DEFAULT RISK PREMIUM AND AGGREGATE FLUCTUATIONS IN A SMALL OPEN ECONOMY

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Abstract

This study investigates the implications of risk premium shocks for aggregate fluctuations in a small open economy with financial and informational frictions. A dynamic, stochastic, general equilibrium framework is developed, where the informational asymmetries among the agents in the model and the uncertainty in the production process necessitate financial intermediation in the economy. The Holmstrom-Tirole type of uncertainty in the production process also leads to collateralized borrowing by firms, with the physical capital stock of firms serving as the collateral as well as the factor of production. There is also a government sector in the economy that borrows domestically with a partial default risk. In order to compensate the lenders for the default risk included in the government bonds, the government has to offer them some risk premium in addition to the exogenously given world interest rate offered by the foreign bond issuers. It is shown that, under certain circumstances, it is possible for the government to reduce its debt and increase its spending in response to a positive, temporary risk premium shock.

Keywords: default risk premium, dynamic stochastic general equilibrium, aggregate fluctuations

JEL classification: F41, G15, G21, H39

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1 Introduction

The implications of default risk premia for the business cycles in emerging economies have started to arouse more interest among researchers over the last decade.\(^1\) One strand of literature focuses on the effects of movements in domestic variables on country risk premia and documents that risk premia respond systematically and counter-cyclically to the business cycles in emerging economies.\(^2\) Another line of research assumes that risk premia are exogenous to the domestic conditions in emerging countries. Authors advocating this argument relate the risk premium and the world interest rate through some exogenous, stochastic process and attempt to partially explain aggregate volatility in small open economies with interest rate fluctuations.\(^3\) This study proposes a theoretical framework incorporating financial and informational frictions as well as uncertainty to examine aggregate fluctuations in response to risk premium shocks in a small open economy.\(^4\) Financial frictions in the model are in the form of restrictions on the composition of deposits held by the domestic financial intermediaries in the economy. More specifically, financial intermediaries are assumed to be able to hold no more than a certain fraction of their total deposits as foreign deposits. Informational asymmetries among the agents in the economy and uncertainty in the production process necessitate financial intermediation and require special attention to the design of the loan contracts between the lenders (financial intermediaries) and the borrowers (firms) in the economy.

In this paper, a dynamic, stochastic, general equilibrium (DSGE, henceforth) framework with financial and informational frictions is developed in order to analyze the impact of risk premium shocks on a small open economy. In other words, aggregate fluctuations in response to risk premium shocks are investigated. It is shown that positive, temporary risk premium shocks lead to an increase in domestic deposits, loans, nominal exchange rate, tax rate and government spending, and a decrease in consumption, output, labor supply, investment and government borrowing. There is a cash-in-advance (CIA) framework, similar to that in Cogley and Nason (1994), modified in such a way that it includes financial and informational frictions. The economy consists of households, firms, foreign lenders, financial intermediaries, the government, the central bank and the financial regulator. The government in the economy is assumed to borrow domestically from the households with an endogenous partial default risk. In order to compensate the households, that also have the options of depositing at the financial intermediaries or holding foreign bonds, for the risk involved in the

\(^1\)See, among others, Arellano (2008).
\(^3\)Neumeyer and Perri (2005) provide evidence for the fact that interest rate shocks constitute an important factor for explaining business cycles in emerging economies.
government bonds, the government offers households some risk premium in addition to the international interest rate prevailing for the foreign bond holdings.

Risk premium is, by definition, relevant for emerging economies that are exposed to risks of default on debt due to their lack of adequately developed financial and macroeconomic infrastructure. Therefore, the implications of risk premium shocks for aggregate fluctuations are of special importance for emerging economies, that are also facing financial frictions. These two crucial aspects of emerging markets; namely, exposure to default risk and financial frictions, are combined in this study in a DSGE framework with financial intermediation.

The default risk has been modeled both exogenously and endogenously in several different frameworks in the literature. Mendoza and Yue (2008) explain output dynamics around defaults, countercyclical spreads, high debt ratios and key business cycle moments in a model with simultaneous default on public and private foreign obligations. They attempt to propose a model that reconciles the business cycle models treating default risk exogenously and the sovereign default models treating output fluctuations exogenously. They develop a model of strategic sovereign default with endogenous output dynamics and examine its quantitative predictions. Bi and Leeper (2010) criticize the strategic default literature due to its inability to match the data. More precisely, they argue that the default frequency is predicted by this literature to be far too high and the level of debt at which default occurs far too low. They propose a dynamic stochastic general equilibrium model, where the perceived riskiness of government debt depends partly on the fiscal environment, to study the tradeoffs between short-run fiscal stimulus and long-run sustainability.

The economy analyzed in this paper features financial frictions restricting the amount of foreign borrowing in the economy, informational asymmetries among the agents in the economy, and imperfections of the Holmstrom-Tirole (1997) type of uncertainty in the production process. Entrepreneurs that run the firms have two different project choices for production, both of which are subject to idiosyncratic risk. The projects yield positive output in the case of success and no output in the case of failure. The projects differ according to their probabilities of success and the private benefits they provide to the entrepreneurs. The project choices of the entrepreneurs are private information, whereas the project outcomes are verifiable by the financial intermediaries. Households and foreign investors are assumed to lack the ability to verify the project outcomes. Therefore, domestic and foreign investors prefer lending to firms indirectly, through financial intermediaries, rather than directly.

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5The strategic default literature has grown out of the papers by Eaton and Gersovitz (1981) and Eaton et al. (1986). This strand of literature models default on external debt as an optimal and strategic decision made by the government. See, among others, Aguiar and Gopinath (2006) for a recent study following this literature.
Exploring the potential impact of financial frictions on aggregate fluctuations has long been a topic of interest in the literature. Financial markets and institutions have been considered to have significant effects on aggregate economic activity. Financial frictions are incorporated into the model here through the introduction of a regulation in the economy that the financial intermediaries can hold no more than a certain fraction of their total deposits as foreign deposits. The parameter representing this fraction is assumed to be controlled by the financial regulator in the economy.

As mentioned above, a DSGE framework with financial intermediation and a government sector is developed in this study in order to analyze aggregate fluctuations in the case of risk premium shocks. As far as models with financial intermediation are concerned, there is a literature following Kiyotaki and Moore (1997) designing the loan contracts between borrowers and lenders with some durable asset, like land, as collateral. In these models, lenders cannot force borrowers to repay debts unless those debts are secured. In such a context, borrowers' assets like land serve both as factors of production and as collateral for new loans. Kiyotaki and Moore (1997) employ such a framework in the dynamic equilibrium model they develop in order to analyze the transmission mechanism in the case of temporary shocks. Kiyotaki and Moore show that small, temporary shocks to technology or income distribution can generate large, persistent fluctuations in output and asset prices. Employing land as collateral, von Hagen and Zhang (2008a) investigate the welfare implications of financial liberalization in a real, small open economy and suggest that financial opening facilitates the inflow of cheap foreign funds and improves production efficiency.

The loan contracts between the firms and the financial intermediaries in the current framework must be designed in such a way that the uncertainty involved in the production process is taken into account. At this point, collateralized borrowing becomes relevant here. However, in the current framework, it is the capital stock of the firms that is suggested to be used as collateral by the firms in the case of failure of their production projects. Therefore, the loan contract specifies the rate of interest on loans that is going to be valid in the case of success and the fraction of the capital stock of the firms to be handed over to the financial intermediaries in the case of failure. In this context, the output produced by the firms using capital and labor as inputs is the return of the projects in the case of success. It is assumed that there is no output in the case of failure. Due to this probability of zero output in the case of failure, firms have to use their capital stock as collateral in order to be able to borrow loans from financial intermediaries.

The structure of the rest of the paper is as follows: Section 2 describes the model and presents the solution of the model that consists of the system of equations includ-

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7The framework developed here is based on the model in Cakici (2011).
8For more information on models with financial intermediation, see Freixas and Rochet (1997).
9For a detailed analysis on the propagation of aggregate fluctuations, see Bernanke et al. (1996).
ing the first-order conditions and the market-clearing conditions. The simulation of the model and the impulse response functions are given in section 3. Finally, concluding remarks are presented in section 4.
2 The Model

The model developed here is one of cash in advance (CIA), similar to the model employed by Cogley and Nason (1994), modified in a way so as to incorporate financial intermediation and a government sector. All decisions are made after, and therefore completely reflect, the current period surprise change in risk premium. For the timing of the stock variables, like money and capital stock, ”stock as of the end of the period” convention is used. For instance, $M_t$ denotes the money stock as of the end of period $t$, that is to be transferred to period $t+1$, and $K_t$ is the capital stock at the end of period $t$. The economy consists of households, firms, foreign lenders, financial intermediaries, the government, the central bank and the financial regulator.

Infinitely-lived households, that are assumed to be the owners of the financial intermediaries, maximize their utility functions that depend on consumption, $C_t$, and hours worked, $H_t$. They decide how much money to deposit at the bank, $DD_t$, in order to earn $R_{Ht} - 1$ of net interest, how much government bonds, $GB_t$, to hold at a gross interest rate of $R^{GB}_t$ and how much foreign bonds, $FB_t$, to hold in return for a gross interest rate of $R^*$, how much to spend on consumption, and how much labor to supply to the firms. At the beginning of each period, households receive their deposits from the previous period plus the interest payment and make current-period deposits at the financial intermediaries. Additionally, they receive payments from the government and the foreign bond issuers for their bond holdings from the previous period inclusive of the interest payments and decide on bond holdings for the current period. They also supply labor, earn wage income and decide how much consumption to make and how much money to transfer to the next period.

Firms are owned by entrepreneurs, who have a finite but stochastic lifetime. Every period, a certain mass of entrepreneurs receives the signal of death and leaves the economy, whereas new entrepreneurs of equal mass enter the economy next period. In the aggregate, the share of entrepreneurs in the society is constant. Entrepreneurs maximize profits, $F_t$, by choosing next period’s capital stock, $K_t$, labor demand, $N_t$, and loans, $L_t$, they borrow from financial intermediaries. At the beginning of every period, the existing entrepreneurs in the economy pay back the loans they borrowed in the previous period from the financial intermediaries including the interest and borrow new loans for the current period. Entrepreneurs then use these loans to hire labor for production. The new entrants, on the other hand, are assumed to bring along some initial wealth with which they can buy the capital stock they need for production from the financial intermediaries. There is Holmstrom-Tirole (1997) type of uncertainty in the production process. Firms have two available project choices to produce goods, both of which are subject to idiosyncratic risk; namely, they yield positive return in the case of success and zero return in the case of failure. The projects differ according to their probabilities of success, with $p^H$ in the case of project ”good” and $p^L$ in the case of project ”bad”. The reason why the entrepreneurs might have incentives to choose project ”bad” is that it yields some private benefits, $PB$, to the entrepreneurs. Project
"good" yields no private benefits. It is assumed that the project outcomes can be perfectly verified by the financial intermediaries, which have the exclusive technology to do so, while the project choices of the entrepreneurs are unobservable. Households and foreign investors are assumed to lack the ability to verify the project outcomes. It is this uncertainty that rationalizes the existence of financial intermediation in the economy.

Foreign investors prefer lending to the firms through the financial intermediaries instead of directly, also due to their limited familiarity with the domestic economy. They are assumed to supply funds, $FD_t$, infinitely elastically at a constant interest rate, $R^*$, that is lower than the domestic loan rate.

Financial intermediaries (FI henceforth) maximize the expected infinite horizon discounted stream of dividends, $B_t$, they pay to households. They receive cash deposits from households, $DD_t$, cash deposits from foreign investors, $FD_t$, and cash injection, $X_t$, from the central bank (which equals the net change in nominal money balances, $M_t - M_{t-1}$). The FI then use these funds to give loans to firms. According to the loan contract between the FI and the firms, the FI gain a net return of $R_{Ft} - 1$ in the case of success of the firms’ projects and a certain fraction, $\mu$, of the capital stock of the firms in the case of failure. The FI are allowed to hold no more than a certain fraction of their total deposits as foreign deposits. In other words, there is an upper limit on the fraction of foreign deposits over total deposits to be held by the FI. At the beginning of each period, financial intermediaries receive the loans they lent to firms in the previous period inclusive of the interest payments, sell out the capital stock they hold due to the failure of the firms’ projects in the previous period, and pay back the domestic and foreign deposits they collected in the previous period together with the interest. Additionally, they accept current-period deposits from households and foreign lenders and give current-period loans to firms.

Finally, the government finances unproductive government purchases, $G_t$, through collecting taxes and issuing one-period government bonds, $GB_t$. It raises tax revenue through a time-varying income tax, $\tau_t$. The government debt is denominated in foreign currency and is subject to partial default risk. More specifically, only with probability $p_t^R$, the government is able to repay its debt totally and with probability $1 - p_t^R$, it can pay only a certain fraction of its debt, determined by the parameter $\chi$, back to the households. In order to compensate the households for the default risk of the government bonds, the government has to offer some risk premium in addition to the world interest rate prevailing for the foreign bond holdings. The risk premium is determined partly by the exogenous risk premium shock and partly by the probability of default by the government, which in turn depends on the value of the government debt.
2.1 Households

A typical infinitely-lived household maximizes an expected utility function of the form

\[ E_0 \left\{ \sum_{t=0}^{\infty} \beta^t [(1 - \phi)\ln C_t + \phi \ln (1 - H_t)] \right\} , 0 < \beta, \phi < 1 \] (1)

where \( \beta \) is the discount factor, subject to the CIA constraint

\[ P_tC_t \leq M_{t-1} + (1 - \tau_t)W_tH_t - DD_t + R_{H,t-1}DD_{t-1} + GB_tE_t - FB_tE_t + FB_{t-1}E_{t-1}R^* \] (2)

where \( Z_t = [p_t^R GB_{t-1}E_tR^{GB}_{t-1} + (1 - p_t^R)\chi GB_{t-1}E_tR^{GB}_{t-1}] \). It is assumed that the money stock transferred from previous period, labour income, interest income on previous-period deposits net of current-period deposits, and interest income on previous-period bond holdings net of current-period bond holdings are available for consumption purchases of households. The maximization problem of households is also subject to the budget constraint

\[ M_t = M_{t-1} + (1 - \tau_t)W_tH_t - P_tC_t + R_{H,t-1}DD_{t-1} - DD_t + Z_t - GB_tE_t + FB_tE_t + FB_{t-1}E_{t-1}R^* - FB_tE_t + B_t \] (3)

and to the nonnegativity constraint

\[ 0 \leq DD_t \] (4)

where \( P_t \) and \( W_t \) denote the price level of the single consumption good and the nominal wage rate, respectively. \( B_t \) refers to the dividends paid to households by financial intermediaries. \( R_{H,t} \) is the gross nominal interest rate on household deposits. \( DD_t \) is denominated in domestic currency whereas \( GB_t \) is denominated in foreign currency.

2.2 Firms

There is a certain mass of entrepreneurs in the population that runs the firms in the economy. Each period, \( \pi \) percent of them dies and new entrants of an equal mass are added to the population such that the size of the population remains the same.\(^{10}\)

At the micro level, this means that, with probability \( \pi \), each entrepreneur receives a signal every period that he will die at the end of the period. Entrepreneurs receiving

\(^{10}\)This assumption is needed in order to be able to prevent the entrepreneurs from accumulating too much profits that would invalidate the borrowing constraint of the firms. In the literature, part of profits accumulated by the firms is, alternatively, distributed to the households, that are assumed to own the firms, as dividends (see, among others, Feldstein and Green (1983)). However, in the current context, there are informational asymmetries in the sense that households are not able to observe firms' profits. Therefore, the assumption that firms are owned by households and that they distribute part of their profits to households as dividends is not reasonable here.
this signal consume everything that is left at the end of the period after paying back
the debt. The new entrants bring along some initial wealth with which they can
obtain the capital stock they need to start production. They buy this capital stock
from the financial intermediaries, which receive capital stock as repayment of loans in
the case of failure of the firms’ projects. Entrepreneurs, therefore, maximize profits
taking into account this probability of death. There is uncertainty involved in the
production process of the firm, resulting from the fact that the entrepreneur has two
available projects to produce the single consumption good11, both of which are subject
to idiosyncratic risk. More specifically, the entrepreneur has two project choices that
differ according to their probabilities of success and the private benefits they provide
to the entrepreneur, and there is positive output in the case of success of the projects
while there is no output in the case of failure. \( p^H \) and \( p^L \) denote the probabilities of
success of the “good” and the “bad” project, respectively, where \( 0 < p^L < p^H < 1 \).
The entrepreneur gets PB amount of private benefits per capital stock if he chooses the
project ”bad” whereas there is no private benefit obtained from the project ”good”.
The entrepreneur’s payoffs from the projects can be summarized as follows:

In the case of ”good project”:

\[
p^H [P_y Y_t - R_{F,t} L_t] + (1 - p^H)[0 - \mu (1 - \delta) K_{t-1} P_t]
\]

In the case of ”bad project”:

\[
p^L [P_y Y_t - R_{F,t} L_t] + (1 - p^L)[0 - \mu (1 - \delta) K_{t-1} P_t] + PB K_t P_t
\]

Entrepreneurs’ profits, \( F_t \), are given as

\[
F_t \leq p^H P_t Y_t - p^H \pi R_{F,t} L_t - p^H (1 - \pi) R_{F,t-1} L_{t-1} - (1 - p^H) \mu (1 - \delta) K_{t-1} P_t
\]  

(5)

where \( \delta \) is the constant physical depreciation rate of capital. Given that the entreprenuer chooses the project ”good” (which will be the case as long as the incentive
constraint stated below holds), with probability \( p^H \) the firm is able to make use of the
loans it borrows from the FI to hire \( N_t \) amount of labor, which it can employ together
with the capital stock it has, \( K_{t-1} \), to produce \( Y_t \) amount of output. The entrepreneur
makes in this case an interest payment to the FI for the loans at the rate specified
in the loan contract, \( R_{F,t} \). In the case of failure, there is no output produced and the
entrepreneur has to transfer a certain amount of its capital stock, which it used as
collateral in order to be able to borrow from the FI, to the FI. \( \mu \) represents the fraction
of the capital stock of the firm guaranteed in the loan contract to be handed over to
the FI in the case of failure. Due to the fact that capital stock is employed in the
production process and therefore subject to depreciation independent of the project
outcome of the firm, it is the net-of-depreciation amount of capital stock, the fraction
of which is to be handed over to the FI in the case of failure. The period t loans of the

\footnote{The consumption good and the capital good are assumed to be identical for analytical purposes.}
firm, $L_t$, and the period $t$ wage payments, $W_t N_t$, cancel out above.

The production function of the firm is given by

$$Y_t = K_{t-1}^{a}(A_t N_t)^{1-a} \quad (6)$$

where $A_t$ denotes technology, the shock process of which is a unit root with drift in the log of technology, given as

$$\ln A_t = \gamma + \ln A_{t-1} + \epsilon_{A,t}, \quad \epsilon_{A,t} \sim N(0, \sigma_A^2) \quad (7)$$

The capital accumulation equation is given as

$$K_t = p^H[\pi \mu (1-\delta)K_{t-1} + (1-\pi)(I_t + (1-\delta)K_{t-1})] + (1-p^H)(1-\mu)(1-\delta)K_{t-1}, \quad 0 < \delta < 1 \quad (8)$$

$K_t$ is the level of physical capital to be employed in the production process at time $t+1$, determined by the entrepreneur at time $t$. Equation (8) is the capital accumulation equation at the macro level in the sense that $\pi$ represents here the mass, out of a group of entrepreneurs, that dies each period, rather than the probability of death of a single entrepreneur, as in the micro sense. Therefore, the first term in the parenthesis on the right-hand side of the equation, $\pi \mu (1-\delta)K_{t-1}$, stands for the capital stock held by the new entrants at the beginning of every period (recall the assumption that an equal mass, $\pi$, of new entrepreneurs enters the economy each period so that the total size of the population remains unchanged). The second term represents the amount of capital stock accumulated by the successful entrepreneurs continuing to live and produce; the net-of-depreciation amount of capital stock of the current period, $(1-\delta)K_{t-1}$, plus the amount of investment, $I_t$. Investment is equal to the real profits made by the successful entrepreneurs in the economy. The last term on the right-hand side of the equation gives the amount of the capital stock of the unsuccessful entrepreneurs that cannot make positive real profits due to the fact that there is no output in the case of failure of the firms’ projects.

It might help to make the capital accumulation process more clear to rewrite the profit and the capital accumulation functions from a micro perspective such that the profit made and the capital stock accumulated are captured separately for the cases of success and failure of the projects. Let $F_t^s$ and $K_t^s$ ($F_t^f$ and $K_t^f$) denote the profit and the capital stock, respectively, accumulated by a single entrepreneur in the case of success (failure) of the projects. Then, the profit made and the capital stock accumulated are given as

$$F_t = F_t^s p^H + F_t^f (1-p^H) \quad (9)$$

and

$$K_t = K_t^s p^H (1-\pi) + K_t^f (1-p^H) \quad (10)$$
The profits made in the case of success of the projects, which are invested, and therefore transferred to the next period, by the entrepreneurs, are given by the following equation:

\[ F_s^t = P_t Y_t - \pi R_{Ft} L_t - (1 - \pi) R_{Ft-1} L_{t-1} \quad (11) \]

The corresponding capital stock in the case of success of the projects is, therefore, given as

\[ K_s^t = \frac{F_s^t}{P_t} + (1 - \delta) K_{t-1} \quad (12) \]

In the case of failure of the firms’ projects, there is no output produced; therefore, profits are

\[ F_f^t = -\mu (1 - \delta) K_{t-1} P_t \quad (13) \]

whereas the capital stock is given as

\[ K_f^t = \frac{F_f^t}{P_t} + (1 - \delta) K_{t-1} = (1 - \mu)(1 - \delta) K_{t-1} \quad (14) \]

Plugging equations (12) and (14) into equation (10), the capital accumulation equation given in equation (8) is obtained at the micro level.

Entrepreneurs’ maximization problem is subject to the constraint reflecting the fact that the firm finances its wage payments with the loans it borrows from the FI. Hence, it obeys

\[ W_t N_t \leq L_t \quad (15) \]

Finally, there is an incentive constraint for the firm to choose the project ”good”:

\[ P_t Y_t - R_{Ft} L_t \geq \frac{PBK_t P_t}{(p_H - p_L)} \quad (16) \]

As long as this constraint holds, the entrepreneur maximizes profits given in equation (5) that is written for the case of project ”good”. The incentive constraint also gives the borrowing constraint of the firm

\[ R_{Ft} L_t \leq P_t Y_t - \frac{PBK_t P_t}{(p_H - p_L)} \quad (17) \]

from which the loan demand, \( L^d_t \), is obtained:

\[ L^d_t = \frac{P_t Y_t - \frac{PBK_t P_t}{(p_H - p_L)}}{R_{Ft}} \quad (18) \]

In the standard collateralized borrowing literature, the maximum amount of loans supplied to the firms by the FIs is determined according to the value of the collateral
firms have. More precisely, the value of the loans the FIs supply does not exceed the value of the collateral of the firms, which is exactly what the collateral constraints imply. However, in those frameworks, the total supply of loans is often fixed (limited). Therefore, it is an optimal allocation problem of loans and assets (that are used as collateral) among agents with differing productivities. In the current framework, the total amount of loans available is determined in part stochastically, due to the fact that it is the sum of total deposits and the monetary injection the FIs hold and that the monetary injection is an exogenous stochastic process. As a result, the optimization problem here has to do with the allocation of the incoming (injected) new loans available to the FIs. In the standard CIA literature, where the total supply of loans is subject to uncertainty similar to the framework here, the allocation problem of the new loans is solved through the adjustment of the loan rate. To be more specific, as the total supply of loans increase, the loan rate falls so that the firms continue to demand the extra loans available, and the FIs continue to supply the extra loans to the firms as long as their return (the loan rate) is positive. However, in those frameworks, there is no uncertainty in the production process; therefore, there is no need for collateralized borrowing. The major novelty of the framework in this paper is to reconcile these two strands of literature with the motivation of analyzing aggregate fluctuations in the case of risk premium shocks for an economy with uncertainty and frictions.

For the FI to continue lending the new resources available as loans to firms, the return on the loans must be positive and greater than the return on any outside option of the FI; that is,

\[ p^H R_{Ft} + (1 - p^H) \mu (1 - \delta) K_{t-1} P_{t+1} \geq R_{Ht} > 0 \]  \hspace{1cm} (19)

where \(R_{Ht}\) represents the return on the outside option of the FI, namely, lending to other FIs.\(^{12}\) The parameters of the model are calibrated in such a way that the inequality holds.

The inequality also gives the loan supply, \(L^s_t\):

\[ L^s_t = \frac{(1 - p^H) \mu (1 - \delta) K_{t-1} P_{t+1}}{(R_{Ht} - p^H R_{Ft})} \]  \hspace{1cm} (20)

The loan market equilibrium condition is obtained through equating the loan supply and the loan demand equations.

\(^{12}\)The return on loans just has to be positive, once the assumption that FIs can lend to and borrow from one another is relaxed.
2.3 Financial Intermediaries

The objective of the FI is to maximize the expected infinite horizon discounted stream of dividends it pays to households:

\[
E_0 \left\{ \sum_{t=0}^{\infty} \beta^{t+1} \frac{B_t}{C_{t+1}P_{t+1}} \right\}
\]

subject to first the budget constraint

\[
B_t \leq \left[ p^H \pi R_{F,t} L_t + p^H (1 - \pi) R_{F,t-1} L_{t-1} \right] + \left[ (1 - p^H) \mu (1 - \delta) K_{t-1} P_t \right] - R_{H,t-1} DD_{t-1} - R^* F D_{t-1} E_{t-1}
\]

where \( FD_t \geq 0 \) is denominated in foreign currency and \( E_t \) is the nominal exchange rate (the domestic currency value of one unit of the foreign currency). Purchasing power parity (PPP) holds so that \( P_t = E_t P_t^* \), with \( P_t^* \) denoting the foreign price level. \( P_t^* \) is normalized to 1; therefore, the fluctuations in the exchange rate in response to the shocks are captured by the movements in the domestic price level. The net present value of future dividends is discounted by the marginal utility of consumption due to the fact that financial intermediaries are owned by households and that an extra unit of dividend is valued by households to the extent that it enables future consumption. The monetary injection, \( X_t \), and the total deposits at the FI in period \( t \), \( D_t \), are used by the FI for the period \( t \) loans, \( L_t \); therefore, they cancel out in the budget constraint of the FI.

\( X_t \) is the monetary injection during date \( t \), \( X_t = M_t - M_{t-1} \), defined similarly to Nason and Cogley (1994).\(^\text{13}\)

The exogenous stochastic process for the growth rate of the monetary injection is given as

\[
\ln m_t = (1 - \rho) \ln m^* + \rho \ln m_{t-1} + \epsilon_{M,t}, \quad \epsilon_{M,t} \sim N(0, \sigma_M^2)
\]

where \( m_t = \frac{M_t}{M_{t-1}} \).

It is therefore an autoregressive stationary process in the growth rate of money, but an AR(2) with a unit root in the log of the level of money. This can be seen from the definition of \( m_t \) which can be rewritten as \( \ln M_t = \ln M_{t-1} + \ln m_t \).

The second constraint the FI faces, namely the balance sheet constraint, requires that the liabilities of the FI are less than or equal to its assets

\[
D_t + X_t \leq L_t
\]

\(^\text{13}\)It can be seen from the households’ budget constraint that \( X_t \) equals the total income of households (labor income+interest on deposits+dividends from the financial intermediaries) minus consumption, which cannot be negative since households are assumed to transfer some cash to the next period.
where \( D_t = DD_t + FD_tE_t \) and \( FD_tE_t = \psi D_t, \) \( DD_t = (1 - \psi)D_t. \)

\( \psi \) represents the financial openness parameter assumed to be controlled by the financial regulator and varies between 0 and 1. Higher levels of \( \psi \) imply higher degrees of financial integration.

### 2.4 Government

The government in the economy finances unproductive government purchases denominated in the consumption good, \( G_t, \) through raising tax revenues and issuing one-period government bonds, \( GB_t. \) Tax revenues are obtained by a time-varying flat rate tax, \( \tau_t, \) on labor income. Government bonds are denominated in foreign currency and government debt is subject to a partial default risk, with an endogenous default probability \( 1 - p_t^R. \) To be more precise, probabilistically, the government might not be able to repay its debt totally. The government budget constraint is then given as

\[
\tau_t W_t N_t + GB_tE_t = [p_t^R GB_{t-1} R_{t-1}^{GB} + (1 - p_t^R)\chi GB_{t-1} R_{t-1}^{GB}] + G_t P_t
\]

(25)

where \( p_t^R \) is the probability with which the government will be able to repay its debt totally and \( \chi \) denotes the percentage of the government debt to be repaid in the case of default. \( R_t^{GB} \) represents the gross nominal interest rate on government bonds, which is determined by the following equation:

\[
R_t^{GB} = R^* + RP_t
\]

(26)

\( RP_t \) stands for the ”risk premium” that the government has to pay in addition to the world interest rate, \( R^*, \) in order to compensate the households, that also have the option of holding foreign bonds, for the risk of default on the government debt. Risk premium is determined partly by the probability of default by the government, \( 1 - p_t^R, \) and partly by a shock to the risk premium:

\[
RP_t = \frac{(1 - p_t^R)}{p_t^R} + \epsilon_{RP,t}, \quad \epsilon_{RP,t} \sim N(0, \sigma_{RP}^2)
\]

(27)

where \( \epsilon_{RP,t} \) denotes the risk premium shock.

The probability of default, \( 1 - p_t^R, \) is a function of the value of the government debt\(^{14} \):

\(^{14}\)Alternative ways have been proposed in the literature to endogeneize default rates. Uribe (2006b), for instance, suggests a framework where future default rates are predicted by current and past fiscal deficits.
\[(1 - p^R_{t+1}) = \frac{e^{GB_t E_t} - 1}{e^{GB_t E_t} + 1} \]  

(28)

The government also follows a simple tax rule, adjusting the tax rate in response to a change in the value of the government debt:

\[ \frac{\tau_t}{\tau_{t-1}} = \lambda \frac{GB_t E_t}{GB_{t-1} E_{t-1}} , \quad 0 < \lambda < 1 \]  

(29)

where \( \lambda \) might be interpreted as the tax adjustment parameter.

### 2.5 System of Equations

In a stochastic setting, the solution of the model is not a series of numbers that match a given set of equations, as in a deterministic setting. In a stochastic environment, the best thing agents can do is to specify a decision, policy or feedback rule for the future, in other words, their optimal actions contingent on each possible realization of shocks. Therefore, it is a function satisfying the model’s equilibrium conditions that is being searched. The system of equations consists of the first-order conditions of the agents’ optimization problems and the market-clearing conditions of the goods, labor, money and credit markets.

The first-order conditions of the household’s optimization problem are given as:

\[ \frac{(1 - \phi)}{C_t} = \frac{\phi P_t}{W_t(1 - H_t)} \]  

(30)

from the maximization of the household’s utility function with respect to consumption,

\[ \frac{\beta R_{Ht}}{W_t(1 - H_{t+1})} = \frac{1}{W_t(1 - H_t)} \]  

(31)

from the maximization with respect to deposits and

\[ \frac{1}{W_t(1 - H_t)(1 - \tau_t)} = \frac{\beta [R^G_{t+1} E_{t+1} (p^R_t - 2 e^{GB_{t+1} E_{t+1}}) + \chi R^G_{t+1} E_{t+1} (1 - p^R_t + 2 e^{GB_{t+1} E_{t+1}})]}{W_t(1 - H_{t+1})(1 - \tau_{t+1})} \]  

(32)

from the maximization with respect to government bonds.

Combining (30) and (31) yields
\[
\frac{1}{C_t P_t} = \frac{\beta R_{HH}}{C_{t+1} P_{t+1}} \quad (33)
\]

From the firm’s optimization problem, there is the binding borrowing constraint

\[
R_{Fl} L_t (p^H - p^L) = P_t Y_t (p^H - p^L) - PBK_t P_t \quad (34)
\]

and the equilibrium condition that the marginal product of labor equals the real wage

\[
K_t \alpha (1 - \alpha) (A_t N_t) - \alpha A_t = W_t P_t \quad (35)
\]

that are among the equations constituting the solution of the model.

Finally, the FI maximizes its dividends with respect to deposits, which leads to the following first order condition:

\[
p^H \pi R_{Fl} P_{t+2} C_{t+2} = \beta P_{t+1} C_{t+1} [(1 - \psi) R_{HH} + \psi R^* - p^H (1 - \pi) R_{Fl}] \quad (36)
\]

As stated above, all markets clear at the equilibrium. The following equations represent equilibrium in the goods, labor, money, and credit markets, respectively:

\[
C_t + I_t + G_t + NX_t = Y_t \quad (37)
\]

\[
N_t = H_t \quad (38)
\]

\[
P_t C_t = M_{t-1} + X_t \quad (39)
\]

\[
D_t + X_t = L_t \quad (40)
\]

\[NX_t\] denotes net exports, which is equal to the net interest payment on foreign borrowing minus the change in the amount of foreign borrowing in a given period. Therefore,

\[
P_t NX_t = (R^* - 1) FD_{t-1} E_{t-1} - [FD_{t-1} E_{t-1} - FD_t E_t] \quad (41)
\]

Combining (15), (30) and (38) gives

\[
\left( \frac{\phi}{1 - \phi} \right) \frac{P_t C_t}{1 - N_t} = \frac{L_t}{N_t} \quad (42)
\]

which constitutes another equation of the solution.
Finally, there is the purchasing power parity (PPP) condition

\[ P_t = E_t P_t^* \] (43)

with \( P_t^* = 1 \), which is used to convert the foreign currency denominated foreign deposits, foreign bond holdings and government bond holdings into domestic currency in the system of equations.

The model, however, needs to be made stationary first so that it can be linearized around the steady-state and that it returns to the steady-state after a shock. The problem of non-stationarity arises because of having stochastic trends in money and technology. In the absence of shocks, real variables grow with \( A_t \) (except \( N_t \) which is stationary since there is no population growth), nominal variables grow with \( M_t \) and prices grow with \( M_t/A_t \). Detrending is carried out as follows (where hats above variables denote stationarity):

For real variables, \( \hat{q}_t = q_t/A_t \) where \( q_t = [Y_t, C_t, K_t, I_t, NX_t, G_t] \). For nominal variables, \( \hat{z}_t = z_t/M_t \) where \( z_t = [W_t, D_t, L_t] \). For prices, \( \hat{P}_t = P_t A_t/M_t - 1 \).

The stationary system of equations is as follows:

\[(1 - \phi)(1 - N_t)L_t m_t = \phi P_tC_tN_t \] (44)

\[ m_tC_{t+1}P_{t+1} = \beta R_{Ht}C_tP_t \] (45)

\[ \frac{1}{W_t(1 - H_t)(1 - \tau_t)} = \frac{\beta[R_t^{GB}E_{t+1}(p_t^H - \frac{2p_t^H E_t G_t E_t}{(G_t E_t + 1)^2}) + \chi R_t^{GB}E_{t+1}(1 - p_t^H + \frac{2p_t^H E_t G_t E_t}{(G_t E_t + 1)^2})]}{W_t+1(1 - H_{t+1})(1 - \tau_{t+1})m_{t+1}} \] (46)

\[ R_{Fl}L_t(p^H - p^L)m_t = P_t Y_t(p^H - p^L) - P BK_tP_t \] (47)

\[ K_{t-1}^\alpha(1 - \alpha)a_t^{-\alpha}N_t^{-\alpha} = W_t m_t \] (48)

\[ p^H \pi R_{Fl}P_{t+2}C_{t+2}m_{t+1} = \beta P_{t+1}C_{t+1}[1 - \psi)R_{Ht} + \psi R^* - p^H(1 - \pi)R_{Fl}] \] (49)

\[ C_t + I_t + G_t + NX_t = Y_t \] (50)

\[ P_tC_t = m_t \] (51)
\[
\frac{DD_t}{(1 - \psi)} + 1 - \frac{1}{m_t} = L_t
\]  
\[W_t N_t = L_t \]  
\[Y_t = K_{t-1}^\alpha a_t^{-\alpha} \widetilde{N}^{1-\alpha} \]

\[K_t a_t = p^H [\pi \mu (1-\delta) K_{t-1} + (1-\pi) (I_t a_t + (1-\delta) K_{t-1})] + (1-p^H)(1-\mu)(1-\delta) K_{t-1} \]  
\[N X_t = \psi [(R^* - 1) DD_{t-1} - DD_{t-1} + DD_t m_t] / (1 - \psi) P_t \]

\[\tau_t W_t N_t m_t + GB_t E_t m_t = [p_t^R GB_{t-1} E_t R_{t-1}^G B + (1 - p_t^R) \chi GB_{t-1} E_t R_{t-1}^{GB}] + G_t P_t \]

\[\frac{\tau_t}{\tau_{t-1}} = \chi (\frac{GB_t E_t}{GB_{t-1} E_{t-1}}) m_t \]

\[R_t^{GB} - R^* = \frac{(1 - p_t^R)}{p_t^R} + \epsilon_{RP,t} \]

\[(1 - p_t^R) = \frac{e^{GB_t E_t} - 1}{e^{GB_t E_t} + 1} \]

\[P_t = E_t \]

Given the equations (44)-(61) and the risk premium shock given in (27), the expected future paths of the variables \(Y_t, C_t, I_t, NX_t, P_t, DD_t, L_t, N_t, K_t, W_t, GB_t, G_t, \tau_t, p_t^R, E_t, R_F, R_H, R_t^{GB} \), namely, the impulse response functions, conditional on temporary risk premium shocks in period 1 are obtained next.
3 Results

3.1 Simulation

The procedure of making the model stationary is followed by linearization and simulation.\textsuperscript{15} The model is linearized up to first order.\textsuperscript{16} The perturbation method employed to solve and to simulate the model can be summarized as follows: The solution to the system of equations obtained in the previous section is a set of equations relating variables in the current period to the past state of the system and current shocks, that satisfy the original system. These are referred to as "the policy functions". In the linearization up to first order, future shocks enter the linearized system of equations only with their first moments (which are zero in expectations); therefore, they drop out when taking expectations of the equations. This is why certainty equivalence holds in the system linearized up to first order. The (approximate) policy functions are obtained by first rewriting the system in terms of past variables, current and future shocks, and then linearizing it around the steady states. Impulse response functions are then acquired simply through iterating the policy functions starting from some initial values (given by the steady states).\textsuperscript{17}

For simulations, the following values are assigned to the structural parameters of the model: $\alpha=0.32$, $\beta=0.99$, $\phi=0.76$, $\delta=0.025$, $\gamma=0.003$, $\rho=0.7$.\textsuperscript{18} The success probability of project "bad", $p^b$, is set equal to 0.1. The fraction of the capital stock to be used as collateral by firms, $\mu$, is taken to be equal to 0.1, whereas the probability of death of firm managers, $\pi$, is set to 0.6. The parameter measuring the private benefits of entrepreneurs from project "bad", $PB$, is taken as 0.5. The fraction of the debt on which the government defaults, $\chi$, is assigned the value 0.7. The tax adjustment parameter, $\lambda$, is calibrated to 0.99.

\textsuperscript{15}The linearization and the simulation of the model are carried out using DYNARE, which is a pre-processor and a collection of MATLAB routines that have been developed to support modern macro modeling.

\textsuperscript{16}In the case of linearization up to the first order, agents behave as if future shocks were equal to zero (since their expectation is null), due to certainty equivalence. In the linearization up to second order, agents make their decisions knowing that the future value of innovations are random but will have zero mean. This is not the same thing because of Jensen’s inequality. For more detailed information, see DYNARE User Guide.

\textsuperscript{17}The impulse response functions presented in the next section depict the responses of the variables in terms of deviations from the steady states.

\textsuperscript{18}For the parameter values, Mendoza (1991), Nason and Cogley (1994) and Dib (2003) are followed. Dib (2003) employs quarterly Canadian data for the calibration and the estimation in his small-open-economy DSGE model.
3.2 Impulse Response Functions

Figure 1 displays the impulse response functions of the variables in the model in the case of a positive, one-time, temporary risk premium shock in period 1. The positive risk premium shock leads directly to an increase in the interest rate on government bonds, which leads to a contraction in the amount of government bonds supplied by the government. The risk premium shock also causes an increase in the nominal exchange rate; that is, a depreciation in the exchange rate. Due to the fact that the tax rate growth is proportional to the growth of the total value of the government debt, according to the simple tax rule followed by the government, the tax rate increases in response to the risk premium shock. Determined partly by the size of the exchange rate depreciation and partly by the tax adjustment parameter, the increase in the government revenue leads consequently to an expansion in government spending.

\[\psi = 0.9, \quad R^* = 1.01, \quad p^H = 0.9 \quad \text{and} \quad \lambda = 0.99.\]
In response to the rise in the price level, households reduce their consumption. Domestic deposit holdings by households, on the other hand, increase since the supply of government bonds is lower due to the higher costs of borrowing for the government. As deposits rise, so do total loans available for firms. However, the increase in the amount of loans does not consequently lead to an increase in employment, and subsequently in production, as a result of the rise in the distorting tax rate creating disincentives to work and thereby causing a fall in the labor supply. Therefore, output also falls in response to the positive risk premium shock. The fall in output and investment in the case of a positive risk premium shock confirms in a monetary framework the findings of the RBC literature for open economies.  

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4 Conclusion

Aggregate fluctuations in the case of risk premium shocks are analyzed in this study for a small open economy. The business cycle in a DSGE framework with financial and informational frictions is investigated in response to positive, temporary risk premium shocks. The risk premium arises in the model due to the existence of a government sector that borrows domestically with a partial default risk. More specifically, in order to be able to compensate the domestic households, that also have the option of holding foreign securities, for the default risk involved in the government bonds, the government offers some risk premium in addition to the international interest rate prevailing for the foreign bonds. A risk premium shock, therefore, affects the investment decisions of households in terms of relative asset holdings. It also has implications for government spending, which is supposed to be financed through domestic borrowing and taxation.

Risk premium is, by definition, an issue of relevance for emerging economies that are exposed to risks of default on debt due to their lack of adequately developed financial and macroeconomic infrastructure. Therefore, the implications of risk premium shocks for aggregate fluctuations are of special importance for emerging economies, that are also facing financial frictions. These two crucial aspects of emerging markets; namely, exposure to default risk and financial frictions, are combined in this paper in a DSGE framework with informational frictions and uncertainty in the production process. Informational asymmetries among the agents in the economy and uncertainty in the production process necessitate financial intermediation in the economy. The Holmstrom-Tirole type of uncertainty in the production process also leads to collateralized borrowing by firms, which requires special attention to the design of the loan contracts between the firms and the financial intermediaries.

The small-open-economy DSGE model developed in this paper is solved and simulated in the case of positive, one-time, temporary risk premium shocks. The model predicts an increase in the interest rate on government bonds and in the nominal exchange rate in response to the shock. The depreciation in the exchange rate increases the value of the government bonds in terms of the domestic currency, which leads to a rise in the tax rate due to the simple tax rule followed by the government. Determined partly by the size of the exchange rate depreciation and partly by the tax adjustment parameter, the increase in the amount of the tax revenue of the government, accompanied by a decrease in the government borrowing due to higher costs of repayment for the government, leads to an increase in the government spending. In response to the rise in the price level, households reduce their consumption. Domestic deposit holdings by households, on the other hand, increase since the supply of government bonds is lower due to the higher costs of borrowing for the government. As deposits rise, so do total loans available for firms. However, the increase in the amount of loans does not consequently lead to an increase in employment, and subsequently in production, because of the fact that the rise in the distorting tax rate creates disincentives to work and thereby causes a fall in the labor supply. Therefore, output also falls in response to the positive risk premium shock.
5 References


