Time-Varying Wage Risk, Incomplete Markets, and Business Cycles

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Question

- How do changes in idiosyncratic labor income risk affect aggregate fluctuations?
- In particular, how is labor market dynamics affected?
Motivation

- Growing interests in fluctuations in uncertainty
  - uncertainty shocks by Bloom 2009
  - various measures of uncertainty rose in the recent financial crisis

- Cyclical variation in idiosyncratic labor earnings risk
  - Storesletten, Telmer, and Yaron 2004, Heathcote, Perri, and Violante 2010
  - previous DSGE analyses typically omit labor supply decisions
    $\implies$ little is known about the impact on labor market dynamics
What this paper does

- Augment DSGE model widely used for labor market analyses
  - idiosyncratic wage/productivity risk
  - incomplete asset markets
  - indivisible labor
  - aggregate shocks
    - TFP shocks
    - uncertainty shocks (fluctuations in idiosyncratic wage risk)
- Analyze the impact of uncertainty shocks on business cycles through stochastic simulation
  - infer the size of uncertainty shocks using individual wage data
Main findings

- Uncertainty shocks move key statistics closer to data
  - $corr(H, Y/H)$ decreases from 0.83 to −0.40
  - $\sigma_{wedge}$ increases from 17% of data to 90%

- Aggregation bias (composition effect)
  - Impacts of uncertainty shocks on employment differ across productivity groups
Varying uninsured idiosyncratic earnings risk


  - exogenous earnings or divisible labor

Uninsured Idiosyncratic wage risk under indivisible labor


  - constant risk
Individuals

- Momentary utility: $u(c, h)$
  - $c$: consumption
  - $h$: labor hours, $h \in \{\bar{h}, 0\}$

- Time-varying uninsured idiosyncratic wage risk
  - **idosyncratic wage risk**: person-specific labor productivity $x$
    \[
    \ln x' = \rho_x \ln x + \varepsilon'_x, \quad \varepsilon'_x \sim \mathcal{N}(0, \sigma^2_{\varepsilon_x})
    \]
  - **uninsured**: single asset $k$ (physical capital), $k \geq k$ ($k \leq 0$)
  - **time-varying**: $\sigma_{\varepsilon_x}$ is a Markov chain
Beginning-of-period value

\[ V(k, x; z, \sigma_{\varepsilon_x}, \mu) = \max\{ V^E(k, x; z, \sigma_{\varepsilon_x}, \mu), V^N(k, x; z, \sigma_{\varepsilon_x}, \mu) \} \]

- Value functions
  - \( V \): beginning-of-period value
  - \( V^E \): employment value
  - \( V^N \): nonemployment value

- State variable
  - \( k \): individual asset holding
  - \( x \): idiosyncratic productivity
  - \( z \): aggregate TFP, AR(1) process
  - \( \sigma_{\varepsilon_x} \): idiosyncratic wage risk, learned one period in advance
  - \( \mu \): individual distribution over \( k \) and \( x \), \( \mu' = \Gamma(z, \sigma_{\varepsilon_x}, \mu) \)
Value of employment

\[ V^E(k, x; z, \sigma_{\varepsilon_x}, \mu) = \max_{c, k'} \left\{ u(c, \bar{h}) \right. \]
\[ + \beta E[V(k', x'; z', \sigma'_{\varepsilon_x}, \mu')|x, z, \sigma_{\varepsilon_x}, \mu] \} \]

s.t. \[ c + k' = w(z, \sigma_{\varepsilon_x}, \mu)x\bar{h} + [1 + r(z, \sigma_{\varepsilon_x}, \mu)]k \]
\[ k' \geq k \]
\[ c \geq 0 \]

Law of motion for \( x, z, \sigma_{\varepsilon_x}, \) and \( \mu \)
Value of nonemployment

\[ V^N(k, x; z, \sigma_{\varepsilon_x}, \mu) = \max_{c,k'} \{ u(c, 0) \}
+ \beta E[V(k', x'; z', \sigma'_{\varepsilon_x}, \mu') | x, z, \sigma_{\varepsilon_x}, \mu] \}
\]

s.t. \[ c + k' = [1 + r(z, \sigma_{\varepsilon_x}, \mu)]k \]
\[ k' \geq k \]
\[ c \geq 0 \]

Law of motion for \( x, z, \sigma_{\varepsilon_x}, \) and \( \mu \)
Employment choice

\[ h = \begin{cases} \overline{h} & \text{if } V^E \geq V^N \\ 0 & \text{otherwise} \end{cases} \]
Representative firm

- Produce the good $Y$
- Rent capital $K$ and labor $L$ from individuals
- Production function
  - $Y = zF(K, L)$
  - constant returns to scale in $K$ and $L$
- Maximize static profits
  \[
  r(z, \sigma_{\epsilon_x}, \mu) = (1 - \alpha)zF_K(K, L) - \delta \\
  w(z, \sigma_{\epsilon_x}, \mu) = \alpha zF_L(K, L)
  \]
- $\delta$ : capital depreciation rate
Equilibrium

A recursive competitive equilibrium consists of a set of functions,

\[(w, r, V^E, V^N, V, c, k', h, K, L, \Gamma),\]

that satisfy the following conditions:

- **Individual optimization**
- **Firm optimization**
- **Market clearing (labor, capital, and good)**
  - Labor: \[L = \int xh(k, x; z, \sigma_{\epsilon_x}, \mu)\mu([dk \times dx])\]
  - Hours: \[H = \int h(k, x; z, \sigma_{\epsilon_x}, \mu)\mu([dk \times dx])\]
- **Law of motion for the distribution across individuals is consistent with individuals’ behavior and the underlying stochastic processes**
### Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.9829</td>
</tr>
<tr>
<td>$B$</td>
<td>1.0203</td>
</tr>
<tr>
<td>$h$</td>
<td>1/3</td>
</tr>
<tr>
<td>$k$</td>
<td>-2.0</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.64</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.025</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>0.950</td>
</tr>
<tr>
<td>$\sigma_{\varepsilon_z}$</td>
<td>0.007</td>
</tr>
</tbody>
</table>

\[
u(c, h) = \begin{cases} 
\ln c - B & \text{when work} \\
\ln c & \text{when not work}
\end{cases}
\]

\[
y = zF(K, L) = zK^{1-\alpha}L^\alpha
\]

\[
\ln z' = \rho_z \ln z + \varepsilon_z', \ \varepsilon_z' \sim N(0, \sigma_{\varepsilon_z}^2)
\]
Parameters on idiosyncratic productivity

\[ \ln x' = \rho_x \ln x + \varepsilon'_x, \quad \varepsilon'_x \sim \mathcal{N}(0, \sigma^2_{\epsilon_x}) \]

- \(\sigma_{\epsilon_x}\) is a 3-state Markov chain
  - \((1 + \lambda)\bar{\sigma}_{\epsilon_x}, \bar{\sigma}_{\epsilon_x}, (1 - \lambda)\bar{\sigma}_{\epsilon_x}\)
  - remain unchanged with prob \(\rho_{\sigma_{\epsilon_x}}\), transition to each of the other states with \((1 - \rho_{\sigma_{\epsilon_x}})/2\), independent of \(z\)

- Parameters
  \[\rho_x, \quad \bar{\sigma}_{\epsilon_x}, \quad \lambda, \quad \rho_{\sigma_{\epsilon_x}}\]
Moments compared between PSID and model

\[
\ln x_{i,t} = \rho_x \ln x_{i,t-1} + \varepsilon_{i,x,t}, \varepsilon_{i,x,t} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_x,t})
\]

\[
\ln w_{i,t} = \ln x_{i,t} + \ln w_t
\]

\[
\ln w_{i,t} = \rho_x \ln w_{i,t-1} + (\ln w_t - \rho_x \ln w_{t-1}) + \varepsilon_{i,x,t}, \varepsilon_{i,x,t} \sim \mathcal{N}(0, \sigma^2_{\varepsilon_x,t})
\]

1. Pooled estimation

\[
\Rightarrow \hat{\rho}_x, \hat{\sigma}_{\varepsilon_x}
\]

- PSID, Model: OLS

2. Year-by-year estimation

\[
\Rightarrow std(\hat{\sigma}_{\varepsilon_x,t}), corr(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1})
\]

- PSID: OLS, controlled OLS, Heckman-type estimation
- Model: OLS
Estimated idiosyncratic wage risk in PSID

![Graph showing the estimated idiosyncratic wage risk from 1970 to 1990. The graph compares OLS, Controlled OLS, and Heckman estimations with different lines for each year. The x-axis represents the years 1970 to 1990, and the y-axis represents the level of estimated idiosyncratic wage risk ranging from 0.2 to 0.35. The graph illustrates the trends and patterns in wage risk across these years.]
Cyclical component of estimated wage risk

<table>
<thead>
<tr>
<th>Year</th>
<th>OLS</th>
<th>Controlled OLS</th>
<th>Heckman</th>
</tr>
</thead>
<tbody>
<tr>
<td>1970</td>
<td>0.032</td>
<td>0.035</td>
<td>0.039</td>
</tr>
<tr>
<td>1975</td>
<td>0.185</td>
<td>0.236</td>
<td>0.056</td>
</tr>
<tr>
<td>1980</td>
<td>0.185</td>
<td>0.236</td>
<td>0.056</td>
</tr>
<tr>
<td>1985</td>
<td>0.185</td>
<td>0.236</td>
<td>0.056</td>
</tr>
<tr>
<td>1990</td>
<td>0.185</td>
<td>0.236</td>
<td>0.056</td>
</tr>
</tbody>
</table>

\[
\text{std}(\hat{\sigma}_{\varepsilon_x,t}) = 0.032, 0.035, 0.039 \\
\text{corr}(\hat{\sigma}_{\varepsilon_x,t}, \hat{\sigma}_{\varepsilon_x,t-1}) = 0.185, 0.236, 0.056
\]
Moments and parameter values

<table>
<thead>
<tr>
<th>Moments (annual)</th>
<th>U.S.</th>
<th>Varying risk</th>
<th>Constant risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\rho}_x$</td>
<td>0.854</td>
<td>0.855</td>
<td>0.855</td>
</tr>
<tr>
<td>$\hat{\sigma}_{\varepsilon_x}$</td>
<td>0.282</td>
<td>0.283</td>
<td>0.279</td>
</tr>
<tr>
<td>$\text{std}(\hat{\sigma}_{\varepsilon_x}, t)$</td>
<td>0.032</td>
<td>0.032</td>
<td>0.008</td>
</tr>
<tr>
<td>$\text{corr}(\hat{\sigma}<em>{\varepsilon_x}, t, \hat{\sigma}</em>{\varepsilon_x}, t-1)$</td>
<td>0.185</td>
<td>0.158</td>
<td>–0.240</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters (quarterly)</th>
<th>Varying risk</th>
<th>Constant risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_x$</td>
<td>–</td>
<td>0.930</td>
</tr>
<tr>
<td>$\bar{\sigma}_{\varepsilon_x}$</td>
<td>–</td>
<td>0.223</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>–</td>
<td>0.090</td>
</tr>
<tr>
<td>$\rho_{\sigma_{\varepsilon_x}}$</td>
<td>–</td>
<td>0.90</td>
</tr>
</tbody>
</table>
Steady state

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini labor income</td>
<td>0.60~0.63</td>
<td>0.60</td>
</tr>
<tr>
<td>Gini wealth</td>
<td>0.78</td>
<td>0.69</td>
</tr>
<tr>
<td>corr(labor income, wealth)</td>
<td>0.23</td>
<td>0.30</td>
</tr>
</tbody>
</table>
### Business cycle statistics

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Constant risk</th>
<th>Varying risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_Y$</td>
<td>1.69</td>
<td>1.37</td>
<td>1.43</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.54</td>
<td>0.32</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>2.85</td>
<td>3.10</td>
<td>3.15</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>1.00</td>
<td>0.57</td>
<td>0.81</td>
</tr>
<tr>
<td>$\sigma_{Y/H}$</td>
<td>0.63</td>
<td>0.48</td>
<td>1.00</td>
</tr>
<tr>
<td>$\text{corr}(Y, C)$</td>
<td>0.78</td>
<td>0.90</td>
<td>0.86</td>
</tr>
<tr>
<td>$\text{corr}(Y, I)$</td>
<td>0.80</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>$\text{corr}(Y, H)$</td>
<td>0.80</td>
<td>0.96</td>
<td>0.41</td>
</tr>
<tr>
<td>$\text{corr}(Y, Y/H)$</td>
<td>0.31</td>
<td>0.95</td>
<td>0.67</td>
</tr>
<tr>
<td>$\text{corr}(H, Y/H)$</td>
<td>$-0.32$</td>
<td>0.83</td>
<td>$-0.40$</td>
</tr>
</tbody>
</table>
One-period increase in idiosyncratic wage risk

Horizontal axis – period
Vertical axis – percent deviation
Underlying two effects

- Uncertainty effect (period 0)
  - uncertainty about future wages rises $\Rightarrow H \uparrow$, $Y/H \downarrow$

- Distribution effect (period 1)
  - the productivity-wealth distribution shifts $\Rightarrow H \downarrow$, $Y/H \uparrow$
Uncertainty effect

- Graph showing density over productivity index and wealth (in(k+2)).
- The graph illustrates the distribution of outcomes under uncertainty.
- The density is shown on a logarithmic scale.

Not Work

Work

Productivity Inx

Wealth in(k+2)
Uncertainty effect

<table>
<thead>
<tr>
<th>Employment</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>low productivity</td>
<td>H</td>
</tr>
<tr>
<td>high productivity</td>
<td>H</td>
</tr>
<tr>
<td>↑↑</td>
<td>↑</td>
</tr>
<tr>
<td>↓</td>
<td>↓</td>
</tr>
</tbody>
</table>

Density

Not Work

Work

Productivity Inx

Wealth ln(k+2)
Uncertainty effect (period 0)
- uncertainty about future wages rises $\Rightarrow H \uparrow, \ Y / H \downarrow$

Distribution effect (period 1)
- the productivity distribution shifts $\Rightarrow H \downarrow, \ Y / H \uparrow$
Distribution effect

![Distribution effect graph](image)

- Productivity Inx
- Density
- Period 0
- Period 1
Distribution effect

<table>
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<tr>
<td>low productivity</td>
<td>H</td>
</tr>
<tr>
<td>high productivity</td>
<td>Y/H</td>
</tr>
</tbody>
</table>

\[ H \leq Y/H \]
Uncertainty versus distribution effects

- Psych risk: only uncertainty effect, individuals receive signals for changes in $\sigma_{\varepsilon_x}$, but those changes in $\sigma_{\varepsilon_x}$ never materialize (Bachmann and Bayer 2013)

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Constant risk</th>
<th>Varying risk</th>
<th>Psych risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_Y$</td>
<td>1.69</td>
<td>1.37</td>
<td>1.43</td>
<td>1.37</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>0.54</td>
<td>0.32</td>
<td>0.33</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>2.85</td>
<td>3.10</td>
<td>3.15</td>
<td>3.10</td>
</tr>
<tr>
<td>$\sigma_H$</td>
<td>1.00</td>
<td>0.57</td>
<td>0.81</td>
<td>0.61</td>
</tr>
<tr>
<td>$\sigma_{Y/H}$</td>
<td>0.63</td>
<td>0.48</td>
<td>1.00</td>
<td>0.52</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>corr($Y, C$)</th>
<th>corr($Y, I$)</th>
<th>corr($Y, H$)</th>
<th>corr($Y, Y/H$)</th>
<th>corr($H, Y/H$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr($Y, C$)</td>
<td>0.78</td>
<td>0.90</td>
<td>0.86</td>
<td>0.89</td>
<td></td>
</tr>
<tr>
<td>corr($Y, I$)</td>
<td>0.80</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>corr($Y, H$)</td>
<td>0.80</td>
<td>0.96</td>
<td>0.41</td>
<td>0.91</td>
<td></td>
</tr>
<tr>
<td>corr($Y, Y/H$)</td>
<td>0.31</td>
<td>0.95</td>
<td>0.67</td>
<td>0.87</td>
<td></td>
</tr>
<tr>
<td>corr($H, Y/H$)</td>
<td>$-0.32$</td>
<td>$0.83$</td>
<td>$-0.40$</td>
<td>$0.58$</td>
<td></td>
</tr>
</tbody>
</table>
Implication for the labor wedge

- Labor wedge is calculated by

\[
\ln \text{wedge} = \ln MPL - \ln MRS = \ln \frac{Y}{H} - \ln B_R H^{1/\gamma} C
\]

\[
U(C, H) = \ln C - \frac{B_R H^{1+1/\gamma}}{1 + 1/\gamma}, \gamma = 1.5
\]

<table>
<thead>
<tr>
<th></th>
<th>U.S.</th>
<th>Constant risk</th>
<th>Varying risk</th>
<th>Psych risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{\text{wedge}})</td>
<td>1.40</td>
<td>0.23</td>
<td>1.26</td>
<td>0.38</td>
</tr>
<tr>
<td>(\text{corr}(H, \text{wedge}))</td>
<td>−0.94</td>
<td>−0.96</td>
<td>−0.84</td>
<td>−0.83</td>
</tr>
</tbody>
</table>

- Fluctuations in the labor wedge arise from those in the deviation of \(w\) and \(MRS\) (Karabarbounis 2014)
Conclusion

- Examine how time-varying idiosyncratic wage risk affects aggregate fluctuations in the heterogeneous-agent model commonly used for labor market analyses.

- Including uncertainty shocks improves the model’s performance concerning labor market dynamics.

- Future work:
  - uncertainty on asset income, endogenous uncertainty
  - other shocks than aggregate TFP and uncertainty shocks
  - home production, family labor supply, and so on
U.S. hours and labor productivity

![Graph showing percent deviation from trend over years](image-url)