , or not relevant. See Murray (2006) for a survey of potential problems with instruments.

We then conduct Monte Carlo experiments in which we sample from the simulated populations according to the different data structures facing researchers. We consider true panels of different lengths, as well as the construction of synthetic cohorts that follow birth cohorts through repeated cross-sectional surveys, such as the U.S. Consumer Expenditure Survey. Synthetic cohorts typically afford a longer time dimension than available panels, and this data structure has been used to estimate Euler Equations (see for example, Blundell, Browning and Meghir, 1994; Attanasio and Weber, 1995; Attanasio et. al. 1999).

In each Monte Carlo experiment, we test the validity and relevance of the instruments typically used in the estimation. We then consider the resulting estimates of the key economic parameters (the elasticity of inter-temporal substitution and the degree of relative prudence). We consider both first- and second-order approximations to the exact Euler Equation. In experiments in which instrument relevance appears to be a concern, we consider limited information maximum likelihood estimation as an alternative to Linear GMM estimation.

In all the models we studied, variables that are orthogonal to the innovation in marginal utility (and so valid instruments for the exact Euler equation) are not valid instruments when a linearized Euler Equation is employed. The lagged interest rate is the only exception. Nevertheless, we find that, across a range of quite different models, the linearized Euler Equation is modestly successful in recovering the EIS when a long panel is available. For example, with a first-order approximation to the Euler equation, a long panel, the standard instrument set, and realistic measurement error, mean bias ranges between 3 and 16% of the true value of the parameter.

Additional findings, however, suggest this fairly positive result is of limited importance. First, with impatient models, even with a long panel, the estimates are quite imprecise. For example, when the true value of the EIS is 0.25, confidence intervals often include both 0 and 0.5. Second, when we move to more realistic data structures, the performance of the linearized Euler Equation deteriorates significantly. With a panel length corresponding to the most frequently employed PSID data, we sometimes find mean bias that is a twice as large as the true parameter. We also show that the 2nd order approximation to the Euler equation does not provide a superior basis for estimation, and in fact suffers from a weak instrument problem: the standard instruments do not have useful predictive power for the second order term.

Although our experiments with synthetic cohorts (which are a realistic route a long time dimension) do confirm that they result in less mean bias than a short panel, the same experiments also reveal a significant loss of precision relative to a panel of the same length. We do find some evidence that synthetic cohorts out-preform true panels in the presence of significant parameter heterogeneity.

For a given data structure, we do find that the effective semi-elasticity predicts the performance of linearized Euler equation estimation across models (albeit, not perfectly).

The remainder of the paper is organized as follows. Section 2 reviews the problems associated with linearized Euler equation estimation. Section 3 introduces the six different life-cycle models we study. Section 4 describes the design of our Monte Carlo studies, and in particular, the different data structures we consider. Section 5 develops our summary measure of the effective semi-elasticity of a consumption function. Section 6 presents the results of our Monte Carlo experiments. Section 7 provides a concluding discussion.

2 The Econometrics of Euler Equation Approximation

Consider a standard life cycle model in which the consumer consumes a single good, has time-separable preferences and holds long and possibly short positions of a single asset. The first order condition from this problem is[3]

$$U^{'}(C_t) = \beta E_t[1 + R_{t+1} U^{'}(C_{t+1})] \tag{1}$$

where $U^{'}$ is the marginal utility of consumption, β is the discount factor and R_{t+1} is the real rate between periods t and $t+1$. A widely used functional form for the sub-utility function is the iso-elastic form:

$$U(C_t) = \frac{C_t^{(1-\gamma)}}{1-\gamma} \tag{2}$$

where the parameter γ is the coefficient of relative risk aversion. Interest usually centers on the reciprocal of this parameter, $(1/\gamma)$, the elasticity of inter-temporal substitution (EIS), or on the coefficient of relative prudence, which is $\frac{\gamma+1}{2}$.

Substituting this utility function into equation (1) yields an exact Euler equation:

$$\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} (1 + R_{t+1})\beta = \varepsilon_{t+1} \tag{3}$$

[3]Of course, if the agent has access to several assets, and she is not at a corner, one can derive a similar condition for each of these assets.

with $E_t(\varepsilon_{t+1}) = 1$ where ε_{t+1} represents the expectation error (the innovation in discounted marginal utility), which the theory implies is orthogonal to variables in the information set at time t. This relationship is the the basis of very many estimates of the preference parameters (β, γ) and tests for the validity of the orthogonality conditions implied by the theory. GMM estimation is based on the orthogonality of the error term to all variables dated "t" or before, such as lagged consumption, interest rate and income variables. As originally emphasized by Hall (1978), this is a very attractive procedure since one can estimate the preference parameters without explicitly specifying the stochastic environment that agents face.

Nonlinear GMM estimation on micro data is inconsistent if the consumption data are measured with error. For example, if we allow for a multiplicative measurement error so that observed consumption is given by:

$$C_t^0 = C_t \eta_t \tag{4}$$

then the exact Euler equation for observable consumption becomes

$$\left(\frac{C_{t+1}^0}{C_t^0}\right)^{-\gamma} (1 + R_{t+1})\beta = \left(\frac{\eta_{t+1}}{\eta_t}\right)^{-\gamma} \varepsilon_{t+1} \tag{5}$$

The problem is that the composite error term does not have a conditional expectation of unity, even if we assume that ϵ_t and η_t are independent[4]

$$E_t \left[\left(\frac{\eta_{t+1}}{\eta_t}\right)^{-\gamma} \varepsilon_{t+1} \right] = E_t \left(\frac{\eta_{t+1}}{\eta_t}\right)^{-\gamma} E_t(\varepsilon_{t+1}) = E_t \left(\frac{\eta_{t+1}}{\eta_t}\right)^{-\gamma} \neq 1 \tag{6}$$

It is now widely accepted that household level consumption data is likely to be very noisy. For example, Runkle (1991) estimates 76% of the variation in the growth rate of food consumption in the PSID is due to measurement error; Shapiro (1984) arrives at an even higher estimate of 92% noise. Using a procedure that allows for preference heterogeneity, Alan and Browning (2010) obtain an estimate of 86%. Dynan (1993) reports the standard deviation of changes in log consumption in the CEX (American Consumer Expenditure Survey) is 0.2, which seems too large for "true" variations. The other widely used data sources are quasi-panels, constructed from cross-section expenditure survey information by taking within-period means following the birth cohorts through time (e.g. taking means over all the 25 year-olds in one year and all the 26 year-olds in the next year). Although this averaging reduces the effect of measurement error, the construction of quasi-panels from samples which change

[4]Note that η_t is not in the agent's information set at time t and cannot be taken outside the conditional expectation.

over time induces sampling error that acts very much like measurement error (Deaton, 1985).

One way to deal with measurement error problem is to linearize the equation (3) and use standard linear IV and GMM techniques. In particular, the convention is to assume measurement error in consumption is multiplicative. Naturally, log-linearization will move such measurement error into the additive residual. Measurement error results in residuals with a MA(1) structure, but variables lagged twice (or more) are still valid instruments. Following the steps in Carroll (2001) we get:

$$\Delta logC_{h,t+1} = \alpha + \frac{1}{\gamma} log(1 + R_{t+1}) + e_{h,t+1} \tag{7}$$

for the first order approximation and

$$\Delta logC_{h,t+1} = \alpha + \frac{1}{\gamma} log(1 + R_{t+1}) + \frac{\gamma + 1}{2} [\Delta logC_{h,t+1}]^2 + v_{h,t+1} \tag{8}$$

For the second order approximation. The constant term α contains the discount rate (β) and the means of the higher order moments consumption growth and interest rates (for the first-order approximation, this would be the second and higher moments; for the second order approximation, this is the third and higher order moments.) In either case the residual term contains (i) the true innovation in marginal utility (or "expectation error") between t and $t + 1$, ε_{t+1}, (ii) the measurement errors at t and $t + 1$, and (iii) an approximation error composed of variation in the higher moments of consumption growth and interest rates (conditional on past information).

3 Life Cycle Models Studied

In order to investigate the problems associated with the estimation of approximate Euler equations, we compare six different life cycle models of consumption which correspond to different economic environments studied in the literature. In all six models, preferences are time-separable and within period utility is iso-elastic with the coefficient of relative risk aversion set to 4 (so that the EIS is 0.25).

Agents face two types of income shocks, permanent and transitory. The income process of agent h is:

$$Y_{h,t} = P_{h,t} U_{h,t} \tag{9}$$

where $U_{h,t}$ is an i.i.d. lognormal transitory shock with unit mean and a constant variance $e^{\sigma_u^2} - 1$ and

$P_{h,t}$ is permanent income which follows a log random walk process:

$$P_{h,t} = GP_{h,t-1}Z_{h,t} \tag{10}$$

where $Z_{h,t}$ is an i.i.d. lognormal permanent shock with unit mean and a constant variance $e^{\sigma_z^2} - 1$. In our simulations we set σ_u to 0.1 and σ_z to 0.05; these values are in line with those used in the literature (they are identical to those used in Attanasio and Low, 2004) and experiments with other values give similar results. We assume that the innovations to income are independent over time and across individuals so that we abstract from aggregate shocks to income. However, there are aggregate shocks in these environments because realizations of the real interest rate are assumed to be the same across agents. The real interest rate follows an AR(1) process with a mean of 0.03, an AR parameter of 0.6 and a standard deviation of the error of 0.025 (see Alan and Browning, 2010). In some models we augment the income process just described with the possibility of a zero income realization (details given below).

Table 1 presents the parameter values assumed for the 6 models. The models differ by degree of impatience (since we are ignoring income growth, the degree of impatience is determined by the difference between real interest rate and individual's discount rate), by the possibility of a zero income realization, and by the type borrowing constraint (either the natural borrowing constraint – the agent can only borrow what she can pay back with certainty – or an explicit, period-by-period constraint.) Table 2 summarizes the distinguishing features of the six models. Model AL-P is similar to the environment studied by Attanasio and Low (2004). Agents' discount rates are equal to the mean real interest rate (0.03) and there is only the natural borrowing constraint. The important feature of this model is that even though borrowing is allowed up to the natural limit, individuals do not borrow because they are quite patient and so have a strong taste for accumulation. The second model, AL-I, is an impatient version of the first model. In this model the discount rate of agents set to 0.07. As a result agents in AL-I borrow especially early in life.

Model C-P and C-I are motivated by Carroll (2001). In these models we augment the income process described above by allowing transitory income shocks to take a '0' value in any given period with small probability (specifically, with probability 0.01) .[5] This addition to the model strengthens agents' precautionary motive. Moreover this assumption, along with backward induction and the fact

[5] Carroll (2001) specifies zero-income probabilities of 0.01, 0.03 and 0.05 in alternative experiments. Attanasio and Low (2004) allow for zero-income with a probability of 0.05 in one of their robustness checks.

that marginal utility of zero consumption is infinite means that agents will not borrow. The resulting consumption functions are very steep at low wealth levels. C-I is very close to the "buffer-stock" model studied by Carroll; C-P retains the income process from Carroll's study but assumes more patient agents.

Finally, we examine two versions of the environment first proposed by Deaton (1991) but not previously considered in the Monte Carlo studies of Euler equation estimation. In these models, individuals are explicitly prevented from borrowing. This assumption (with a lower bound for labour income) leads to a kink in the consumption function. We have two motivations for including these models. First, households with zero assets are observed in real data and these models (particularly the impatient version) can replicate that fact, while the models described above do not. Second, a comparison of D-P and AL-P will help clarify our arguments below regarding the key features of the economic environment.

We solve these 6 different life cycle models using standard methods. Further details of our specification, solution and simulation of the life cycle models are given in the Appendix. After solving for the consumption function of a generic household for 60 periods for each model, we simulate 60-period consumption paths for 10,000 ex-ante identical agents (households) and discard the first 10 and last 10 periods. We then use the resulting simulated data (for each model) as the basis of our Monte Carlo experiments.

4 Monte Carlo Design

Our Monte Carlo experiments draw repeatedly from the simulated population of agents (ie., consumption paths) described above. This is done 1000 times. In each case a sample of 1000 agents is drawn from the population of 10,000 with replacement. However, we mimic three different data structures. The first is a long panel. Each of the 1000 agents is followed for 40 periods. As emphasized by Chamberlain (1984) and others (see Attanasio and Low, 2004) the orthogonality conditions implied by theory hold in with long T . Thus a long panel is a best case scenario for Euler equation estimation. However, typical lengths of household panels are much shorter, and very long panels will inevitably suffer from attrition and other problems. We therefore also consider shorter panel data sets with $T = 14$. This length roughly mimics the available PSID data on food expenditure from 1974 to 1987[6] which has been

[6]Although the PSID began in 1969 and continues, the food variables expenditure are very hard to interpret prior to 1974, and food related questions are suspended for several years after 1987. This case illustrates typical practical problems with true panels. It is not sufficient for the panel to continue for many periods. It is also necessary that the

much used in Euler Equation estimation (see Alan and Browning, 2010, and the references therein).

Finally, one way in which researchers have tried to get around the short length of available panels is to construct synthetic cohorts from repeated cross-sectional surveys. Some repeated cross-sectional surveys, such as the Family Expenditure Survey (UK) or the Consumer Expenditure Survey (US) are available for many years. They do not allow for individual agents to be followed over time. However, cohorts defined by fixed characteristics can be followed by time and, as shown by Browning et al. (1985) and Deaton (1985) this allows for the estimation of linear models (of individual agent behaviour) in differences at the aggregate cohort level. Thus such data can be used to estimate linearized Euler equations. Note that this is not the same as estimating an Euler equation on aggregate consumption data; the individual-agent-level equation is explicitly aggregated, avoiding the problems identified by Attanasio and Weber (1993).[7]

Attanasio and Low (2004) emphasize that aggregation to the cohort level can have additional benefits (in addition to long T); in particular, there can be some averaging-out of the measurement error in household level consumption data. However, there are downsides to aggregation as well. While no variation in aggregate level instruments (the lagged interest rate) is lost, potentially useful within-cohort variation in other instruments (lagged consumption growth, lagged income) is lost. Moreover, as shown in Deaton (1985), constructing cohort means year by year from fresh samples is subject to sampling error, and this sampling error effectively induces a cohort-level measurement error in the means. Thus it is of interest to compare the performance of linearized Euler equation estimation on synthetic cohort data to the estimation of the same equation on true panels of different lengths.

In our Monte Carlo experiments on synthetic cohort estimation, we construct a synthetic cohort of length $T = 40$ from our simulated populations. For our synthetic cohort, we draw, with replacement, 1000 agents for *each* period ($t = 1,\ 2,\ 3 \ldots 40$); thus the synthetic cohort is constructed from different samples of individuals for each period, as would be the case in constructing a synthetic cohort from the U.S. Consumer Expenditure Survey or the U.K. Family Expenditure Survey.

For each of these data structures (long panel, short panel, synthetic cohort), our baseline experiments are conducted twice: first on the simulated data, and then on the simulated data with measurement error added. Experiments on the simulated data without measurement error allow us to isolate the effect of approximation bias and are comparable to previous papers in the literature.

consumption information be collected continuously and in a consistent fashion.

[7]As pointed out in Attanasio and Low (2004), estimation on synthetic cohorts requires an equation that is linear in parameters. Estimation on aggregate data requires a household level equation that is also linear in variables.

So they are a useful starting point. Of course, in the absence of measurement error, and with true panel data, there is no reason to work with the linear approximation to the Euler equation, as one could estimate the exact Euler equation by nonlinear GMM (as noted above, to use a synthetic cohort requires the linear approximation). When we add measurement error to the simulated data, we create a scenario which mimics the one faced by researchers using actual consumption data. Again, when we work with synthetic cohort, this measurement error will be averaged out, but a second source of measurement error (the sampling variation in cohort means) is introduced.

The measurement error that we add to the simulated data at the individual agent level is i.i.d log normal with a unit mean and a variance of 0.004 such that the approximately 75% of the period to period variance in consumption growth is due to the noise, close to the estimate for the PSID in Runkle (1991).[8] When working with the unadulterated simulated data, we lag instruments once, but when we add measurement error to the simulated data we lag consumption instruments twice, because i.i.d measurement error in consumption levels induces measurement error with an MA(1) structure in consumption changes. We also use twice-lagged consumption growth and income instruments when working with synthetic cohorts, because of the measurement error in cohort means induced by sampling. All the different data structures we consider are summarized in Table 3.

In each of our experiments, we consider two things. As in previous papers in this literature, we examine the actual estimates in repeated samples. We report the mean finite sample bias in 1000 replications, and also the mean standard error. However, we also go beyond the previous literature and look directly at the properties of the instrument in the simulated data. The instruments we consider are the ones used extensively in the literature: lagged interest rates, lagged consumption growth and lagged income.[9] We add lagged consumption growth squared to the instrument set for the second order approximation. We consider the validity of these instruments – whether they are uncorrelated with the residual in the log-linearized Euler equation (Equation number here) – and the relevance of these instruments – the strength with which they predict the endogenous variables in the estimating equation.

Note that because we know the true value of the preference parameter (γ) in our simulated data, we can use the true parameter value to construct the true residuals in the linearized Euler equation

[8] This is the same measurement error structure as assumed in Alan, Attanasio and Browning (2009). We also repeated our experiments with a measurement error variance that implied that approximately 50% of the period to period variance in consumption growth is due to the noise (i.e. with a smaller measurement error variance.) The results were similar to those obtained under our baseline measurement error assumption, and so for sake of brevity they are not reported, but are available upon request.

[9] We use the same instrument set as Attanasio and Low (2004).

(plus a constant). That is, inverting equations (7) and (8) and evaluating at $\gamma = 4$ gives:

$$\Delta logC_{h,t+1} - \frac{1}{4}log(1 + R_{t+1}) = \alpha + e_{h,t+1} \tag{11}$$

for the first order approximation and

$$\Delta logC_{h,t+1} - \frac{1}{4}log(1 + R_{t+1}) - \frac{4+1}{2}(\Delta logC_{h,t+1})^2 = \alpha + v_{h,t+1} \tag{12}$$

for the second order approximation. All the terms on the left-hand side of equations (11) and (12) are observed in our simulated data, so we can calculate the right-hand side for each observation in the data. Those quantities will not be mean zero (because they contain the constant from the linearized Euler equation). As described in Section 2, the variation in those terms across individuals and time comes from innovations to marginal utility, measurement error, and approximation error (where the latter comprises variation in higher-order moments in consumption growth and interest rates. The constructed residuals on the right-hand side of equations (11) and (12) may or may not be correlated with the instruments. We will assume that the measurement error is orthogonal to the instruments. Theory indicates that the expectation error - but and not necessarily the approximation error - should be orthogonal to lagged variables. But with the constructed residuals in hand, the issues is open to direct empirical investigation.

Our instrument validity test is then a t-test obtained from the regression of these constructed residuals on our instruments. The null hypothesis is that the residuals are uncorrelated with the instruments: a significant t-statistic would suggest that instruments are not valid. Estimation of linearized Euler equations (with the standard instruments) will not be a promising strategy if we get many rejections of this null hypothesis. We report the fraction of t-statistics in the repeated samples of our Monte Carlo experiments that exceed 1.96 in absolute value (corresponding to a 5% test). It is important to note that this is not a standard over-identification test. Because we can construct the appropriate residuals, we can test the validity of the instruments even when the equation of interest is just identified.

In each sample in each experiment we also conduct the "first stage" regression(s) of the endogenous regressors on the instruments to examine instrument relevance (that is, to check for weak instruments). The endogenous regressors are the contemporaneous interest rate, and, in the case of the 2nd order approximation, the square of consumption growth. We establish instrument relevance using the Cragg-

Donald F statistic.[10] The null hypothesis of this test is instruments are jointly weak. Estimation of linearized Euler equations (with the standard instruments) can only be a promising strategy if we get many rejections of this null hypothesis. Stock and Yogo (2005) calculated the critical values for this test as a function of the number of included endogenous regressors, the number of instrumental variable, and the desired maximal bias of the IV estimator relative to OLS. In the case for one endogenous variable, allowing for a maximum relative bias of 10% compared to OLS, and at the 5% significance level and with three of instruments (as in our experiments with the first-order linearized Euler equation), the critical value is 9.08 for Linear GMM, and 6.46 for LIML. When there are two endogenous variables, a maximum relative bias of 10% compared to OLS, a 5% significance level and four instruments (as in our experiments with the second-order linearized Euler equation), the critical value is 7.56 for Linear GMM , and 4.72 for LIML.

In our experiments realizations of the interest rate process are common to all agents. This means that we have an aggregate variable on the right-hand side of equations (7) and (8) and the usual formulas may significantly underestimate the standard error of the estimates (Moulton, 1986). Accordingly, we cluster the standard error on the time period.

Finally, we note that when estimating on data simulated from models D-P and particularly D-I the Euler equation does not hold in situations where the explicit borrowing constraint is binding. Thus we delete form the estimation sample all observations with cash on hand strictly equal to zero at $t-1$. Note that the conditional expectation expressed in Equation (1) which underpins Euler equation still holds conditional on this selection: we are selecting on the basis of a variable in the information set.

5 The Consumption Function and Linearized Euler Equations

Before turning to the Monte Carlo results, it is useful to develop some hypotheses regarding when the estimation of linearized Euler Equations is likely to perform well, and when it is likely to fail.

The potential problem with the approximate Euler equation (Equation 7 or 8) is that variation in the higher order moments of consumption growth (and consumption growth and interest rates) that are subsumed into the residual term may be correlated with lagged variables, leaving researchers without any valid instruments. For example, there is no theoretical reason that the lagged consumption growth should be uncorrelated with conditional skewness. We refer to the resulting inconsistency of the estimate as approximation bias.

[10]This is the F statistics from the first stage when there is only one endogenous regressor.

Of course, this is more likely to be a problem the larger (and more variable) the higher moments are. For example, a comparison of the first- and second- order approximations in Section 2 (Equation 7 or 8) shows that the key omitted variable in the first-order linearized Euler Equation is the variance of consumption growth, which may be correlated with the lagged consumption growth.

The models we consider are homogeneous with respect to the stochastic processes. For a given model, all agents face the same income and interest rate processes. Across models, the interest rate process is the same, and the only difference across models in the income process is the small positive probability of zero income realization in models three and four. Thus differences in the variance of consumption growth are driven not by these stochastic processes but rather by the sensitivity of log consumption to realizations of the state variables. This in turn is controlled by the semi-elasticity of consumption function (policy rule) with respect to the state variables. The semi-elasticity with respect to cash-on-hand turns out to be critical. A large semi-elasticity implies that shocks to cash-on-hand will pass through to greater variability in consumption growth. This has two consequences. First, from the point of view of estimating interest rate responses, greater variability in consumption growth coming from income shocks means more noise and hence less precision. Second, when the higher order moments of consumption growth are larger, there is more scope for them to vary and potentially correlate with the instruments.[11]Thus we expect that when the semi-elasticity is large, estimates based on linearized Euler equations will be certainly be less precise, and they may be more biased.

Figure 1 illustrates these key characteristics of our models. For each model, the numerically solved consumption function for age 40, and the (simulated) distribution of normalized cash-on hand at the same age are plotted. Of course the consumption function also depends on the interest rate. We solve the model for a stochastic interest rate but for comparability we simulate all 6 models with a common vector of interest rate realizations. In all our simulations the interest rate realization at age 40 is 0.033 (recall that the long run average of the interest rate process is 0.03.)

In most of the models we study, the policy functions (that is, consumption functions) are nonlinear, and broadly similar: steep (and curved) at low cash-on-hand levels but much flatter (and near-linear) at high cash-on-hand levels. The consumption function for AL-P is distinctive in that it is linear, while the consumption for D-P is distinguished by a sharp kink.

What determines the degree of approximation bias is not overall shape of the policy function (which

[11]If the higher moments of consumption growth were large but varied neither through time or across agents, then of course they would be subsumed in the intercept of the linearized Euler Equation, and could not be correlated with the instruments. The point here is that larger second and higher moments give greater scope for potential correlation.

is common across many models) but the semi-elasticity of the consumption function in the region of the state space in which agents operate. It is clear from the figures that cash-on-hand distributions of patient models (Models AL-P, C-P and D-P) are located at higher cash-on-hand levels. As a result, for agents in these models, the steep parts of the consumption function are irrelevant and their weighted average semi-elasticity is low. On the contrary, in impatient models (Models AL-I, C-I and D-I) agents accumulate very little wealth. They operate on the steep part of their consumption function, and the weighted average semi-elasticity is high. The consumption growth of these agents will thus be much more sensitive to shocks to cash on hand, and the resulting higher variability of consumption growth gives greater scope for approximation bias.

To summarize the effective semi-elasticity of the consumption function for each of our models, we use our simulated data to estimate, non-parametrically, the weighted average semi-elasticity of the consumption function. This statistic weights the semi-elasticity of the underlying consumption function (at age 40 and the corresponding realized interest rate) at every point by the ex-post density of normalized cash-on-hand.[12] To do this, we allow log consumption values from each age-40 consumption function to be a flexible function of the normalized cash-on-hand (x).[13] That is, first we non-parametrically estimate:

$$E[logc_i|x_i; A, r] = g_{A,r}(x_i) \tag{13}$$

on the simulated data. We do this using local polynomial regression, which provides convenient estimates of $\frac{\partial g_{A,r}}{\partial x}(x)$ at each evaluation point. In practice, we evaluate at each x point (cash-on-hand level) that arises in the simulated data, for age 40. Next, we take a weighted average of this measure, where the weights are the age-specific empirical density $f(\hat{x})$ of cash-on-hand in the simulated data at each evaluation point.[14] Thus we calculate:

$$effective\ semi-elasticity = \int \frac{\partial \hat{g_{A,r}}}{\partial x}(x)f(\hat{x})dx \tag{14}$$

The effective semi-elasticity for each model is given in each panel of Figure 1.

A comparison of AL-P and D-P illustrates our point. These models are superficially quite different in the sense that borrowing is allowed in AL-P but not in D-P and the shapes of the consumption functions for these two models are very different. However, in the region of the state space where

[12]This is the ratio of cash on hand to permanent income.

[13]We could, of course, calculate the same statistic at different ages. The value of the statistic changes marginally, but the ordering of the models does not.

[14]For more on average derivative estimation see, for example, Deaton and Ng. (1998).

agents operate, as indicated by the empirical density of cash on hand, the consumption functions are very similar, and this is reflected in almost identical values for the effective semi-elasticity.

Our hypothesis is that estimation of linearized Euler Equation will perform poorly when the effective semi-elasticity is large, as it is in the impatient models (AL-I, D-I, and C-I) and that among impatient models, performance may be worse when the effective semi-elasticity is higher. For example, performance is expected to be worse in D-I and C-I than in AL-I. Among apparently different models with very similar effective semi-elasticity, such as AL-P and D-P, we would expect performance to be quite similar. Of course, the effective semi-elasticity is not an object that we would calculate ex-ante on real data (in which cash-on-hand is typically not observed).[15] We calculate it here on the simulated data to clarify and test intuition about the performance of linearized Euler equations in different environments.

6 Monte Carlo Results

6.1 Baseline Experiments

We begin our discussion of the results of the Monte Carlo experiments with Table 4. Here we consider estimation of a first order approximation the Euler equation. Table 4 reports evidence on instrument validity and instrument relevance. As described above, instrument validity is assessed by regressing the error term from the linearized Euler equation (plus a constant) on the instruments. The error term is calculated (without estimation) from the simulated interest rate and consumption growth, and the true value of the preference parameter, γ (see equation 10). Instrument relevance is assessed by regressing the endogenous explanatory variable - here the interest rate - on the instrument set. Again the instrument set includes lagged interest rate, lagged consumption growth and lagged income.

The final column of Table 4 reveals that in these models, using the first order approximation as a basis for estimation, the standard instrument set has very good predictive power for the interest rate. There is no issue of weak instruments. The table also shows that the lagged interest rate is always a valid instrument. In no model and no sample does it have a significant correlation with the error term.[16] In impatient models (AL-I, C-I, and D-I) we have significant validity problems with the other instruments. For example, in AL-I, without measurement error, we find that lagged consumption growth is significantly correlated with the error term in 985 out of 1000 samples; For

[15] For example, in the PSID, wealth is observed only every five years.

[16] The complete absence of rejections suggests that our clustered standard errors are too conservative.

C-I lagged consumption growth is significantly correlated with the error term in all 1000 samples. This suggests that lagged consumption growth and lagged income are not valid instruments for the estimation of the first-order linearized Euler equation (even though the model we simulate implies that they must be uncorrelated with the innovation in marginal utility). Interestingly, adding realistic measurement error to consumption appears to improve the situation because it weakens the correlation between these instruments and the error terms (without much cost in terms of instrument relevance.) For example, in the C-I model, the frequency with which lagged consumption growth is significantly correlated with the error term falls from 100% to 15%. This could be because the error terms now contain the measurement error (in addition to the variation in approximation error and the innovations to marginal utility) or it could be because, in the presence of measurement error we use second (rather than first) lags of consumption growth in the instrument. Nevertheless, the results in this table strongly suggest that only the lagged interest rate should be used in the instrument set.

Table 5 reports the estimates of the EIS that result from estimating the first-order linearized Euler Equation on these models. The left-hand column gives the results with unadulterated consumption data; the second column gives the results when realistic measurement error is added to the data. The true value of the EIS is 0.25. For each model, and in each column, we report three numbers. First, at the top, the mean estimate in 1000 samples. Second, in round parentheses, the mean standard error of the estimate across the samples. Note that the standard errors calculated in each sample are robust to heteroskedasticity and clustered on time period. Third, in square parentheses the mean of the percentage bias across the samples.[17]

In the left-hand column, without measurement error, we see the mean bias is negative in all models the first-order linearization of the Euler equation tends to lead to underestimates of the EIS. As expected the problem is much more severe in impatient models. In patient models, the mean bias is about 5% of the true value. In impatient models, this rises to 12 to 22% of the true value.

The right-hand column of Table 5 reveals that measurement error sometimes worsens things, but not always. There is a loss of precision in all cases, but this is sometimes small. The mean percentage bias sometimes rises, and sometimes falls. The pattern of the estimation strategy performing much better in models with patient agents is no longer so sharp.

Another aspect of Table 5 is that the effective semi-elasticity is moderately successful at predicting performance of the linearized Euler Equations. For example, the AL-P and D-P models are quite

[17] We also calculated the median percentage bias in each case. These were very similar to the mean percentage bias and are available on request.

different (and have quite different consumption functions - see Figure 1), but they have very seminar effective semi-elasticities and the the performance of the linearized Euler equation is quite similar across the two models. More broadly, models in which agents are more impatient have higher effective-elasticities and we find that with data from this models estimates of the EIS are less precise, and generally more biased.

Tables 6 and 7 parallel Tables 4 and 5, but for the second-order approximation of the Euler Equation. The first thing to note in Table 6 is that there are now strong signs of weak instrument problems (recall that critical value for two endogenous and four instruments at the 5% significance level is 7.56 for Linear GMM and 4.72 for LIML). The extra endogenous variable here is the square of consumption growth, and the extra instrument the square of lagged consumption growth, as is standard in the literature, but the instrument set has insufficient predictive power. In commenting on the poor performance of the second-order linearization to of the Euler Equation, Attanasio and Low (2004) postulated that the problem was the lack of a good instrument for the 2nd order term. The statistical tests reported in the final column of Table 6 confirm that this is the case.

Turning to instrument validity, we see similar patterns as in Table 4. The lagged interest rate is always a valid instrument, whereas the lags of consumption growth (and its square), and lagged income, often are not. Problems of invalid instruments are greater in models with impatient agents and high effective semi-elasticities, but are somewhat ameliorated by measurement error in consumption (and the consequent use of second lags as instruments).

Table 7 reports the estimates. There are two coefficients to estimate in the 2nd-order approximate Euler equation. The coefficient on the interest rate is the EIS, while the coefficient on the square of consumption growth is the coefficient of relative prudence. For each we report the mean coefficient, the mean standard error, and the mean percentage bias. As before all calculations are replicated with and without measurement error in consumption.

There are two important questions here. First, do the estimates of the coefficient of relative prudence contain any useful information. The answer is plainly no. In almost all models the estimates are very imprecise and suffer from substantial mean bias. The second question is whether the inclusion of the 2nd order term in consumption growth improves the estimate of the EIS, by reducing the scope for approximation bias. Comparing Table 7 with Table 5 suggest no strong evidence of improvement.

We would summarize the result of these section as follows. With a long panel and sufficient variation (recall that $T = 40$ here, and the auto-correlation coefficient in the interest rate process is 0.6), the linearized Euler Equation is modestly successful in recovering the EIS. Across a range of quite different models, and with realistic measurement error in consumption, the mean bias in estimates of the EIS ranges from 3% to 16% (see the 2nd column of Table 5). However, note that in impatient models, the estimates are quite imprecise. For example, with the data from the AL-I model, with realistic measurement error and the first-order approximation, the mean width of a confidence interval for the EIS is 0.76, which would include both 0 and 0.5. There is no advantage to the 2nd order approximation to the first-order approximation. In all the models we studied, the lagged interest rate is the only valid instrument. It is the aggregate shocks to interest rates that identify the parameter of interest.[18]

These baseline estimates are based on a true annual panel of 40 years in length. While this is useful "best case" scenario, given the cost of panel surveys, as well as problems with attrition, this is not realistic. In the next subsection we consider two more realistic data structures.

6.2 Alternative Data Structures

In Tables 8 and 9 we report experiments with two more realistic data structures. As described in Section 4 these are a short annual panel of 14 years, and a synthetic cohort observed for 40 years. In this subsection our intention is to be as realistic as possible and so we always experiment with consumption data which includes measurement error. To facilitate comparison, we present results based on a long true panel with measurement error (repeated from the previous section) alongside the results for more realistic data structures. We focus on the AL and C models (both patient and impatient versions) and only employ the first-order approximation to the Euler equation, as our baseline experiments indicated no advantage to the 2nd-order approximation.

Table 8 reports our findings on instrument validity and instrument relevance with these alternative data structures. There are two points to note. First, problems with the validity of lagged consumption growth and lagged income as less apparent with the synthetic cohort. Thus it is largely the individual (cross-sectional) variation in these instruments, rather than the time-series variation, that undermines validity in the (true) panel data. Second, turning to instrument relevance, the instruments are much weaker at the cohort level and in many case are below the critical value of the Cragg-Donaldson F-

[18]In our experiments, all agents face the aggregate interest rate, and the lagged aggregate interest rate is a very strong instrument. In practice, there may be individual variation in the inter-temporal price which agents face which is not easily observable to the researcher. On the other hand, our experimental setup presumes aggregate exogenous shocks to interest rates. Part of the argument in Carroll (2001) is that in an equilibrium of a standard closed model, the interest rate is not exogenous. We nevertheless think that it is reasonable to proceed on the basis that there are shocks to aggregate interest rates that can be thought of as exogenous to typical consumers is a given country. Given the results in Table 5, we did experiment with the just identified model (with the lagged interest rate as only instrument). This gave very similar results and the details are available upon request.

test (recall that critical value for one endogenous and three instruments at the 5% significance level is 9.58 for Linear GMM and 6.46 for LIML) For this reason, when estimating the approximate Euler equation these experiments, we used both linear Generalized Method of Moments (GMM) and Limited Information Maximum Likelihood (LIML). The latter is known to perform better when instruments are weak (see, eg., Andrews and Stock, 2005; Hahn, Hausman, and Kuersteiner, 2003).

Table 9 summarizes the estimates from these Monte Carlo experiments. As in Tables 5 and 9 we report the mean estimates (in 1000 replications) the mean standard error and the mean percentage bias. Estimation on the short panel is, in general, not successful. The extent of mean bias, particularly on the impatient models, is unacceptable. For example, when the data are generated by AL-I, and estimation is by linear GMM, the *mean* bias is 224% of the true parameter value. These results suggest that 14 annual observations - as are afforded by the PSID - are not adequate for linearized Euler Equation estimation.

We would expect LIML to perform better than GMM on the synthetic panel, where the instruments are weak. LIML does in fact lead to more reliable estimates when the data are generated by AL-I, and used to construct a synthetic panel. The mean bias is just 1.2% of the true parameter value, as opposed to 27% when linear GMM is employed on the same 1000 data sets.

In terms of mean bias the synthetic cohort produces results that compare quite well to the long panel, particularly when LIML is employed. The cost though is a loss of precision: the mean standard error is about twice as large in the synthetic panel. Here we are estimating with a single cohort, and some additional precision might come from following multiple birth cohorts through the data.

A summary of this section is that with realistic data structures, linearized Euler Equation estimation seems much less promising.

6.3 The Effects of Parameter Heterogeneity

A final issue which we investigated is the effects of parameter heterogeneity. Emerging evidence strongly suggests that the key structural parameters of the consumption Euler Equation are heterogeneous in the population (see for example, Guvenen (2006) and Alan and Browning (2010)). Heterogeneity in the EIS_i $(1/\gamma_i)$ implies heterogeneity in the slope(s) of the linearized Euler equation. As noted in Section 2, the discount rate is subsumed in the constant of the linearized Euler equation, so that heterogeneity in the discount ratesβ_i implies heterogeneous intercepts. Let $E\bar{I}S$ be the mean of EIS and $\bar{\beta}$ be the mean discount factors in the population under study. Then the first-order approximation to the Euler equation is:

$$\begin{aligned}\Delta logC_{i,t+1} &= \alpha_i + EIS_i log(1+R_{t+1}) + e_{i,t+1} \\ &= \bar{\alpha} + E\bar{I}S log(1+R_{t+1}) + [(EIS_i - E\bar{I}S) log(1+R_{t+1}) + (\alpha_i - \bar{\alpha}) + e_{i,t+1}] \quad (15)\end{aligned}$$

Heterogeneity in preference parameters now enters the composite error term and it is immediately obvious that many of the usual instruments are now unlikely to be valid for a second reason. Lagged consumption growth will certainly be correlated with heterogeneity in discount rates or the EIS; lagged income may be as well. In contrast the aggregate interest rate has only time series variation, and so is orthogonal to time-invariant individual preference heterogeneity by construction. Thus preference heterogeneity is a further reason why aggregate shocks to the inter-temporal price must play a central role in identifying the parameters of interest. In the presence of preference heterogeneity, the synthetic cohort data structure may have the further advantage that such heterogeneity, like measurement error in consumption, is averaged out.

Table 10 reports a set of experiments designed to explore this issue. In the top panel of of Table 10 we report results from Monte Carlo experiments on data simulated from models. The first two have been presented previously in Table 5: AL-P and AL-I with $\gamma = 4$ (and hence an EIS of 0.25). The third is a variant of AL-P with $\gamma = 6$ (and hence an EIS of 0.167). In all three cases the simulated data have been augmented with realistic measurement error. Estimation is based on the first-order approximation to the Euler equation and is by linear GMM. With more curvature in utility (and hence a lower EIS), the AL-P model has higher effective semi-elasticity of 0.0498 (as opposed to 0.0425 when $\gamma = 4$). The top panel of Table 5 shows that with this additional curvature in utility, linearized Euler equation estimation performs somewhat less well, at least in the AL-P model. There is more suggestion of potential approximation bias: the lagged interest rate is often no longer valid. The mean percentage bias doubles, from 6% to 12%.

The key results of this section, are in the next two panels of Table 10. Here, in each replication we mix data from the two different models. In the middle panel we consider heterogeneity in the EIS by mixing data from two versions of the AL-P. Half the agents are drawn from data generated by the model with $\gamma = 4$, while the other half are drawn from data generated by the model with $\gamma = 6$. The issue here is whether linearized Euler equation estimation recovers the mean EIS, which is 0.2085. We considered two data structures: the long panel ($T = 40$) and a synthetic cohort of the same length.

Linearized Euler equation estimation does reasonably well here, and recovers the mean EIS with about as much accuracy and precision as with data from a homogenous model. There is no strong evidence of an additional advantage to the synthetic cohort data structure. In fact, given the same T, the two procedures exhibit similar performance, with mean bias of 7 or 8 percent. Again, there is a lack of precision with the synthetic panel relative to the long panel.

The third panel we consider discount rate heterogeneity by mixing simulated data from models AL-I and AL-P. Again, discount heterogeneity generates heterogeneous intercepts in the linearized Euler equation. Here estimation on linearized Euler Equations does a poor job of recovering in the EIS with mean bias of 20% or more. Note however, that this is not much worse than with data from the homogenous AL-I model. The poor performance may be partly a consequence of the inclusion of data from impatient agents, as well as of preference heterogeneity. Nevertheless, in this panel we do show some evidence that the synthetic cohort performs a bit better, at least in terms of mean bias, than a true panel of the same length. This is presumably due to the averaging out of individual heterogeneity. As always, though, the synthetic cohort estimates are less precise.

Overall, the results of this subsection do not significantly alter the findings that emerged in the previous subsections. The first-order Euler equation leads to biased estimates of the EIS, but that bias is perhaps not too large when there is a sufficient time dimension to the data, and sufficient variation in interest rates. A sufficient time dimension can only realistically be achieved with a synthetic cohort. Estimates are unlikely to be very precise, and bias will be worse the more impatient agents are.

7 Discussion

A large empirical literature investigates the preference parameters that control the sensitivity of consumption (or, equivalently, saving) to interest rates (the elasticity of inter-temporal substitution) and to uncertainty (prudence). A common approach to estimating these parameters employs the (Euler equation) derived from the dynamic optimization problem of a consumer. Often these non-linear equations are log-linearized (approximate Euler equations) so that linear instrumental variables methods can be used to deal with measurement errors in consumption. However, it has recently been argued that the approximation bias induced by linearization may be worse than the problems that linearization is intended to solve. Consequently, the usefulness of the Euler equation approach has been debated. We have explored this issue with a series of simulation studies. Our findings offer a reconciliation of the debate.

On the one hand, our results confirm the finding of Attanasio and Low (2004) that with sufficiently long time series and sufficient variation in the interest rate, linearized Euler equation estimation works reasonably well. We strongly affirm their emphasis on the central role that variation in the intertemporal price must play in estimating the elasticity of consumption with respect to that price. We find no evidence that the particular models studied by Attanasio and Low (2004) generate data which is particularly suitable for linearized Euler equation estimation. We also find that the second-order approximation to the Euler equation provides no advantage over the first-order approximation and directly confirm Attanasio and Low's suggestion that the key problem with the second-order approximation is that the conventional instruments have insufficient predictive power in this case. In other words, we showed that estimation of the second-order approximation suffers from a weak instrument problem.

More broadly, though, our findings support Carroll (2001) and others in this literature who are skeptical of the usefulness of linearized Euler equation estimation. We find that the performance of linearized Euler equation estimation declines, with both greater mean bias and a loss of precision, when agents are moderately impatient. Perhaps more importantly, Linearized Euler equation estimation does not appear to work well on any realistic data structure. A sufficient time dimension can only realistically be achieved with a synthetic cohort, and our experiments suggest that estimates from synthetic cohorts of sufficient length, while often exhibiting small mean bias, are quite imprecise.

How then, can we learn about the important structural parameters such as the EIS? One reaction to these results would be to move toward the estimation of more fully specified structural models. We have some sympathy with this view. At the same time, however, it is important not to lose sight of what is learned from the data, and what is assumed by the model.

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Appendix: Specification, Solution and Simulation of Life-Cycle Models

We assume that the utility function is intertemporally additive and the sub-utilities are iso-elastic. The problem of the generic household is:

$$\max_{C_t} E_t \sum_{t=1}^{T} \beta^t u(C_t)$$

$$s.t.\ X_{t+1} = (1 + R_{t+1})(X_t - C_t) + Y_{t+1}$$

where C_t is nondurable consumption in period t, X_t is cash-on-hand (total financial and nonfinancial wealth) and Y_t is current labor income. We assume that durable consumption and leisure are separable from the nondurable consumption. The income process is assumed as follows:

$$Y_{t+1} = P_{t+1} U_{t+1}$$

$$P_{t+1} = G P_t N_{t+1}$$

where G is predictable permanent income growth, P_t is permanent income which is subject to log-normally distributed shocks N_t with mean unity and variance $(e^{\sigma_n^2} - 1)$, current income Y_t equals permanent income multiplied by a transitory shock, U_t , which is distributed lognormally with mean one and variance $(e^{\sigma_u^2} - 1)$. The interest rate series is assumed to be generated by a stationary first order autoregressive process with long-run mean μ and autoregressive coefficient ρ. Interest rates shocks ε_{t+1} are assumed to be white noise with variance, σ_ε^2. The process is:

$$R_{t+1} = (1 - \rho)\mu + \rho R_t + \varepsilon_{t+1}.$$

The inter-temporal model described above does not have an analytical solution due to the assumed income uncertainty. Therefore we utilized the standard numerical dynamic programming methods to obtain a solution. Since the utility function is additive over the life cycle we solved the model recursively starting from the last period of life. We assume away any bequest motive so that consumption in period T is:

$$C_t(x_T) = x_T.$$

The problem is solved via policy function iteration using the terminal value condition. Having a nonstationary income process makes the problem harder to solve since the range of possible income values is too large. Instead, we redefine all relevant variables in terms of their ratios to permanent income and solve for the consumption to income ratio. By doing this we reduced the number of state variables to two, namely the cash on hand to income ratio and the interest rate. Moreover, we obtain an iid income process which can be approximated by standard Quadrature methods. Given the redefinition of the variables, the Euler equation can be written as:

$$\theta_t(\omega_t, R_t)^{-\gamma} - \beta E_t[(1 + R_{t+1})\theta_{t+1}(\omega_{t+1}, R_{t+1})^{-\gamma} n_{t+1}^{-\gamma} = 0$$

where $\theta_t = \frac{C_t}{P_t}$, $\omega_t = \frac{x_t}{P_t}$. At the terminal date T, consumption to income ratio is a function of only the cash on hand to income ratio and since the bequest motive is assumed away it follows that $\theta_T = \omega_T$.

For the income process, we use 10 point Gaussian Quadrature and we approximate the interest rate process by forming a 10 point first order discrete Markov process. We use a cubic spline to approximate the consumption function at each iteration. The agent is allowed to borrow the amount he can pay back with certainty. In practice this constraint will never bind because the functional form of the utility function implies that zero consumption results in infinite marginal utility. In models where we do not assume an explicit borrowing limit (model 1, 2, 3 and 4), the consumption functions are continuously differentiable. In fact, in our case where agents have iso-elastic preferences and income uncertainty, consumption functions are strictly concave.

In order to solve the problem, we define an exogenous grid for the cash on hand to income ratio: $\{x_j\}_{j=1}^{J}$. It is important to adjust the grid as the solution goes back in time. The algorithm finds the consumption level that makes the standard Euler equation hold for each value of x and r. We made the grid for x finer at lower levels in order to capture the curvature of the consumption function. After solving for the consumption function of a generic household for 80 periods, we simulate consumption paths for 10,000 ex-ante identical households facing the same interest rate realizations. We use only the middle 40 periods for the estimations. Table 1 presents the assumed parameter values for our experiments.

Table 1: Parameter Values

Parameter	Value
Coefficient of Risk Aversion γ	4, 6
Discount Rate $\left(\delta = (\frac{1}{\beta}) - 1\right)$	0.03 and 0.07
Standard Deviation of Permanent Income Shocks (	0.05
Standard Deviation of Transitory Income Shocks σ_u	0.1
Unconditional Mean of Interest Rate Process μ	0.03
AR(1) Coefficient of Interest Rate Process ρ	0.6
Standard Deviation of Interest Rate Process σ_ε	0.025
Probability of Zero Income (Models 3 and 4)	0.01

Table 2: Models

Model	Impatient	Patient	Borrowing Constraint	Zero Income Probability
AL-P	-	Yes	Life Time	No
AL-I	Yes	-	Life Time	No
C-P	-	Yes	Life Time	Yes
C-I	Yes	-	Life Time	Yes
D-P	-	Yes	Explicit	No
D-I	Yes	-	Explicit	No

Table 3: Data Structures

	Measurement Error	True Panel	N	T	Cohorts
Long Panel	N	Y	1000	40	-
Long Panel	Y	Y	1000	40	-
Short Panel	Y	Y	1000	14	-
Synthetic Cohort	Y	N	1000	40	1

Table 4: First Order Approximation
Instrument Validity and Relevance Results, 1000 replications

Model (effective semi-elasticity)	Measurement Error	Instrument Validity — *First Order* : – Fraction of t stats with absolute value greater than 1.96 — ***Lag Interest Rate***	***Lag Consumption Growth***	***Lag Income***	Instrument Relevance (First Stage) — F stat — Mean [10% , 90%]
AL-P *(0.0425)*	*No*	0	0.035	0.066	7000 [6996 , 7003]
	Yes	0	0.084	0.002	5524 [5522 , 5526]
AL-I *(0.0542)*	*No*	0	0.985	0.989	7458 [7450 , 7470]
	Yes	0	0.134	0.732	7145 [7140 , 7151]
C-P (0.0427)	*No*	0	0.078	0.019	7362 [7320 , 7402]
	Yes	0	0.109	0.004	7076 [7044 , 7113]
C-I *(0.1099)*	*No*	0	1	0.338	7362 [7320 , 7402]
	Yes	0	0.150	0.020	7074 [7035 , 7108]
D-P *(0.0426)*	*No*	0	0.041	0.022	7438 [7431 , 7447]
	Yes	0	0.092	0.004	7147 [7140 , 7157]
D-I *(0.3922)*	*No*	0	0.124	0.160	6961 [6853 , 7076]
	Yes	0	0.061	0.045	6682 [6573 , 6775]

Notes to Table 4:

1. The average semi elasticity of each model is reported in parentheses in the first column. All other numbers are result of 1000 Monte Carlo replications.
2. For the first order approximation instruments, without measurement error case instruments used are lag interest rate, lag consumption growth and lag income.. For the measurement error cases, we use the twice lagged consumption instrument with the lag interest rate and lag income.
3. Instrument validity test is a t-test obtained from the regression of constructed residuals on instruments.
    a. Instrument validity columns report for each instrument the fraction of t stats with absolute value greater than 1.96 (critical value at 5% significance level)
    b. Corresponding mean values of R^2 for each regression are reported.
    a. Instrument Relevance column reports the Cragg-Donald F (CDF) statistic from the first stage of IV. For the first order approximation interest rate is the only endogenous variable and lagged interest rate, (twice)lagged consumption growth, lagged income are instruments. Mean values of CDF are reported. CDF values at 10 and 90 percent are in parentheses.
    b. Stock and Yogo (2002) - test for weak instrument H_0 =bias of two stage estimation relative to OLS is greater than 10%
        i. Critical Value at 5% significance level when the number of instruments is 3 and endogenous variable is 1 for Linear GMM = 9.08, for LIML=6.46.
4. .The measurement error that we add to the simulated data is i.i.d log normal with a unit mean and a variance of 0.004.

Table 5: Estimates of the EIS, 1000 replications			
		First Order Approximation	
True Value		EIS: $\left(\frac{1}{\gamma}\right) = 0.25$	
Model (effective semi-elasticity)		**Base**	**With Measurement Error**
AL-P	*mean coefficient*	0.234	0.290
(0.0425)	*(mean std. error)*	(0.078)	(0.105)
	(mean bias as % of true parameter value)	[-6.4]	[16]
AL-I	*mean coefficient*	0.194	0.261
(0.0542)	*(mean std. error)*	(0.192)	(0.194)
	(mean bias as % of true parameter value)	[-22.4]	[4.4]
C-P	*mean coefficient*	0.242	0.258
(0.0427)	*(mean std. error)*	(0.041)	(0.048)
	(mean bias as % of true parameter value)	[-3.2]	[3.2]
C-I	*mean coefficient*	0.219	0.216
(0.1099)	*(mean std. error)*	(0.039)	(0.044)
	(mean bias as % of true parameter value)	[-12.4]	[-13.6]
D-P	*mean coefficient*	0.238	0.244
(0.0426)	*(mean std. error)*	(0.060)	(0.063)
	(mean bias as % of true parameter value)	[-4.8]	[-2.4]
D-I	*mean coefficient*	0.218	0.236
(0.3922)	*(mean std. error)*	(0.109)	(0.175)
	(mean bias as % of true parameter value)	[-12.8]	[-5.6]

1. For each model and estimation strategy, the first number is the mean value of the estimate of the EIS in 1000 Monte Carlo replications. The second number, in parenthesis, is the mean robust standard errors clustered on period. The third number is the mean bias as the percentage of true parameter value. The effective semi-elasticity of each model is reported in parentheses in the first column.

Table 6: Second Order Approximation

Instrument Validity and Relevance Results, 1000 replications

Second Order :		**Instrument Validity** –	——			**Instrument Relevance (First Stage)**
		Fraction of t stats with absolute value greater than 1.96				CDF stat
Model (effective semi-elasticity)	Measurement Error	***Lag Interest Rate***	***Lag Consumption Growth***	***Lag Income***	***Lag Consumption Growth Square***	Mean [10% , 90%]
AL-P	*No*	0	0.048	0.027	0.062	3.24
(0.0425)						[0.21, 9.78]
	Yes	0	0.073	0.012	0.051	1.18
						[0.01 , 4.49]
AL-I	*No*	0	0.487	0.554	0.690	93.6
(0.0542)						[42.7 , 176]
	Yes	0	0.056	0.143	0.055	3.82
						[0.26 , 9.85]
C-P	*No*	0	0.074	0.007	0.065	401
(0.0427)						[3.65 , 5149]
	Yes	0	0.116	0.003	0.028	1.0151
						[0.21 , 2.058]
C-I	*No*	0	0.450	0.050	0.113	2008
(0.1099)						[483 , 5461]
	Yes	0.003	0.053	0.012	0.041	8.81
						[0.9 , 19.54]
D-P	*No*	0	0.043	0.011	0.053	7.53
(0.0426)						[2.01 , 15.31]
	Yes	0	0.084	0.005	0.018	1.09
						[0.20 , 2.57]
D-I	*No*	0	0.055	0.129	0.061	2.59
(0.3922)						[0.08 , 9.61]
	Yes	0	0.060	0.031	0.050	0.95
						[0.18 , 3.38]

Notes to Table 6:

1. The effective semi elasticity of each model is reported in parentheses in the first column. All other numbers are result of 1000 Monte Carlo replications.
2. For the second order approximation, without measurement error case instruments are lag interest rate, lag consumption growth and lag income and lagged consumption growth square. For the measurement error cases, we use the twice lagged consumption instruments with the lag interest rate and lag income.
3. Instrument validity test is a t-test obtained from the regression of constructed residuals on instruments.
    a. Instrument validity columns report for each instrument the fraction of t stats with absolute value greater than 1.96 (critical value at 5% significance level)
    b. Corresponding mean values of R^2 for each regression are reported.
4. Instrument Relevance column reports the Cragg-Donald F (CDF) statistic from the first stage of IV. For the first order approximation interest rate is the only endogenous variable and lagged interest rate, twice lagged consumption growth, lagged income are instruments. For the second order, endogenous variables are interest rate and lagged consumption growth square and instruments are lagged interest rate, twice lagged consumption growth, twice lagged consumption growth square and lagged income.
    c. Mean values of CDF are reported. CDF values at 10 and 90 percent are in parentheses.
    d. Stock and Yogo (2002) - test for weak instrument H_0 =bias of two stage estimation relative to OLS is greater than 10%
        i. Critical Value at 5% significance level when the number of instruments is 4 and endogenous variable is 2 for Linear GMM = 7.56, for LIML =4.72

Table 7: Second Order Approximation, Estimates of EIS and Prudence, 1000 replications

	True Value	EIS: $\left(\frac{1}{\gamma}\right) = 0.25$		PRUDENCE: $\left(\frac{\gamma+1}{2}\right) = 2.5$	
Model (effective semi-elasticity)		**Base**	**With Measurement Error**	**Base**	**With Measurement Error**
AL-P	*mean coefficient*	0.243	0.291	2.90	0.662
(0.0425)	*(mean std. error)*	(0.080)	(0.119)	(5.36)	(6.093)
	(mean bias as % of true parameter value)	[-3.6]	[16.4]	[16]	[-73.5]
AL-I	*mean coefficient*	0.189	0.201	5.595	5.30
(0.0542)	*(mean std. error)*	(0.197)	(0.202)	(0.896)	(2.93)
	(mean bias as % of true parameter value)	[-24.4]	[-19.6]	[124]	[112]
C-P	*mean coefficient*	0.243	0.245	2.109	1.14
(0.0427)	*(mean std. error)*	(0.040)	(0.063)	(2.031)	(5.87)
	(mean bias as % of true parameter value)	[-2.8]	[-2]	[-15.6]	[-54.4]
C-I	*mean coefficient*	0.237	0.228	2.468	1.212
(0.1099)	*(mean std. error)*	(0.041)	(0.059)	(0.364)	(2.779)
	(mean bias as % of true parameter value)	[-5.2]	[-8.8]	[-1.28]	[-51.5]
D-P	*mean coefficient*	0.224	0.242	1.93	0.585
(0.0426)	*(mean std. error)*	(0.058)	(0.080)	(3.22)	(6.347)
	(mean bias as % of true parameter value)	[-10.4]	[-3.2]	[-22.8]	[-76.6]
D-I	*mean coefficient*	0.240	0.239	6.23	0.820
(0.3922)	*(mean std. error)*	(0.123)	(0.131)	(7.188)	(6.77)
	(mean bias as % of true parameter value)	[-4]	[-4.4]	[149.2]	[-67.2]

1. For each model and estimation strategy, the first number is the mean value of the estimate of the EIS and Prudence in 1000 Monte Carlo replications. The second number, in parenthesis, is the mean robust standard errors clustered on period. The third number is the mean bias as the percentage of true parameter value. The effective semi elasticity of each model is reported in parentheses in the first column.

Table 8 : Different Data Structures
First Order Approximation with Measurement Error
Instrument Validity and Relevance Results.

		Instrument Validity			Instrument Relevance
		Fraction of t stats bigger than 1.96 (2.024)			*F-stat*
Model (effective semi-elasticity)		***Lag Interest Rate***	***Lag Consumption Growth***	***Lag Income***	Mean [10% , 90%]
AL-P *(0.0425)*	*Long Panel*	0	0.084	0.002	5524 [5522 , 5526]
	Short Panel	0	0.076	0.204	497 [496 , 500]
	Synthetic Cohort	0.006	0.068	0.032	5.95 [4.92 , 11.53]
AL-I *(0.0542)*	*Long Panel*	0	0.134	0.732	7145 [7140 , 7151]
	Short Panel	0	0.138	0.250	497 [496 , 500]
	Synthetic Cohort	0	0.037	0.064	7.97 [6.36 , 12.9]
C-P *(0.0427)*	*Long Panel*	0	0.109	0.004	7076 [7044 , 7113]
	Short Panel	0.014	0.135	0.024	491 [483 , 495]
	Synthetic Cohort	0.017	0.092	0.039	7.90 [6.36 , 15.23]
C-I *(0.1099)*	*Long Panel*	0	0.150	0.013	7074 [7035 , 7108]
	Short Panel	0.094	0.102	0.020	491 [485 , 497]
	Synthetic Cohort	0.011	0.085	0.041	7.70 [6.37 , 14.87]

Notes to Table 8:

1. Table reports the first order approximation results using the simulated data with measurement error. The measurement error that we add to the simulated data is i.i.d log normal with a unit mean and a variance of 0.004.

2. The average semi elasticity of each model is reported in parentheses in the first column. All other numbers are result of 1000 Monte Carlo replications.

3. For long panel and short panel data structures, we use the twice lagged consumption instrument with the lag interest rate and lag income. For the synthetic panel data, we also twice lagged the income.

4. Instrument validity test is a t-test obtained from the regression of constructed residuals on instruments.
    a. For long panel and short panel data structures; instrument validity columns report for each instrument the fraction of t stats with absolute value greater than 1.96 (critical value at 5% significance level)
    b. For synthetic panel data, instrument validity columns report for each instrument the fraction of t stats with absolute value greater than 2.024 (critical value at 5% significance level)

5. Instrument Relevance column reports the Cragg-Donald F (CDF) statistic from the first stage of IV. For the first order approximation interest rate is the only endogenous variable and lagged interest rate, twice lagged consumption growth, (twice) lagged income are instruments..
    a. Mean values of CDF are reported. CDF values at 10 and 90 percent are in parentheses.
    b. Stock and Yogo (2002) - test for weak instrument H_0 =bias of two stage estimation relative to OLS is greater than 10%
        i. Critical Value at 5% significance level when the number of instruments is 3 and endogenous variable is 1 for Linear GMM = 9.08, for LIML=6.46.

Table 9 : Different Data Structures, First Order Approximation with Measurement Error
Estimates of EIS, 1000 Replications

Model (effective semi-elasticity)		**GMM**	Mean bias as % of true parameter value	**LIML**	Mean bias as % of true parameter value
AL-P *(0.0425)*	*Long Panel*	0.290 (0.105)	[16]	0.291 (0.109)	[16.4]
	Short Panel	0.208 (0.11)	[-17]	0.208 (0.114)	[-17]
	Synthetic Cohort	0.261 (0.159)	[4.4]	0.295 (0.20)	[18]
AL-I *(0.0542)*	*Long Panel*	0.261 (0.194)	[4.4]	0.240 (0.198)	[-4.4]
	Short Panel	0.81 (1.007)	[224]	0.979 (1.113)	[290.8]
	Synthetic Cohort	0.183 (0.197)	[-26.8]	0.252 (0.240)	[-1.2]
C-P *(0.0427)*	*Long Panel*	0.258 (0.048)	[3.2]	0.251 (0.049)	[0.4]
	Short Panel	0.483 (0.366)	[93.2]	0.478 (0.406)	[91.2]
	Synthetic Cohort	0.245 (0.099)	[-2]	0.254 (0.119)	[1.6]
C-I *(0.1099)*	*Long Panel*	0.216 (0.044)	[-13.6]	0.223 (0.049)	[-10.8]
	Short Panel	0.476 (0.305)	[90.4]	0.481 (0.354)	[92.4]
	Synthetic Cohort	0.209 (0.104)	[-16.4]	0.220 (0.150)	[-12]

Notes to Table 9:

1. Table reports the first order approximation results using the simulated data with measurement error. The measurement error that we add to the simulated data is i.i.d log normal with a unit mean and a variance of 0.004.

2. The average semi elasticity of each model is reported in parentheses in the first column. All other numbers are result of 1000 Monte Carlo replications. For each model and estimation strategy, the first number is the mean value of the estimate of the EIS in 1000 Monte Carlo replications. The second number, in parenthesis, is the mean robust standard errors clustered on period. The third number which is in the square brackets is the mean bias as the percentage of true parameter value.

3. For long panel and short panel data structures, we use the twice lagged consumption instrument with the lag interest rate and lag income. For the synthetic panel data, we also twice lagged the income.

Table 10 : Heterogeneity
First Order Approximation with measurement error, 1000 replications

		Instrument Validity			Instrument Relevance	Estimates of EIS
		Fraction of t stats bigger than 1.96 (2.024)			*CDF –stat*	
Model		***Lag Interest Rate***	***Lag Consumption Growth***	***Lag Income***	[10% , 90%]	GMM
AL-P (γ =4, EIS=0.25)	*Long Panel*	0	0.084	0.002	5524 [5522, 5526]	0.234 (0.078) [-6.4]
AL-I (γ =4, EIS=0.25)	*Long Panel*	0	0.985	0.989	7458 [7450 , 7470]	0.194 (0.192) [-22.4]
AL-P (γ =6) (γ =4, EIS=0.167)	*Long Panel*	0.374	0.083	0.003	5522 [5521 , 5527]	0.147 (0.048) [-11.9]
CRRA Heterogenity						
AL-P (γ =4, 6; mean EIS=0.2085)	*Long Panel*	0	0.185	0.014	5520 [5518, 5523]	0.224 (0.078) [7.4]
AL-P (γ =4, 6; mean EIS=0.2085)	*Synthetic Cohort*	0.014	0.084	0.039	6.03 [4.92 , 11.63]	0.192 (0.144) [-7.9]
Discount Rate Heterogenity						
AL-P & AL-I (γ =4, EIS=0.25)	*Long Panel*	0.001	0.242	0.666	6980 [6936, 6993]	0.189 (0.113) [-24.4]
AL-P & AL-I (γ =4, EIS=0.25)	*Synthetic Cohort*	0	0.062	0.066	7.48 [6.37, 12.36]	0.201 (0.141) [-19.6]

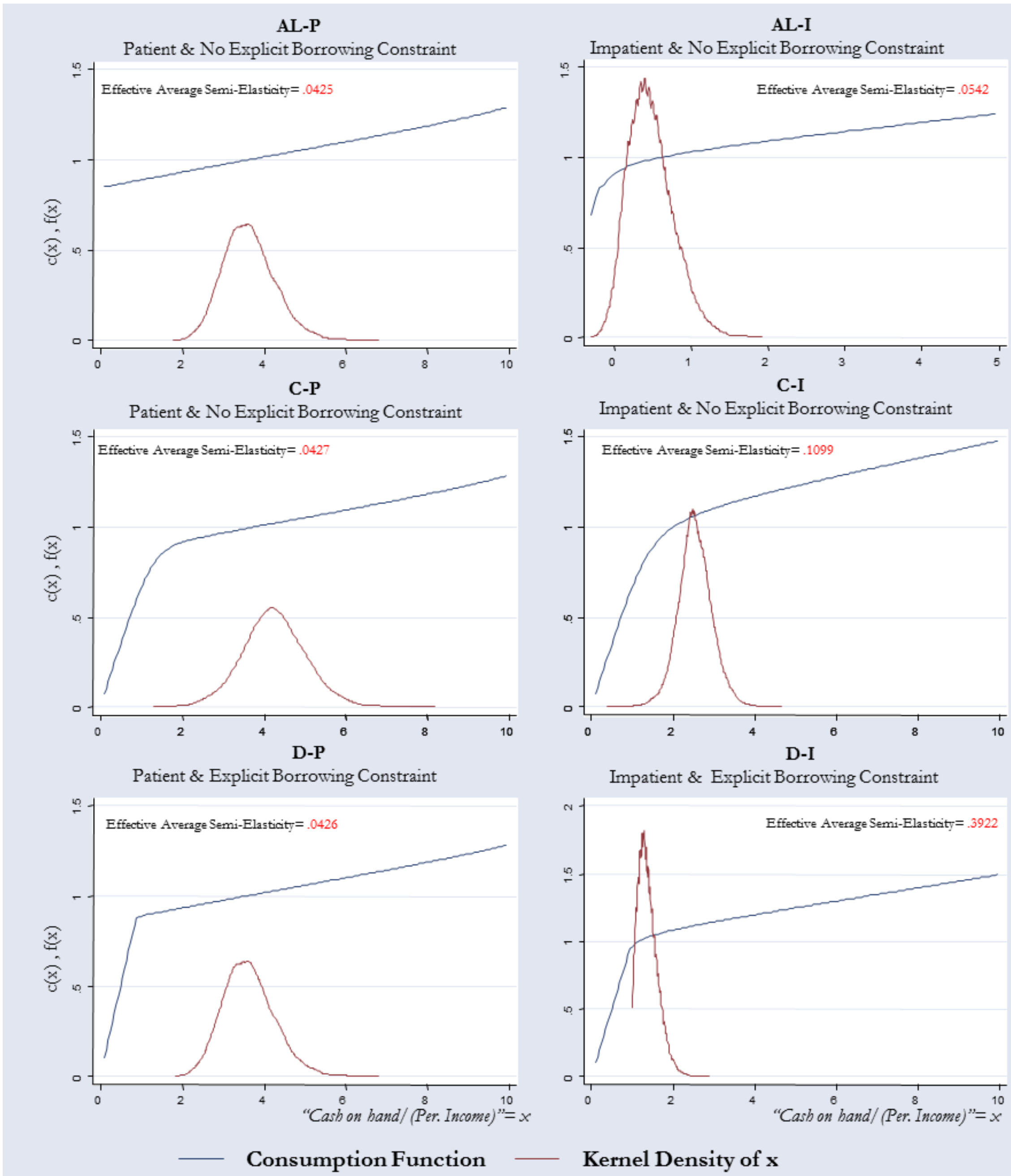
Notes to Table 10:

1. For long panel and short panel data structures, we use the twice lagged consumption instrument with the lag interest rate and lag income. For the synthetic panel data, we also twice lagged the income.

2. Instrument validity test is a t-test obtained from the regression of constructed residuals on instruments.
    a. For long panel and short panel data structures; instrument validity columns report for each instrument the fraction of t stats with absolute value greater than 1.96 (critical value at 5% significance level)
    b. For synthetic panel data, instrument validity columns report for each instrument the fraction of t stats with absolute value greater than 2.024 (critical value at 5% significance level)

3. Instrument Relevance column reports the Cragg-Donald F (CDF) statistic from the first stage of IV. For the first order approximation interest rate is the only endogenous variable and lagged interest rate, twice lagged consumption growth, (twice) lagged income are instruments..
    a. Mean values of CDF are reported. CDF values at 10 and 90 percent are in parentheses.
    b. Stock and Yogo (2002) - test for weak instrument H_0 =bias of two stage estimation relative to OLS is greater than 10%
        i. Critical Value at 5% significance level when the number of instruments is 3 and endogenous variable is 1 for Linear GMM = 9.08, for LIML=6.46.

Figure 1: Consumption Functions and Distribution of Cash-on-Hand



Note to Figure 1:

- Consumption Function and distribution of normalized cash-on-hand at the age of 40 . Non-parametrically calculated effective semi elasticity measures are in parentheses. X axis is cash-on-hand to permanent income ratio