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Rhetoric in Legislative Bargaining with Asymmetric Information[1]

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Abstract

In this paper we analyze a legislative bargaining game in which parties privately informed about their preferences bargain over an ideological and a distributive decision. Communication takes place before a proposal is offered and majority rule voting determines the outcome. When the private information pertains to the ideological intensities but the ideological positions are publicly known, it may not be possible to have informative communication from the legislator who is ideologically distant from the proposer, but the more moderate legislator can communicate whether he would "compromise" or "fight" on ideology. If instead the private information pertains to the ideological positions, then all parties may convey whether they will "cooperate," "compromise," or "fight" on ideology. When the uncertainty is about ideological intensity, the proposer is always better off making proposals for the two dimensions together despite separable preferences, but when the uncertainty is about ideological positions, bundling can result in informational loss which hurts the proposer.

JEL classification: C78, D72, D82, D83

1 Introduction

Legislative policy-making typically involves speeches and demands by the legislators that shape the proposals made by the leadership. For example, in the recent health care overhaul, one version of the Senate bill included $100 million in Medicaid funding for Nebraska as well as restrictions on abortion coverage in exchange for the vote of Nebraska Senator Ben Nelson. As another example, consider the threat by seven members of the Senate Budget Committee to withhold their support for critical legislation to raise the debt ceiling unless a commission to recommend cuts to Medicare and Social Security is approved.[1] Would these senators indeed let the United States default on its debt, or was their demand just a bluff? More generally, what are the patterns of demands in legislative policy-making? How much information do they convey? Do they influence the nature of the proposed bills? Which coalitions form and what kind of policies are chosen under the ultimately accepted bills?

To answer these questions, it is necessary to have a legislative bargaining model in which legislators make demands before the proposal of bills. One approach is to assume that the role of demands is to serve as a commitment device, that is, the legislators refuse any offer that does not meet their demands.[2] While this approach offers interesting insights into some of the questions raised above, it relies on the strong assumption that legislators commit to their demands.[3] In this paper, we offer a different approach that allows legislators to make speeches but do not necessarily commit to them when casting their votes. The premise of our approach is that only individual legislators know which bills they prefer to the status quo. So even if the legislators do not necessarily undertake what they say, their demands can be meaningful rhetoric in conveying private information and dispelling some uncertainty in the bargaining process.

We model rhetoric as cheap-talk messages as in Matthews (1989). In our framework

[1]http://thehill.com/homenews/senate/67293-sens-squeeze-speaker-over-commission

[2]This is the approach taken by Morelli (1999) in a complete information framework. He does not explicitly model proposal making and voting stage. As such, the commitment assumption is implicit.

[3]Politicians often carry out empty threats, for example, http://thehill.com/homenews/news/14312-gopsays-it-can-call-reids-bluffs.

(1) three legislators bargain over an ideological and a distributive decision; (2) bargainers other than the proposer are privately informed about their preferences; (3) communication takes place before a proposal is offered; (4) majority rule voting determines whether the proposal is implemented. By introducing communication into legislative bargaining, our goal is take a step towards answering fundamental questions of political economy, "who gets what, when and how" (Lasswell, 1958), together with fundamental questions of communication theory, "who says what to whom in what channel with what effect" (Lasswell, 1948).

We begin by analyzing the case in which the legislators' positions on a unidimensional ideological spectrum are publicly known, but their trade-offs between the ideological dimension and the distributive dimension are private information. So the proposer of a bill (also referred to as the chair) is unsure how much private benefit he has to offer to a legislator to gain support on a policy decision. When no equilibrium coalition is a surplus coalition (i.e., at most one legislator other than the proposer gets positive private benefit), we obtain two main findings: (1) the rhetoric of the legislator who is ideologically more distant from the proposer is not informative in equilibrium; (2) it is possible for the more moderate legislator to have meaningful rhetoric.

To establish these results, we first explore the legislators' expected payoffs in different coalitions. Suppose one legislator is offered positive private benefit while the other is offered none (call this a minimum winning coalition). Then the legislator who is excluded strictly prefers the status quo and will vote against the proposal whereas the legislator who is included becomes pivotal and hence can guarantee a payoff at least as high as the status quo. Alternatively, suppose no legislator is offered private benefit (call this a minority coalition). Then the chair's optimal proposal is the one that makes the moderate legislator just willing to accept. Hence in a minority coalition the more moderate legislator gets a payoff equal to the status quo but the more distant legislator is made worse off than the status quo. It follows that the more distant legislator would like to maximize his chance to be included in a coalition, thereby undermining the credibility of his rhetoric. As to the more moderate legislator, it is possible for him to have (at most) two equilibrium messages signaling his ideological intensity. When he puts a relatively high weight on the ideological dimension, he sends the "fight" message, and the chair responds with a minority coalition that excludes both legislators as their demands indicate that there is no room for making a deal. When he puts a relatively low weight on the ideological dimension, he sends the "compromise" message and the chair responds by offering some private benefits in exchange for moving the policy closer to his own ideal. The threshold type is indifferent between sending the "fight" and the "compromise" messages because either way he gets a payoff equal to the status quo, and a single-crossing property of the utility function guarantees that other types' incentive constraints are satisfied as well. It is impossible for even the moderate legislator to convey more precise information about his ideological intensity. In particular, once the chair believes that the moderate legislator puts a relatively low weight on ideology and hence includes him in a minimum winning coalition, the legislator now has the incentive to exaggerate his ideological intensity and demand a better deal from the chair, but this undermines the credibility of his demands. Somewhat ironically, the proposal of a minority coalition induced by the "fight" message always passes in equilibrium, but the minimum winning coalition induced by the "compromise" message may fail to pass.

Next, we consider the case in which the legislators' ideological intensities are known, but their ideological positions are uncertain. The setup is related to Matthews (1989) which models presidential veto threats as cheap talk in a bilateral bargaining game over a unidimensional policy and assumes that the president's position is his private information. Our model differs from Matthews (1989) by having multiple senders and a distributive dimension in addition to an ideological dimension. In this case, we find that equilibrium demands from either legislator may convey limited information about their preferences. In particular, legislators can signal whether they will "cooperate," "compromise" or "fight." If either legislator makes a cooperative speech, the chair responds by proposing his ideal policy and a minority coalition in which he extract all the surplus . If both legislators make tough demands by sending the "fight" message, the chair gives up on the ideological issue and again does not give out any private benefits. Otherwise, he proposes a compromise policy, which depends on whether one or both legislators signal willingness to compromise. Again, only the minimum winning coalition induced by a "compromise" message may fail to pass in equilibrium whereas the minority coalitions induced by the "cooperate" or "fight" message always get passed in equilibrium.

Since the legislators in our model bargain over both an ideological dimension and a distributive dimension, a natural question arises as to whether it is better to bundle the two issues together in one bill or negotiate over them separately. An obvious advantage of bundling the two issues together is that the chair can exploit difference in the other legislators' trade offs between the two dimensions and use private benefits as an instrument to make deals with them on policy changes that he wants to implement. Indeed, when the uncertainty is about the ideological intensities of the legislators, we find that it always benefits the chair to bundle the two dimensions together. But bundling may also result in informational loss when the uncertainty is about ideological positions: once side payments become a possibility, it might be too tempting for a legislator to declare that his position is not especially close to the chair's in the hope that the chair will respond with a more attractive deal. This incentive to distort one's demand may result in less information transmitted in equilibrium, hurting the legislators in the end. (By contrast, if the uncertainty is about ideological intensity, then rhetoric does not matter if the two dimensions are separated and hence bundling never results in information loss.) If we interpret bundling as the possibility of using pork barrel spending to gain support on policy reform, our finding points out a potential harm of pork barrel spending that, to our knowledge, was not pointed out before.

Before turning to the description of our model, we briefly discuss the related literature. Starting with the seminal work of Baron and Ferejohn (1989), legislative bargaining models have become a staple of political economy and have been used in numerous applications. The literature is too large to list comprehensively here. The papers most closely related to ours are Austen-Smith and Banks (1988), Banks and Duggan (2000), Jackson and Moselle (2002), and Diermeier and Merlo (2004), which include an ideological dimension and a distributive dimension. All these papers (and others that build on Baron and Ferejohn) take the form of sequential offers but do not incorporate demands. A smaller strand of literature, notably Morelli (1999), instead model legislative process as a sequential demand game where the legislators commit to their demands.[4] With the exceptions of Tsai (2009), Tsai and Yang (2009a, b), who do not model demands, all of these papers assume complete information.

Existing cheap talk literature has largely progressed in parallel to the bargaining literature. Exceptions are Farrell and Gibbons (1989), Matthews (1989), and Matthews and Postlewaite (1989). Of these Matthews (1989) is the most closely related, but as discussed earlier, there are a number of important differences between his model and ours. Our paper is also related to cheap talk games with multiple senders (see, for example, Gilligan and Krehbiel (1989), Austen-Smith (1993), Krishna and Morgan (2001), Battaglini (2002) and Ambrus and Takahashi (2008)). Our framework differs from these papers because it has voting over the proposal made by the receiver and also incorporates a distributive dimension.

In the next section we describe our model. In Section 3, we analyze the bargaining game when the legislators' ideological intensities are uncertain. In Section 4, we analyze the bargaining game when the legislators' ideological positions are uncertain. We discuss extensions and generalizations in Section 5.

2 Model

Three legislators play a three-stage game to collectively decide on an outcome that consists of an ideological component and a distributive component, for example, setting the level of environmental regulation and dividing government spending across states. We assume that legislator 0 is the chair (proposer) of the legislature in charge of formulating a proposal. Denote an outcome by a vector $z = (y; x)$ where y is an ideological decision and $x = (x_0, x_1, x_2)$ is a distributive decision. The set of feasible ideological decisions is $Y \subset \mathbb{R}$, and the set of distributions is denoted by X. For $x \in X$, x_i denotes the cake share of legislator i. Suppose $c\,(\geq 0)$ is the size of the cake for division and a proposal $(y; x)$ satisfies $\sum_{i=0}^{2} x_i = c$. We also assume that the chair cannot give negative share to the other legislators, so $x_i \geq 0$ for $i = 1, 2$. The status quo allocation is denoted by

[4] See also Vidal-Puga (2004), Montero and Vidal-Puga (2007), Breitmoser (2009).

$s = (\tilde{y}; \tilde{x})$. We assume that $\tilde{y} \in Y$ and normalize $\tilde{x}$ to be $(0, 0, 0)$. The set of possible outcomes is thus $Y \times X$ where $X = \{(x_0, x_1, x_2) : \sum_{i=0}^{2} x_i = c, x_1 \geq 0, x_2 \geq 0\} \cup \tilde{x}$.

The payoff of each legislator i depends on the ideological decision and his own cake share. We assume that the legislators' preferences are separable over the two dimensions. Specifically, legislator i has a quasi-linear von Neumann-Morgenstern utility function

$$u_i(z, \theta_i, \hat{y}_i) = x_i + \theta_i v(y, \hat{y}_i),$$

where $z = (y; x)$ specifies the outcome, $\hat{y}_i$ denotes the ideal policy of legislator i and $\theta_i \in [0, \infty)$ is a parameter that measures the intensity of legislator i's preference over the ideological dimension relative to the distributive dimension.[5] When $\theta_i = 0$, legislator i cares about only the distributive dimension. In the other extreme, when $\theta_i \to \infty$, legislator i cares about only the ideological dimension.

We make the following assumptions on the function $v(\cdot, \cdot)$: (1) $v(\cdot, \cdot)$ is continuous; (2) $v(\cdot, \hat{y}_i)$ is single-peaked at $\hat{y}_i$; (3) $v(\cdot, \cdot)$ satisfies the single-crossing property, i.e., if type $\hat{y}_i$ is indifferent between two policies y' and y and $y' > y$, then the higher types prefer y' and the lower types prefer y. Formally, if $v(y', \hat{y}_i) = v(y, \hat{y}_i)$ and $y' > y$, then $(\hat{y}'_i - \hat{y}_i)(v(y', \hat{y}'_i)) - v(y, \hat{y}'_i)) > 0$.

We assume that the ideological intensity θ_i and the ideal policy $\hat{y}_i$ of legislator $i \neq 0$ are observed only by legislator i and let $t_i = (\theta_i, \hat{y}_i)$ denote the type of legislator i. All other legislators believe that t_i is a random variable, independent of all other t_j for $j \neq i$, with a distribution function F_i whose support is $T_i = [\underline{\theta}_i, \overline{\theta}_i] \times [\underline{y}_i, \overline{y}_i] \subset \mathbb{R}^+ \times \mathbb{R}$, possibly with $\underline{y}_i = \overline{y}_i$ or $\underline{\theta}_i = \overline{\theta}_i$. For simplicity, we assume that the preference of the chair is commonly known, that is, the chair's ideal policy $\hat{y}_0$ and ideological intensity $\theta_0 \in (0, \infty)$ are observed by all legislators. Without loss of generality, we assume $\hat{y}_0 < \tilde{y}$ so that the chair would like to lower the status quo policy.

The bargaining game consists of three stages. In the first stage, each legislator $i \neq 0$ observes his type t_i and simultaneously sends a message $m_i \in M_i$ to the chair. We

[5] The model can be easily extended to allow for a more general $v_i(y)$ in the place of $v(y, \hat{y}_i)$. For expositional convenience, we use the simpler form and let $\hat{y}_i$ parameterizes legislator i's ideological preference.

assume, without loss of generality, that $M_i = T_i$ for $i = 1, 2$. In the second stage, the chair makes a proposal in $Y \times X$. In the last stage, the legislators vote on the proposal. Without loss of generality we assume that the chair always votes for the proposal. The voting rule is majority rule, so if at least one of legislators 1 and 2 votes for the proposal, then it is accepted. Otherwise, status quo $s = (\tilde{y}; \tilde{x})$ prevails.

A strategy for legislator $i \neq 0$ consists of a message rule in the first stage and an acceptance rule in the third stage. A message rule $m_i : T_i \to M_i$ for legislator i specifies what message he sends as a function of his type. An acceptance rule for legislator i is a function $\gamma_i(., t_i) : Y \times X \to \{0, 1\}$ that specifies how he votes when his type is t_i: he votes for a proposal $z = (y; x)$ if $\gamma_i(z, t_i) = 1$, and he votes against it if $\gamma_i(z, t_i) = 0$. A strategy set for legislator i consists of the measurable pairs of functions (m_i, γ_i) satisfying these properties. The chair's strategy set consists of all proposal rules $\pi : Y \times X \times M_1 \times M_2 \to [0, 1]$ where $\pi(z, m)$ is the probability that he offers $z = (y; x)$ when the message profile is $m = (m_1, m_2)$.

Following Matthews (1989), we define the equilibria directly in terms of the derived properties of perfect Bayesian equilibria.[6] An equilibrium is a strategy profile (m, γ, π) such that the following conditions hold for all $i \neq 0$, $t_i \in T_i$, $y \in Y, x \in X$ and $m \in M_1 \times M_2$:

$$(E1) \;\; \gamma_i(z, t_i) = \begin{cases} 1 & \text{if } u_i(z, t_i) \geq u(s, t_i), \\ 0 & \text{if } u_i(z, t_i) < u(s, t_i); \end{cases}$$

(E2) if $\pi(z, m) > 0$ then $u_0(z, t_0) \geq u_0(s, t_0)$.

If in addition $\int_{\{m_i(t_i) = m_i\}} dF_i(t_i) > 0$ for all $i \neq 0$, then

$$z \in \arg\max_{z' \in Y \times X} u_0(z', t_0)\beta(z'|m) + u_0(s, t_0)\left(1 - \beta(z'|m)\right),$$

where

$$\beta(z|m) = 1 - (1 - \beta_1(z|m_1))\left(1 - \beta_2(z|m_2)\right)$$

[6] The formal definition of perfect Bayesain equilibrium requires only that the optimality conditions hold for *almost all* types and pairs of messages. This would not change any of our results.

is the conditional probability that z is accepted and $\beta_i(z|m_i)$ is the conditional probability that legislator $i \neq 0$ votes to accept the proposal:

$$\beta_i(z|m_i) = \frac{\int_{\{m_i(t_i)=m_i\}} \gamma_i(z,t_i) dF_i(t_i)}{\int_{\{m_i(t_i)=m_i\}} dF_i(t_i)}.$$

Otherwise, $\pi(z,m) = \pi(z,m')$ for all z where m' is any arbitrary message profile such that such that $\int_{\{m_i(t_i)=m'_i\}} dF_i(t_i) > 0$ for all $i \neq 0$.

(E3) if $m_i\,(t_i) = m_i$, then $m_i \in \arg\max_{m'_i} V(m'_i)$ where

$$V(m'_i) = E\left[\beta_j(z|m_j)u_i\,(z,t_i) + (1-\beta_j(z|m_j)) \max\{u_i\,(z,t_i)\,, u_i(s,t_i)\}\right]$$

where $j \neq i$ and the expectation is taken over z using $\pi(.,m'_i,m_j)$, over m_j using $m_j(t_j)$, and over t_j using F_j.

Condition (E1) is subgame perfection: it requires the legislators to accept proposals that they prefer to the status quo.[7] Condition (E2) requires that the equilibrium proposals maximize the payoff of the chair and the belief be consistent with Bayes' rule. Condition (E3) requires that the legislators demand only their most preferred proposals among the ones that are possible in equilibrium (in the sense that there is some demand that generates it), taking into account the acceptance rule of the other legislator.

Say that a proposal z is induced by a message profile m if $\pi\,(z,m) > 0$. A proposal z is an equilibrium proposal if it is induced in equilibrium with positive probability. Given an equilibrium strategy profile (m,γ,π), a proposal z is induced by message m_i if there exists a message m_j with $\int_{\{m_j(t_j)=m_j\}} dF_j(t_j) > 0$ such that $\pi\,(z,m_i,m_j) > 0$. Call a proposal a minimum winning coalition if either $x_1 > 0$ or $x_2 > 0$ but not both, i.e., only one legislator (other than the chair) is given positive private benefit, a minority coalition if $x_1 = 0$ and $x_2 = 0$, and a surplus coalition if both $x_1 > 0$ and $x_2 > 0$. Also, say that a proposal $(y;x)$ includes legislator i if $x_i > 0$ and excludes legislator i if $x_i = 0$.

[7] We assume that a legislator accepts a proposal whenever he is indifferent. This assumption simplifies the exposition but is not necessary. It is easy to show that this assumption must be satisfied in any equilibrium.

3 Uncertain ideological intensity

We start by analyzing the case in which legislator i's ($i = 1, 2$) ideological position, $\hat{y}_i$, is commonly known,[8] but his ideological intensity, θ_i, is his private information. Given this restriction, we redefine legislator i's type to be θ_i and assume that the distribution of θ_i has full support on $[\underline{\theta}_i, \bar{\theta}_i]$ for $i = 1, 2$.

If $v(\hat{y}_0, \hat{y}_i) \geq v(\tilde{y}, \hat{y}_i)$ for either $i = 1$ or $i = 2$, i.e., legislator i prefers the chair's ideal to the status quo policy, then the chair's problem is trivial: he proposes his ideal and leaves all the private benefit to himself, forming a minority coalition. To avoid triviality, assume $v(\hat{y}_0, \hat{y}_i) < v(\tilde{y}, \hat{y}_i)$ for $i = 1, 2$ for the remainder of the section. Also, without loss of generality, assume that $\hat{y}_0 \leq \hat{y}_1 \leq \hat{y}_2$, so legislator 1 has an ideal closer to the chair's than legislator 2 has.

Say legislator i's message rule is informative if and only if there exist two messages m_i and m_i' that are sent by legislator i with positive probability and induce different distributions of proposals. To see whether an informative equilibrium exists, we first establish a few lemmas on properties of equilibrium proposals.

Lemma 1. *Suppose an equilibrium proposal $(y; x)$ includes i but excludes j, i.e., $x_i > 0$ and $x_j = 0$, then legislator j strictly prefers the status quo to $(y; x)$ and rejects the proposal, and legislator i is pivotal.*

Proof. We first show that legislator j prefers the status quo to $(y; x)$. Suppose to the contrary that some type of legislator j prefers $(y; x)$ to the status quo. Note that legislator j's preference over $(y; x)$ and $(\tilde{y}; \tilde{x})$ is independent of θ_j as both give him zero private benefit. So if any type of legislator j prefers $(y; x)$ to $(\tilde{y}; \tilde{x})$, he will accept $(y; x)$ with probability 1. But then the chair can make an alternative proposal with $x_i = 0$ (not giving i any private benefit either) and still have it accepted by legislator j. This alternative proposal gives the chair a strictly higher payoff, a contradiction. So legislator j rejects the equilibrium proposal $(y; x)$ if j is not included and it follows that legislator i is pivotal. □

[8] So the support of $\hat{y}_i$ is degenerate: $\underline{y}_i = \bar{y}_i$.

The next lemma shows that if a minimum winning coalition is formed with legislator i when he sends message m, then for the highest type who sends m, the proposal cannot be strictly better than the status quo,[9] but since legislator i is pivotal, he can guarantee the status quo payoff.

Lemma 2. *Suppose legislator i's message m induces a proposal $(y;x)$ with $x_i > 0, x_j = 0$. Let $\theta_i^* = \sup\{\theta_i : m_i(\theta_i) = m\}$. Then type θ_i^* must weakly prefer the status quo to the proposal $(y;x)$ and type θ_i^*'s payoff when inducing $(y;x)$ is equal to his status quo payoff.*

Proof. For $x_i > 0$, if $x_i + \theta_i v(y, \hat{y}_i) \geq \theta_i v(\tilde{y}, \hat{y}_i)$, then either (i) $v(\tilde{y}, \hat{y}_i) - v(y, \hat{y}_i) > 0$ and therefore $x_i + \theta_i' v(y, \hat{y}_i) > \theta_i' v(\tilde{y}, \hat{y}_i)$ for $0 \leq \theta_i' < \theta_i$ or (ii) $v(\tilde{y}, \hat{y}_i) - v(y, \hat{y}_i) \leq 0$ and $x_i + \theta_i' v(y, \hat{y}_i) > \theta_i' v(\tilde{y}, \hat{y}_i)$ for $0 \leq \theta_i' < \theta_i$. So if type θ_i of legislator i prefers a proposal $(y;x)$ to the status quo, then any lower type θ_i' must prefer $(y;x)$ to the status quo as well. Suppose the chair proposes $(y;x)$ in response to m that has $x_i > 0$, $x_j = 0$ and makes type θ_i^* strictly better off than the status quo. Then there exists a proposal $(y;x')$ with $x_i' < x_i$, $x_j' = 0$ that still makes type θ_i^* strictly better off than the status quo and hence is accepted by legislator i with probability 1. But the chair strictly prefers $(y;x')$ to (y,x), a contradiction. So type θ_i^* must weakly prefer the status quo $(\tilde{y};\tilde{x})$ to the proposal $(y;x)$. Since legislator i is pivotal when he is included and legislator j is excluded and he can always reject the proposal, type θ_i^* gets a payoff equal to the status quo payoff. $\square$

The next lemma establishes some properties of minority coalitions.

Lemma 3. *Suppose the chair proposes a minority coalition in equilibrium. If $\hat{y}_1 \geq \tilde{y}$, then the ideological outcome is $\tilde{y}$. If $\hat{y}_1 < \tilde{y}$, the proposed y satisfies $y < \tilde{y}$, $v(y, \hat{y}_1) = v(\tilde{y}, \hat{y}_1)$ and is accepted by legislator 1, but legislator 2 prefers the status quo to the proposal (strictly if $\hat{y}_1 < \hat{y}_2$).*

Proof. In a minority coalition, $x_1 = 0, x_2 = 0$. Since $\hat{y}_1 \leq \hat{y}_2$, it follows that if $\hat{y}_1 \geq \tilde{y}$, then both legislators will vote against any $y < \tilde{y}$. Hence the resulting ideological outcome

[9] More precisely, the highest type who sends m may not exist, so we define θ_i^* as the lowest upper bound in the lemma. Although the message sent by type θ_i^* may not be m, by continuity his equilibrium payoff must be the same as what he gets if he sends m.

is $\tilde{y}$. If $y_1 < \tilde{y}$, then legislator 1 accepts any y that satisfies $v(y, \hat{y}_1) \geq v(\tilde{y}, \hat{y}_1)$. Since $v(\hat{y}_0, \hat{y}_1) \leq v(\tilde{y}, \hat{y}_1)$ and $v(\cdot,\cdot)$ is single-peaked, it is optimal for the chair to propose y that satisfies $v(y, \hat{y}_1) = v(\tilde{y}, \hat{y}_1)$. Since $v(\cdot,\cdot)$ satisfies the single-crossing property and $\hat{y}_1 \leq \hat{y}_2$, legislator 2 prefers the status quo to the proposal and strictly so if $\hat{y}_1 < \hat{y}_2$. $\square$

In what follows, we derive results regarding the legislators' equilibrium message rules under the assumption that no equilibrium coalition is a surplus coalition. This is satisfied under reasonable conditions on the type distributions. For example, it is satisfied if the density of θ_i is weakly increasing (e.g., uniform distribution), as is typically assumed in related cheap-talk models.[10] Moreover, if only one legislator has an unknown type, then no equilibrium coalition is a surplus coalition (see discussion on page 18).

The first proposition shows that legislator 2 cannot convey meaningful information in equilibrium if his ideal point is not the same as legislator 1's and legislator 1 wants to lower the status quo as the chair does.[11]

Proposition 1. *If $\hat{y}_1 < \tilde{y}$ and $\hat{y}_1 < \hat{y}_2$, then legislator 2's message rule is not informative in equilibrium.*

Proof. Suppose to the contrary that there exist two messages m' and m'' that are sent with positive probability in equilibrium by legislator 2 and induce different distributions of proposals. As Lemma 3 shows, if a proposal excludes both legislators, then legislator 2's payoff is strictly lower than the status quo. Let $\theta_2' = \sup\{\theta_2 : m_2(\theta_2) = m'\}$ and $\theta_2'' = \sup\{\theta_2 : m_2(\theta_2) = m''\}$. As Lemma 1 shows, if a coalition includes legislator 1 but excludes 2, then legislator 2's payoff is strictly lower than the status quo. Moreover, since legislator 1 is pivotal in this case, the proposal does not depend on legislator 2's message. If a coalition includes legislator 2 but excludes 1, then legislator 2 is pivotal and as Lemma 2 shows, the payoff of type θ_2' (θ_2'') is the same as his status quo payoff.

[10] In a supplementary appendix not intended for publication (also available on our web pages), we provide sufficient conditions for no equilibrium coalition to be a surplus coalition.

[11] If $\hat{y}_1 \geq \tilde{y}$, i.e., both legislators' ideals are higher than the status quo, then in a minority coalition $y = \tilde{y}$ and both legislators 1 and 2's payoffs are equal to the status quo. In this case, for certain parameters we can construct an equilibrium in which legislator 2's messages induce different proposals. It has the same properties as the informative message rule described in Proposition 3.

So, the probability of legislator 2 being included in a coalition must be the same for messages m' and m'' because otherwise either type θ_2' or type θ_2'' would have an incentive to deviate to the message that induces a higher probability of inclusion. Note also that conditional on legislator 2 being included in a coalition, the proposal does not depend on legislator 1's message. Since the inclusion probabilities are the same for m' and m'', to prevent deviation messages m' and m'' must induce the same proposals when legislator 2 is included. But this implies that m' and m'' induce the same distribution of proposals for legislator 2, a contradiction. $\square$

Now that we have established that legislator 2, whose ideology is furthest from the chair's, cannot convey useful information in equilibrium, we would like to see whether the message of legislator 1, whose position is closer to the chair's, can be informative. We will call legislator 1 the moderate legislator.

Suppose legislator 1's message rule takes the following form:[12] send message m^k if $\theta_1 \in (\theta_1^k, \theta_1^{k+1}]$ $(k = 1, ..., K)$ where $\underline{\theta_i} = \theta_i^1 < \theta_i^2 < ... < \theta_i^{K+1} = \bar{\theta}_i$. We refer to K as the "size" of legislator 1's message rule. If $K = 1$, then legislator 1's speech is uninformative, but if $K > 1$, then legislator 1's speech conveys information about his preference.

Lemma 4. *If m^k induces a proposal that includes legislator 1 who accepts it with positive probability, then any proposal induced by m^l where $l < k$ must include legislator 1.*

[12] This is the form of equilibrium message rules in both the classic model of Crawford and Sobel (1982) and the related model of Matthews (1989). In our model, it is without loss of generality to consider a message rule of this form if the chair plays a pure strategy. To see this, first note that there can be at most one equilibrium proposal that does not include legislator 1 because such a proposal depends only on 2's message, but as already shown, 2 sends the same message in equilibrium. If such a proposal is induced in an equilibrium, then there exists a type θ_1 such that only types higher than θ_1 induces it. Next, consider two messages that are sent in equilibrium by legislator 1: m and m'. Suppose m induces z, m' induces z' and both z and z' include legislator 1. Without loss of generality, assume that $x_1 > x_1'$. Let θ_1^* be the type who is indifferent between z and z'. Then any type $\theta_1 < \theta_1^*$ strictly prefers z to z' and any type $\theta_1 > \theta_1^*$ strictly prefers z' to z. Since this holds for any pair of messages m and m', it follows that an equilibrium message rule has the partition form. An argument similar to that in the proof of Proposition 3 shows that there can be at most one equilibrium proposal that includes legislator 1.

Proof. Denote by z^k the proposal induced by m^k that includes legislator 1 and is accepted with positive probability. Since z^k is a best response for the chair, it must give the chair an expected payoff at least as high as any proposal that excludes legislator 1. Also, since z^k is accepted with positive probability, type θ_1^k must strictly prefer z^k to the status quo and therefore any type $\theta_1 < \theta_1^k$ strictly prefers z^k to the status quo. Hence, for any m^l with $l < k$, there exists a proposal $z^l = (y^l; x^l)$ such that $y^l = y^k$ and $0 < x_1^l < x_1^k$ and z^l is accepted with probability 1 by legislator 1. Moreover, z^l gives the chair an expected payoff strictly higher than any proposal that excludes legislator 1. It follows that any proposal induced by m^l must include legislator 1. $\square$

The next proposition shows that in an informative equilibrium, the ideologically more distant legislator is never included in a coalition.

Proposition 2. *If legislator* 2 *is included in a proposal with positive probability in equilibrium, then legislator* 1*'s message rule is not informative.*

Proof. Suppose to the contrary that $K \geq 2$. Let m^k be the highest message sent by legislator 1 that induces a proposal, denoted by z^k, that includes legislator 2 and excludes legislator 1.

Suppose $m^k = m^1$. Then by Lemma 4, any proposal induced by m^2 must exclude legislator 1. Since m^k is the highest message that induces a proposal that includes 2, it follows that no proposal induced by m^2 includes 2. Hence the proposal induced by m^2 excludes both legislators, and by Lemma 3, legislator 1's payoff is equal to his status quo payoff. Since type θ_1^{k+1}'s payoff is lower than the status quo payoff from the proposal z^k, type θ_1^{k+1} (and types in the neighborhood below it) has an incentive to deviate and send m^2, a contradiction.

Suppose $m^k > m^1$. We next show that any proposal induced by m^k does not include legislator 1. Suppose not. Then by Lemma 4, any proposal induced by message m^l with $l < k$ includes legislator 1 and hence gives legislator 1 a payoff at least as high as his status quo payoff. Since type θ_1^{k+1}'s payoff is equal to his status quo payoff when legislator 1 is included and strictly lower than his status quo payoff when legislator 2 is included, it follows that type θ_1^{k+1} (and types in the neighborhood below it) has an incentive to deviate and send m^l, a contradiction. So if a message m^k of legislator 1 induces a proposal that includes legislator 2, then any proposal induced by m^k must exclude legislator 1. Now consider a message $m^n \neq m^k$ sent by legislator 1. Recall that legislator 1's payoff is strictly lower than his status quo payoff when legislator 2 is included and weakly higher than the status quo payoff when legislator 2 is excluded. So, to prevent profitable deviations, any proposal induced by m^n must also exclude legislator 1 and the probability that legislator 2 is included must be the same for messages m^k and m^n. Since a proposal does not depend on legislator 1's message if legislator 2 is included, it follows that messages m^k and m^n induce the same distribution of proposals, a contradiction. $\square$

It follows from Proposition 2 that in an informative equilibrium, the ideologically more distant legislator is always excluded in a coalition. The next proposition describes what messages are sent and what proposals are induced in an informative equilibrium. Let $y_1^* = \min\{y : v(y, \hat{y}_1) = v(\tilde{y}, \hat{y}_1)\}$, i.e., the lowest y that gives legislator 1 the status quo payoff.

Proposition 3. *If legislator* 1*'s message rule is informative in equilibrium, then it must have size two: message* m^1 *induces a proposal* (y, x) *with* $y < y_1^*$*,* $x_1 > 0$ *and* $x_2 = 0$*, which is accepted with positive probability by legislator* 1*; message* m^2 *induces the proposal* $(y_1^*; c, 0, 0)$*, which is accepted by legislator* 1 *with probability* 1*. In an informative equilibrium, legislator* 1*'s payoff is higher than the status quo and legislator* 2*'s payoff is lower than the status quo.*

Proof. We first show that if a message m^k induces a proposal z^k that includes legislator 1 and is accepted with positive probability, then $m^k = m^1$, i.e., the lowest message. Suppose not, then m^{k-1} exists and by Lemma 4, any proposal induced by message m^{k-1} must include legislator 1. Among all proposals induced by m^{k-1}, let z^{k-1} denote the proposal that is accepted with the highest probability. Let θ_1' denote the type who is indifferent between z^{k-1} and the status quo. Note that $\theta_1' \in (\theta_1^{k-1}, \theta_1^k]$. Let θ_1'' denote the type who is indifferent between z^k and the status quo. Note that $\theta_1'' \in (\theta_1^k, \theta_1^{k+1}]$. Next, we show that type θ_1' strictly prefers z^k to z^{k-1}. Since type θ_1' is indifferent between

z^{k-1} and the status quo, we have $x_1^{k-1} + \theta_1' v\left(y^{k-1}, \hat{y}_1\right) = \theta_1' v\left(\tilde{y}, \hat{y}_1\right)$, which implies that $x_1^{k-1} = \theta_1'\left(v\left(\tilde{y}, \hat{y}_1\right) - v\left(y^{k-1}, \hat{y}_1\right)\right)$. Similarly, since type θ_1'' is indifferent between z^k and the status quo, we have $x_1^k = \theta_1''\left(v\left(\tilde{y}, \hat{y}_1\right) - v\left(y^k, \hat{y}_1\right)\right)$. So

$$\begin{aligned} u_1\left(z^k, \theta_1', \hat{y}_1\right) - u_1\left(z^{k-1}, \theta_1', \hat{y}_1\right) &= x_1^k + \theta_1' v\left(y^k, \hat{y}_1\right) - \theta_1' v\left(\tilde{y}, \hat{y}_1\right) \\ &= \left(\theta_1'' - \theta_1'\right)\left(v\left(\tilde{y}, \hat{y}_1\right) - v\left(y^k, \hat{y}_1\right)\right). \end{aligned}$$

Since $\theta_1'' > \theta_1'$ and $v\left(\tilde{y}, \hat{y}_1\right) - v\left(y^k, \hat{y}_1\right) > 0$, we have $u_1\left(z^k, \theta_1', \hat{y}_1\right) - u_1\left(z^{k-1}, \theta_1', \hat{y}_1\right) > 0$, i.e., type θ_1' strictly prefers z^k to z^{k-1}. So the expected payoff that type θ_1' gets by sending m^{k-1} is equal to the status quo and the expected payoff he gets by sending m^k is strictly higher than the status quo. Hence type θ_1' (and types in the neighborhood immediately below it) has an incentive to deviate, a contradiction. So if a message m^k induces a proposal z^k that includes legislator 1 and is accepted with positive probability, then $m^k = m^1$. The argument also implies that there can be at most one message above m^1 and the proposal it induces does not include legislator 1. Since in an informative equilibrium, legislator 2 is always excluded, the proposal induced by m^2 excludes both legislators. Since the chair's optimal proposal when both legislators are excluded is $(y_1^*; c, 0, 0)$, it follows that m^2 induces $(y_1^*; c, 0, 0)$. Since legislator 1 is pivotal in all equilibrium coalitions, his payoff is higher than the status quo payoff. Legislator 2 is always excluded and his payoff is therefore lower than the status quo payoff. □

Proposition 3 says that legislator 1 may be able to convey limited information about his ideological intensity. When legislator 1 is intensely ideological, he sends a "fight" message, signaling that the chair will not be able to "buy" his vote on an ideological compromise through private benefits, and the chair responds with a proposal of minority coalition and a policy that makes legislator 1 just willing to accept. When legislator 1 is not intensely ideological, he sends a "compromise" message and the chair responds with a proposal of a minimum winning coalition that includes legislator 1 and a policy closer to the chair's ideal.

Proposition 3 shows that in an informative equilibrium, the chair forms a coalition only with the legislator whose ideological position is closer to his own. It is worth noting that in an uninformative equilibrium, it is possible that the more distant legislator is included in a coalition. This happens if his position is not too extreme relative to the other legislator and the chair believes that he puts much less weight on ideology than the other legislator and hence it is less costly to gain his support.

To illustrate what an equilibrium looks like when legislator 1's message is informative, we provide the following example.

Example 1. *Suppose* $\tilde{y} = 0$, $\hat{y}_0 = -1$, $\hat{y}_1 = -\frac{1}{5}$, $\hat{y}_2 = \frac{1}{2}$, $c = 1$. *Also, assume that legislator i's utility function is* $x_i - \theta_i (y - \hat{y}_i)^2$, $\theta_0 = 1$, *and* θ_1, θ_2 *are both uniformly distributed on* $[\frac{1}{4}, 4]$.

Consider the following strategy for legislator 1: send m^1 if $\theta_1 \in [\frac{1}{4}, \theta_1^2]$ and m^2 if $\theta_1 \in (\theta_1^2, 4]$; and the following strategy for legislator 2: always send the same message for any θ_2.

To find the chair's best response, note that if the chair wants to make a proposal that legislator 1 accepts when $\theta_1 \leq \theta_1^*$, then he would propose (y, x) so as to leave type θ_1^* indifferent between the status quo and the proposal (y, x). If $x_1 > 0$, we have $x_1 - \theta_1^*(y - \hat{y}_1)^2 = -\theta_1^*(\tilde{y} - \hat{y}_1)^2$. So $x_1 = \theta_1^*(y - \tilde{y})(y + \tilde{y} - 2\hat{y}_1)$ and the chair solves

$$\max_{y \in [\underline{y}, \tilde{y}]} (c - \theta_1^*(y - \tilde{y})(y + \tilde{y} - 2\hat{y}_1)) - \theta_0(y - \hat{y}_0)^2$$

subject to

$$\theta_1^*(y - \tilde{y})(y + \tilde{y} - 2\hat{y}_1) \geq 0.$$

If an interior solution exits, then $y = \frac{\theta_0\hat{y}_0 + \theta_1^*\hat{y}_1}{\theta_0 + \theta_1^*}$. Since the chair can always propose $y = 2\hat{y}_1 - \tilde{y}$, $x_1 = 0$ and have it accepted by legislator 1 with probability 1, we must have $y = \min\{\frac{\theta_0\hat{y}_0 + \theta_1^*\hat{y}_1}{\theta_0 + \theta_1^*}, 2\hat{y}_1 - \tilde{y}\}$.

Given the strategy of legislator 1, when the chair receives message m^1, he infers that $\theta_1 \in [\frac{1}{4}, \theta_1^2]$. Using the calculation above, the proposal $y = \frac{\theta_0\hat{y}_0 + \theta_1^2\hat{y}_1}{\theta_0 + \theta_1^2}$, $x_1 = \theta_1^2(y - \tilde{y})(y + \tilde{y} - 2\hat{y}_1)$ will be accepted with probability 1. Suppose $\theta_1^2 = 2$.[13] Plugging in the numbers,

[13] There are many other values of the threshold θ_1^2 that satisfy the equilibrium conditions. We pick $\theta_1^2 = 2$ just as an example. What is important is that the chair's optimal proposal when receiving m^2 involves a minority coalition and his optimal proposal when receiving m^1 involves a minimum winning coalition with legislator 1.

we find that if the chair wants to make a proposal that is accepted by all $\theta_1 \in [\frac{1}{4}, \theta_1^2]$, then the optimal proposal is $y = -\frac{7}{15}$ and $x_1 = \frac{14}{225}$. Calculation shows that indeed this is the chair's optimal proposal for $\theta_1 \in [\frac{1}{4}, 2]$ and $\theta_2 \in [\frac{1}{4}, 4]$. In particular, any proposal that is not accepted with probability 1 by legislator 1 is suboptimal and it is also optimal to exclude legislator 2.[14]

When the chair receives message m^2 from legislator 1, he infers that $\theta_1 \in (\theta_1^2, 4]$. Calculation shows that when $\theta_1^2 = 2$, it is optimal to propose $y = \tilde{y} - 2\hat{y}_1 = -\frac{2}{5}$ and $x_1 = 0, x_2 = 0$, and legislator 1 accepts the proposal with probability 1.

We next check that legislator 1's incentive constraints are satisfied. Let z^1 denote the proposal induced by m^1 and z^2 denote the proposal induced by m^2, i.e., $z^1 = \left(-\frac{7}{15}; \frac{211}{225}, \frac{14}{225}, 0\right)$ and $z^2 = \left(-\frac{2}{5}; 1, 0, 0\right)$. Type θ_1^2 is indifferent between z^1 and z^2 because he gets the status quo payoff either way. Since $y^1 \left(= -\frac{7}{15}\right) < y^2 \left(= -\frac{2}{5}\right)$ and $x_1^1 \left(= \frac{14}{225}\right) > x_1^2 \,(= 0)$, types below θ_1^2 (who puts relatively low weight on ideology) prefer z^1 to z^2 and types above θ_1^2 (who put relatively high weight on ideology) strictly prefer z^2 to z^1.

In this equilibrium, when legislator 1 puts relatively low weight on ideology and high weight on private benefit, he sends message m^1, which can be interpreted as signaling willingness to form a coalition with the chair. The chair responds to m^1 with a proposal that includes legislator 1 in the coalition and moves the policy towards the chair's ideal. When legislator 1 puts relatively high weight on ideology and low weight on private benefit, he sends message m^2. The chair responds with a proposal of a minority coalition because it is too costly to form a coalition with either legislator: legislator 1 is too intensely ideological and legislator 2 has an ideological position that is too far away.

Next, we use our results to shed light on a number of interesting questions.

Seniority and uncertainty: What happens if only one of the legislators has private information on his preference? This applies if one legislator is a senior member of the legislature whose preference has been revealed from past experience and the other legislator is a junior member whose ideological intensity remains uncertain. To see what

[14] In this example, the minimum winning coalition that the chair proposes in response to m^1 is accepted with probability 1. This is a special feature of the example and does not hold in general, i.e., it may happen that the minimum winning coalition fails to pass with positive probability.

happens in this case, first note that no surplus coalition is ever proposed in equilibrium because there is no uncertainty about whether the senior legislator will vote for any given proposal. In particular, if it is optimal for the chair to include the senior member in a coalition, it must be optimal for him to propose a policy that ensures the senior member's vote and also leave the junior member out of the coalition.

Is it possible for the junior member's speech to be informative in equilibrium? As this is a special case of the preceding general analysis (in particular, if $\underline{\theta_i} = \bar{\theta}_i$, then there is no uncertainty about legislator i's preference), we can apply the results to obtain the following observations. Whether the junior legislator's speech can be informative depends on the relative positions of the legislators on the ideological spectrum. If the junior member has an ideological position closer to the chair's, then it is possible for his messages to be informative. The message rule and the resulting outcomes are similar to those described in Proposition 3. By contrast, if the senior member is the one whose ideological position is closer to the chair's, then it is impossible for the junior member's message to be informative in equilibrium, as shown in Proposition 1.

Benefits of bundling the two dimensions: Since the legislators bargain over both an ideological dimension and a distributive dimension, a natural question to ask is whether the proposer is better off bundling the two dimensions together or negotiating them separately. When the uncertainty is about the trade-off between the two dimensions, the answer is unambiguous: the chair (weakly) prefers to bundle the two dimensions together. To see why, note that if the two dimensions are bargained over separately, then the legislators' private information is irrelevant since it is about how they trade off one dimension for the other, not about their preferences on each dimension. The resulting outcome is $(y_1^*; c, 0, 0)$, i.e., the chair gets all the private benefit and the policy he proposes is the one that make the closest legislator just willing to accept. When the two issues are bundled together, the proposal $(y_1^*; c, 0, 0)$ is still feasible and will pass with probability 1. It immediately follows that bundling can never make the chair worse off. In fact, there are two potential advantages from bundling: (1) Useful information may be revealed in equilibrium, as seen in Proposition 3. (2) Given the information he has, the chair can use private benefit as an instrument to make better proposals that exploit the difference in how the players trade off the two dimensions.

This result that legislators prefer to make proposals for the two dimensions together despite separable preferences is analogous to the finding in Jackson and Moselle (2002), although their model of legislative bargaining has no asymmetric information or communication. But as we will see in the next section, this result is sensitive to the nature of the uncertainty. When legislators' ideological positions, rather than intensities, are uncertain, it may be better for the chair to separates the two dimensions.

4 Uncertain ideological position

Suppose θ_i is commonly known,[15] but legislator i ($i \neq 0$) is privately informed about his ideological position $\hat{y}_i$. As such, we redefine legislator i's type to be his ideal point, i.e. $t_i = \hat{y}_i$, and assume that the distribution of t_i has full support on $[\underline{y_i}, \bar{y}_i]$ for $i = 1, 2$.

As before we analyze equilibria in which the message rule for legislator i ($i \neq 0$) takes the following form: send message m_i^k if and only if $t_i \in (\tau_i^k, \tau_i^{k+1}]$ ($k = 1, ..., K_i$) where $\underline{y_i} = \tau_i^1 < \tau_i^2 < ... < \tau_i^{K_i+1} = \bar{y}_i$. So K_i is the size of legislator i's message rule. What follows is a number of useful lemmas about equilibrium proposals.

Lemma 5. *Any equilibrium proposal $(y; x)$ satisfies $y \leq \tilde{y}$. If an equilibrium proposal $(y; x)$ satisfies $y = \tilde{y}$, then x must be $(c, 0, 0)$ and is accepted with probability 1.*

Proof. Since the proposal $(\tilde{y}; c, 0, 0)$ is accepted with probability 1 and $\hat{y}_0 < \tilde{y}$, the chair will never make a proposal with $y > \tilde{y}$. Hence any equilibrium proposal $z = (y; x)$ satisfies $y \leq \tilde{y}$. Suppose an equilibrium proposal $z = (y; x)$ satisfies $y = \tilde{y}$, but $x \neq (c, 0, 0)$. Then $x_0 < c$ and there exists a legislator $i\,(\neq 0)$ with $x_i > 0$ and hence will vote yes on z. But since the status quo is $(\tilde{y}; 0, 0, 0)$, the chair can instead propose $z' = (\tilde{y}; x')$ with $0 < x'_i < x_i$. Legislator i will still vote for the proposal, but the chair is better off with z', a contradiction. $\square$

[15] So the support of θ_i is degenerate: $\underline{\theta}_i = \overline{\theta}_i$.

Lemma 6. *Suppose a proposal $(y; x)$ with $y < \tilde{y}$ is induced by $\left(m_1^{k_1}, m_2^{k_2}\right)$ and accepted with positive probability by legislator i in equilibrium. Then, type $\tau_i^{k_i}$ of legislator i strictly prefers $(y; x)$ to the status quo.*

Proof. Since $v(y, t_i)$ is single-peaked and satisfies the single-crossing property in (y, t_i) and u_i is quasi-linear, it follows that if type t_i weakly prefers a proposal $z' = (y'; x')$ to $z = (y; x)$ and $y' > y$, then type $t_i' > t_i$ must strictly prefer z' to z. Assume, without loss of generality, that $(y; x)$ is accepted by legislator 1 with positive probability. Suppose type $\tau_1^{k_1}$ weakly prefers $(\tilde{y}; \tilde{x})$ to $(y; x)$. Since $y < \tilde{y}$, this implies that if $t_1 > \tau_1^{k_1}$, then type t_1 must strictly prefer $(\tilde{y}; \tilde{x})$ to $(y; x)$, contradicting the assumption that $(y; x)$ is accepted by legislator 1 with positive probability. Hence, type $\tau_1^{k_1}$ strictly prefers $(y; x)$ to the status quo. The same argument applies if $(y; x)$ is accepted by legislator 2 with positive probability. □

Lemma 7. *Suppose a proposal $(y; x)$ with $y < \tilde{y}$ is induced by m_i^k $(k \geq 2)$ and accepted with positive probability by legislator i in equilibrium. Then any equilibrium proposal induced by m_i^{k-1} must be $(\hat{y}_0; c, 0, 0)$.*

Proof. Lemma 6 implies that type τ_i^k of legislator i strictly prefer $(y; x)$ to the status quo. In equilibrium type τ_i^k must be indifferent between sending m_i^k and m_i^{k-1} and therefore indifferent between the distributions of outcomes induced by the messages. Next, we show that all the equilibrium proposals induced by m_i^{k-1} must be $(\hat{y}_0; c, 0, 0)$. Suppose not. Then at least one proposal induced by m_i^{k-1} is not $(\hat{y}_0; c, 0, 0)$. Among these proposals find the one that gives type τ_i^k the highest payoff, and denote it by $(y'; x')$. Consider the following two cases. (i) Suppose type τ_i^k prefers $(\hat{y}_0; c, 0, 0)$ to $(y'; x')$. Then it follows that type τ_i^k strictly prefers $(\hat{y}_0; c, 0, 0)$ to $(\tilde{y}; \tilde{x})$. Because $\hat{y}_0 < \tilde{y}$, u_i is quasi-linear and $v(y, t_i)$ satisfies the single-crossing property in (y, t_i), it follows that for any type $t_i < \tau_i^k$, legislator i strictly prefers $(\hat{y}_0; c, 0, 0)$ to $(\tilde{y}; \tilde{x})$ and therefore will vote for it. Since $(\hat{y}_0; c, 0, 0)$ gives the chair the highest possible payoff, his optimal response to m_i^{k-1} must be $(\hat{y}_0; c, 0, 0)$, a contradiction. (ii) Suppose type τ_i^k prefers $(y'; x')$ to $(\hat{y}_0; c, 0, 0)$. Then $(y'; x')$ is the proposal among those induced by m_i^{k-1} that type τ_i^k likes best. It follows that type τ_i^k (and all the lower types) strictly prefers $(y'; x')$ to

$(\tilde{y};\tilde{x})$. This implies that instead of proposing $(y';x')$, the chair can propose a policy lower than y' or shares lower than x' to the other legislators and still get the proposal accepted with probability 1, but this alternative proposal makes the chair strictly better off, a contradiction. □

Let $\tilde{k}_i = \max\{k : m_i^k$ induces a proposal other than $(\tilde{y};c,0,0)$ and it is accepted with positive probability by legislator $i\}$. Lemma 7 implies that $\tilde{k}_i \leq 2$ since any equilibrium message m_i^k with $k < \tilde{k}_i$ must induce $(\hat{y}_0;c,0,0)$.

Lemma 8. *Suppose an equilibrium message m_i^k either induces $(\tilde{y};c,0,0)$ or induces a proposal that is rejected by legislator i. Then $k = K_i$.*

Proof. Suppose to the contrary that $k < K_i$. Suppose $(y;x)$ is a proposal induced by m_i^{k+1} and accepted by legislator i with positive probability. If $y < \tilde{y}$, then by Lemma 7, m_i^k must induce $(\hat{y}_0;c,0,0)$. Since m_i^k induces $(\tilde{y};c,0,0)$, we must have $y = \tilde{y}$. It follows from Lemma 5 that $x = (c,0,0)$. Hence a proposal induced by m_i^{k+1} must either be $(\tilde{y};c,0,0)$ or rejected by legislator i. Note that when a proposal is rejected by legislator i, then it depends only on legislator j's message. Hence messages m_i^k and m_i^{k+1} induce the same distribution of proposals, a contradiction. □

It follows from Lemma 8 that there can be at most one equilibrium message m_i^k with $k > \tilde{k}_i$. Since $\tilde{k}_i \leq 2$, the maximum number of equilibrium messages for legislator i is 3, i.e., $K_i \leq 3$. To summarize, when the legislators' ideological positions are uncertain, it is possible for their demands to affect the proposals and outcomes in equilibrium, but the information conveyed is still limited.

Proposition 4. *An equilibrium message rule has at most size three, i.e., $K_i \leq 3$ ($i = 1,2$). For a size-three message rule $m_i(\cdot)$, m_i^1 induces the proposal $(\hat{y}_0;c,0,0)$; m_i^3 either induces a proposal $(\tilde{y};c,0,0)$ or induces a proposal that legislator i rejects; compromise proposals with $y \in (\hat{y}_0,\tilde{y})$ are induced only by m_i^2.*

It is useful to interpret m_i^1 as the "cooperate" message, m_i^2 as the "compromise" message and m_i^3 as the "fight" message. Legislator i sends message m_i^1 only when his ideology is sufficiently aligned with the chair's and in particular, he prefers the chair's ideal to the status quo policy. By sending the "cooperate" message, he signals his willingness to vote for the chair's most preferred policy even without getting any private benefit. With the assurance of this cooperative ally (one such legislator is enough under the majority rule), the chair proposes his ideal policy without giving out any private benefit to the other legislators. By contrast, when legislator i has an ideological position that is distant from the chair's, he send message m_i^3 to signal a tough stance on policy change. If both legislators send the "fight" message, the chair realizes that both legislators' ideals are too far from his own and the best proposal he can put forward is to keep the status quo policy unchanged and give out no private benefit. When a legislator has an ideological position that is somewhat aligned with the chair's, he sends a "compromise" message and the chair responds with a policy that is in between the status quo and his own ideal unless the other legislator indicates willingness to cooperate.

To illustrate what an equilibrium with meaningful rhetoric looks like, we provide the following example.

Example 2. *Suppose $\tilde{y} = 0$, $\hat{y}_0 = -1$, $c = 1$, legislator i's utility function is $x_i - (y - t_i)^2$ and t_i ($i = 1, 2$) is uniformly distributed on $[-2, 2]$ for $i = 1, 2$.*[16]

Consider the following strategy profile. The message rule for legislator $i = 1, 2$ has size three: specifically, $m_i(t_i) = m_i^1$ if $t \in [\tau_i^1, \tau_i^2]$, $m_i(t_i) = m_i^2$ if $t \in (\tau_i^2, \tau_i^3]$ and $m_i(t_i) = m_i^3$ if $(\tau_i^3, \tau_i^4]$ where $\tau_i^1 = -2 < \tau_i^2 < \tau_i^3 < \tau_i^4 = 2$. The chair's strategy is the following: if $m_i = m_i^1$ for either $i = 1$ or 2, propose $(\hat{y}_0; 1, 0, 0)$; if $m_i = m_i^3$ for both $i = 1, 2$, propose $(\tilde{y}; 1, 0, 0)$; if $m_1 = m_1^2$ and $m_2 = m_2^3$, propose $(y'; x'_0, 1 - x'_0, 0)$; if $m_1 = m_1^3$ and $m_2 = m_2^2$, propose $(y'; x'_0, 0, 1 - x'_0)$; if $m_i = m_i^2$ for both $i = 1, 2$, propose $(y''; x''_0, 1 - x''_0, 0)$ with probability $\frac{1}{2}$ and propose $(y''; x''_0, 0, 1 - x''_0)$ with probability $\frac{1}{2}$.

By using the indifference condition of type τ_i^2 and type τ_i^3 and conditions for the chair's proposals to be optimal conditional on the messages received, we find that $\tau_i^2 \approx -0.80$, $\tau_i^3 \approx 0.54$, $y' \approx -0.23, x'_0 \approx 0.7, y'' \approx -0.45, x''_0 \approx 0.87$.

[16] For expositional simplicity, we assume that legislators 1 and 2 are ex ante identical in this example, but equilibrium message rules of size-three may still exist even when the legislators are not ex ante identical.

In this equilibrium, if at least one of the legislators signals his willingness to cooperate by sending message m_i^1, then the chair proposes his ideal policy $\hat{y}_0$ and keeps the whole cake to himself. Because legislator i's ideal is in $[\tau_i^1, \tau_i^2] = [-2, -0.80]$, he is willing to go along with the chair's ideal policy even without any transfer of private benefit. This proposal of a minimum winning coalition passes with probability 1.

If both legislators act tough and send m_i^3, then the chair proposes the status quo policy $\tilde{y} = 0$ and still keeps the whole cake to himself. Since both legislators' ideal policies are high ($t_i \in (0.54, 2])$), it is too costly (i.e., the cake shares needed in exchange for their votes are too large) for it to be optimal for the chair to try to change the status quo policy. Legislators 1 and 2 are indifferent between voting for and against the proposal. In equilibrium they vote yes and this proposal of a minority coalition passes with probability 1.

If legislator i signals willingness to compromise by sending m_i^2 while legislator j sends the tough message m_j^3, then the chair tries to gain the vote from legislator i while giving up on legislator j. He proposes a compromise policy ($y' \approx -0.23$) and forms a minimum winning coalition with legislator i ($x_i' \approx 0.3$) to legislator i and zero share to legislator j. The proposal is rejected by legislator j, but is accepted by legislator i since any $t_i \in (\tau_i^2, \tau_i^3]$ strictly prefers it to the status quo.

Perhaps the most interesting case is when both legislators signal willingness to compromise by sending m_i^2. In the equilibrium we constructed, it is equally costly (in expectation) for the chair to win the vote of either legislator. So he randomizes with equal probability between two proposals that involve the same policy ($y'' \approx -0.45$) and the same cake share ($x_0'' \approx 0.87$) for himself, but differ with respect to which legislator he chooses to form a coalition with. Compared with the case in which only one legislator signals willingness to compromise while the other shows a tough stand, here the compromise policy is even closer to the chair's ideal and the cake share that the chair keeps for himself is also larger. Intuitively, when both legislators signal willingness to compromise, they create competition between themselves. Since the chair needs only one vote to have a proposal passed, his optimal proposal involves less ideological compromise and less distributive concession. It is interesting to observe that in equilibrium the legislator who is excluded from the coalition still votes for the proposal if he finds it more attractive than the status quo. In particular, suppose legislator i gets a positive share while j does not. Then legislator i votes for the proposal if $u_i\left(y'', 1 - x''_0, t_i\right) \geq u_i\left(\tilde{y}, 0, t_i\right)$, i.e., if $t_i \in [-0.80, -0.075]$ and legislator j votes for the proposal if $u_j\left(y'', 0, t_j\right) \geq u_i\left(\tilde{y}, 0, t_j\right)$, i.e., if $t_j \in [-0.80, -0.224]$. Although the probability that legislator j votes for the proposal is lower compared to legislator i, both vote for the proposal with positive probability. Moreover, this proposal is rejected with positive probability in equilibrium.

To compare the informative size-three equilibrium with the uninformative babbling equilibrium (which always exists), note that in the babbling equilibrium, what the chair should propose becomes a simple optimization problem. Calculation shows that in this example the chair's optimal proposal is $(0; 1, 0, 0)$, i.e., he gets the whole cake and implements the status quo policy. So the chair's equilibrium payoff in the babbling equilibrium is $1 - (0+1)^2 = 0$. Since the chair always benefits from more information revelation, his expected payoff is higher in the size-three equilibrium than in the babbling equilibrium. Calculation confirms that his expected payoff in the size-three equilibrium is 0.56, higher than that in the babbling equilibrium. Interestingly, the other legislators also have higher expected payoffs in the size-three equilibrium (-2.662) than they do in the babbling equilibrium (-2.666). So they also benefit from more information transmission.

Seniority and uncertainty: One special case of our analysis is when only one legislator's preference is uncertain, perhaps because the other legislator is a senior member with known preference, as discussed in section 3. If the senior member prefers the chair's ideal to the status quo policy, then the chair can propose $(\hat{y}_0; c, 0, 0)$ and have it passed. In this case, the junior legislator's message has no effect on equilibrium outcome. The interesting case is when the senior member prefers the status quo to the chair's ideal. The message rule of the junior legislator still has at most size three. To illustrate with an example, suppose legislator 2 is the senior member with a known ideological position $\hat{y}_2 = -0.2$ but otherwise keep the parametric assumptions in Example 2. We find that legislator 1 has an equilibrium message rule of size three with the cutoffs $\tau_1^1 = -2, \tau_1^2 = -0.87, \tau_1^3 = -0.3$ and $\tau_1^4 = 2$. The chair responds to the "cooperate" message with the proposal $(\hat{y}_0; 1, 0, 0)$, the "compromise" message with the proposal

$(y'; 1 - x_1', x_1', 0)$ where $y' = -0.65$, $x_1' = 0.0325$ and "fight" message with the proposal $(y''; 1 - x_2'', 0, x_2'')$ where $y'' = -0.6$ and $x_2'' = 0.12$. A few things are worth noting. First, when the junior member sends the "fight" message, the chair does not respond with the minority coalition of $(\tilde{y}; c, 0, 0)$ because it is better to form a coalition with the senior legislator. The proposal will be accepted by the senior legislator, but rejected by the junior one. This observation holds in general as long as the senior member's preference is sufficiently close to the chair's so that it is better for the chair to form a coalition with him than to implement $(\tilde{y}; c, 0, 0)$. Second, the senior legislator is included in a coalition only when the junior legislator sends the "fight" message. To see this, note that if the chair were to respond to the "compromise" message with a coalition that includes the senior legislator, then the "fight" message and the "compromise" message induce the same proposal $(y''; 1 - x_2'', 0, x_2'')$, effectively becoming the same message. So when the junior legislator sends the "compromise" message in equilibrium, the chair always proposes a minimum winning coalition that includes the junior legislator.

Disadvantages of bundling the two dimensions: As we have seen in section 3, bundling the ideological and distributive dimensions together affords the legislators the flexibility of trading private benefits for policy compromises and is always better for the chair when the uncertainty is about the legislators' ideological intensities. As will be shown in the discussion that follows, however, when the uncertainty is about the legislators' ideological positions, bundling has two potential disadvantages. The first disadvantage is that by combining the two dimensions together, the chair risks losing the cake if negotiation breaks down whereas no such risk exists if the legislators bargain over the distributive dimension separately.

This disadvantage can be easily illustrated using Example 2. Suppose the legislators negotiate over the two dimensions separately. Then bargaining over the distributive dimension becomes a simple ultimatum game and the chair keeps the whole cake to himself. As to the ideological dimension, it is straightforward to show that the most-informative equilibrium is the following[17]: both legislators 1 and 2 play the message

[17]Matthews (1989) considers a bargaining game between two legislators over ideology and shows that an equilibrium has at most "size two." Although there are three legislators in our game, we can modify

rule such that $m_i(t_i) = m_i^1$ if $t \in [\tau_i^1, \tau_i^2]$, $m_i(t_i) = m_i^2$ if $t \in (\tau_i^2, \tau_i^3]$ where $\tau_i^1 = -2 < \tau_i^2 < \tau_i^3 = 2$; the chair proposes his ideal $\hat{y}_0$ if at least one of the legislators send m_i^1, and he proposes a compromise policy y if both legislators send m_i^2. The indifference condition of type τ_i^2 implies that $\tau_i^2 \approx -0.81$. If both legislators send m_i^2, the chair responds with the proposal $y \approx -0.62$, and the legislator i votes for the proposal if and only if $t_i \leq \frac{y+\tilde{y}}{2} \approx -0.31$. Calculation shows that the chair's expected equilibrium payoff on the policy dimension is -0.36. Since he keeps the whole cake, his payoff on the distributive dimension is 1. So the chair's total expected payoff is $1 - 0.36 = 0.64$, higher than his expected payoff ($= 0.56$) in the size-three equilibrium if the two dimensions are bundled together in negotiation. The reduction of payoff from bundling comes from the loss of the cake when the legislators fail to reach an agreement, which happens when both legislators send the "compromise" message but their ideological positions are too far from the chair's for them to find the resulting proposal attractive enough. Although failure to reach an agreement also happens even if the legislators negotiate the ideological dimension separately, the distributive dimension is shielded from such failure.

Another, perhaps less obvious, disadvantage of bundling is the information loss that may result from bargaining the two dimensions together (or, as another interpretation, the chair's lack of commitment of not using private benefit in exchange for votes on ideological decisions). This matters even if there is no risk of "losing the cake." To illustrate, suppose $c = 0$, so the break-down of agreement does not result in the dissipation of private benefits. In this case, we can interpret bundling of the two dimensions together as (the possibility of) using side payments to gain support on an ideological decision and separation of the two dimensions as the unavailability of side payments. As shown in the previous paragraph, if no side payments are allowed, then there exists an informative equilibrium in which the legislators send m_i^1 if t_i is below τ_i^2 and send m_i^2 if t_i is above τ_i^2. When side payments are allowed, however, this is no longer an equilibrium strategy if the chair puts a relatively low weight on the distributive dimen-

the argument provided in Matthews (1989) to show that each legislator's message rule has at most size two when they bargain over just the ideological dimension. Because it does not provide much new insight, we omit the details of the modified argument here.

sion. For example, suppose $\theta_0 = 9$, but keep all the other parametric assumptions in Example 2 unchanged. If the legislators were to follow the message rule with the cutoff $\tau_i^2 = -0.81$, then, upon receiving m_i^2 from both legislators, the chair would optimally respond by proposing y equal to his ideal point $\hat{y}_0$ and a side payment ($= 3.\,841\,5$) to one of the legislators. But this undermines the legislator i' incentive to send m_i^1 because it induces the chair's ideal without the possibility of getting any side payment whereas the other message m_i^2 induces the chair's ideal with a side payment as well. Indeed, in this example there exists no informative equilibrium when the chair can use side payments in his proposals. Because of this informational loss, the chair's equilibrium payoff is lower when side payments are allowed.[18]

5 Concluding remarks

In this paper, we develop a new model of legislative bargaining that incorporates private information about preferences and allows speech making before a bill is proposed. Although the model is simple, our analysis generates interesting predictions about what speeches can be credible even without commitment and how they influence proposals and legislative outcomes. We are also able to provide new insight into when legislators should make proposals for different issues together and when they should make proposals separately.

We believe that both private information and communication are essential elements of the legislative decision making process. Our paper has taken a first step in understanding their roles in the workings of a legislature. There are many more issues to explore and many ways to generalize and extend our model and what follows is a brief discussion of some of them.

Our motivation for incorporating private information into legislative bargaining is that individual legislators know their preferences better than others. Another possible

[18] Harstad (2007) shows in another bargaining game that side payments may be harmful because they increase conflict of interest and incentive to signal, resulting in more delay. The reason for side payments to be harmful is different here.

source of private information is that some legislators may have better information (perhaps acquired through specialized committee work or from staff advisors) regarding the consequences of certain policies, which is relevant for all legislators. Although the role of this kind of "common value" private information in debates and legislative decision making has been studied in the literature (e.g. Austen-Smith (1990)), it is only in the context of one-dimensional spatial policy making. It would be interesting to explore it further when there is trade off between ideology and distribution of private benefits.

In our model the chair does not have private information about his preference, consistent with the observation that bill proposers are typically established members with known positions. But sometimes legislators can be uncertain about what exactly the legislative leaders' goals are, in particular, how much compromise the leaders are willing to make to accommodate their demands in exchange for their votes. In this case, apart from speeches, the proposal that the chair puts on the table may also reveal some of his private information. This kind of signaling effect becomes particularly relevant when the legislators have interdependent preferences or when the proposal is not an ultimatum but can be modified if agreement fails.

We have focused on a specific extensive form in which the legislators send messages simultaneously. It would be interesting to explore whether and how some of our results change if the legislators send messages sequentially. In that case, the design of the optimal order of demands (from the perspective of the proposer as well as the legislature) itself is an interesting question. Another design question with respect to communication protocol is whether the messages should be public or private. Although this distinction does not matter for our model because we assume simultaneous speeches and one round of bargaining, it would matter if either there were multiple rounds of bargaining or the preferences were interdependent.

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