Informal Sector, Government Policy and Institutions*

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Abstract

We document cross-country differences in informal activity, government policies and institutions using a data set covering 127 countries. We develop a general equilibrium model where households optimally choose the extent of informal activity and the government optimally chooses policies, both taking as given the institutions of the economy. The model is able to account for the cross-country differences in policies, and other key facts that emerge from the data.

Key Words: Ramsey problem, Friedman rule, inflation, taxation

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1 Introduction

There is considerable heterogeneity across countries regarding the sources of revenues of the government, in particular the use of income taxes versus inflation tax. In this paper we put forth a model that focuses on the effect of institutions on the optimal decisions of governments via their impact on the level of informal activity in the country.[1] We demonstrate that our model is able to account for the cross-country heterogeneity in policies both qualitatively and quantitatively.

To document the cross-country differences in institutions, government policies and levels of informal activity we compile a comprehensive data set of 127 countries. Five key facts emerge from this data set. Better institutions are associated with lower inflation, higher income tax rates and less informal activity. Related to these, we also find that higher levels of informal activity is associated with lower income tax rates and higher inflation.

In order to account for these facts, we use a general equilibrium model that generates both the government policies and the extent of informal activity *endogenously,* taking as given the institutional structure of the country. Our maintained hypothesis is that all countries in the world are populated by identical people and these people and the governments respond optimally to economic incentives stemming from, among others factors, the institutions of the country.

There are three key components of our model. First, we explicitly model the private sector's informal activity choice. Facing a risk of a tax audit (and a punishment if found evading taxes), and taking into account the government's income tax and inflation policies, the agents in the economy optimally choose the level of informal activity. Second, we consider institutions of the country as exogenous. In our model, these institutions will determine the implicit cost of evading taxes (through the tax audit). Third, we consider a benevolent and optimizing government whose objective is to raise a given amount of revenue in the least distorting way, in the tradition of Ramsey (1927). As in many similar optimal policy problems, the government strikes a balance between inflation and taxation. The additional wrinkle in this model comes from the fact that these policies also affect the tax evasion incentives for the private sector – higher inflation deters and higher taxes encourage informal

[1] By institutions we refer to the set of rules that determine how economic activity is conducted. In our empirical analysis we use "rule of law" to measure institutions. For our purposes the terms "unofficial", "informal" or "shadow" economy refer to the same phenomenon, which is any economic activity that is done outside the reach of the government and therefore is not subject to taxation. A key characteristic of informal activity is that it is typically cash-intensive.

activity.

The second and third components, the exogeneity of institutions and the presence of a benevolent and optimizing government deserve some elaboration. Acemoglu, Johnson and Robinson (2005) argue that the institutions of a country should be thought of as endogenously determined in a dynamic model, along with a number of economic and political outcomes. However, as they and many others emphasize, institutions are very persistent and thus evolve slowly – certainly much slower than the two government policies we consider. As such, one can think of our model as embedded within one period of the dynamic framework in Acemoglu, Johnson and Robinson (2005), during which institutions are constant and therefore given.

Considering the large set of countries we have in our dataset, the assumption of a benevolent and optimizing government choosing policies is clearly a stretch. In many countries factors other than the ones considered in our model, such as political considerations are key in determining policies. We view this as a test of our theory. We expect our theory to *fail* in such countries as much as to *succeed* in those where the political considerations are of secondary importance.

To understand how the mechanism in our model works, consider two countries A and B which are identical in all aspects except that country B has "better" institutions. The citizens of country B will choose to do less informal activity than those of country A because the cost of tax evasion is higher. In turn, the government of country B will be less inclined to use inflation to discourage informal activity and needs to use higher income taxes to finance its expenditures.[2] As a result, we find that country B has less informal activity, lower inflation and higher taxes which shows that our model can *qualitatively* explain the five facts we obtained from the data.

We then take our model to the data and calibrate it for each of the 127 countries in our sample, maintaining the assumption that abilities and desires of households are identical across countries. As we hinted above, our calibration is successful for the majority of the countries, but fail for others. For the latter set of countries, we are not able to find a combination of taxes and inflation that generates sufficient revenue for the government while keeping the informal sector active. Focusing on the former set of countries, we generate model-based measures of the size of the informal sector and government policies. The measures of the size of the informal sector are of interest on their own as they are generated

[2]Even though we describe the events in equilibrium as sequential, all decisions are of course simultaneous.

without any *a priori* assumptions about the determinants of the informal sector. Using the measures obtained from our model, we show that our model can also *quantitatively* explain the five facts we obtained from the data. Turning to the countries for which the calibration fails, we find that these countries are the poorest, has worse institutions than the median country and, most importantly are classified as either authoritarian or a hybrid regime in a widely-used ranking of countries with respect to their democracy status. As we argued above, this is a positive result for our model since our key assumption of a benevolent and optimizing government is clearly least valid for these set of countries.

Our work is related to a number of different strands in the literature. There is a vast empirical literature that focuses on the causes of informal activity. Johnson, Kaufmann and Schleifer (1997), Johnson, Kaufmann and Zoido-Lobaton (1997) and Friedman et al. (2000) provide empirical results suggesting that large informal markets are typically associated with high tax burden and institutional factors such as regulation, poor enforcement of law and corruption. These results are useful in establishing the link between institutions and informal activity, which is one of the key facts we exploit, but are silent in the relationship between informal activity and government policies both of which are endogenous.

There is also a very large literature on political-economy explanations of cross-country differences in government policies. One of the seminal papers, Alesina and Drazen (1991), links the delay in stabilization policies to a war of attrition between different socioeconomic groups in the country which may be affected asymmetrically from the stabilization. Until the appropriate policies are enacted, the economy continues in a volatile path which is also typically associated with high inflation. Their model implies, among other things, that political polarization of a country would be associated with longer periods of instability, hence higher likelihood of high inflation. In a recent paper Alesina, Ardagna and Trebbi (2006) test the implications of the mechanism in Alesina and Drazen (1991) and find that the delay in stabilization is shorter when the ruling executive has more control over the legislative body of the country or the executive has more institutional constraints. Albanesi (2007) also provide a political-economy explanation for differences in inflation that relies on exogenous differences in labor productivity. In her model, the poor, those with lower labor productivity, hold more of their wealth as currency and thus are more vulnerable to inflation. If a country has a more unequal income distribution, the political bargaining process favors the rich and equilibrium inflation is higher. She shows that her model is able to account for a large fraction of the positive correlation between inflation and inequality. Explanations for cross-country

differences in policies based on the conflict between heterogenous segments of the society have also been provided by Fernandez and Rodrik (1991), Cukierman, Edwards and Tabellini (1992) and Laban and Sturzenegger (1994). As we conjectured above, *a priori* we expect our model to fail for countries where the above-mentioned political-economy considerations are most important. To the extent that it helps explain the facts we document for some countries, we think our theory complements the political-economy theories we outlined.

Our work is also linked to a small theoretical literature that considers the determinants of informal activity. Nicolini (1998) is one of the first to show theoretically in the context of a cash-good-credit-good model that tax evasion due to informal activity is a motive for inflation under optimal policy. He finds, however, that this is not quantitatively sufficient to explain the high inflation in Peru. Yesin (2004, 2006) considers the optimal policy in the same model when the government faces (exogenous) tax collection costs and finds some success in explaining different policies for a small set of countries. The extent of informal activity (the set of goods that are formal versus informal) is assumed to be exogenously fixed in these papers. Koreshkova (2006) also models the trade-off that an optimizing planner faces between taxation (and evasion) and inflation in a cash-in-advance model with costly credit. The size of the informal sector in her model is directly linked to the assumed productivity differences across formal and informal production and as such can be considered exogenous. None of the mentioned papers consider the effect of institutions on government policies through its impact on the incentives of the private sector. Ahiabu (2006) explores the trade-off between tax rates in the formal sector and audits (punishments) in the informal sector but does not conduct an optimal policy exercise. Finally, Kuehn (2007) considers the mechanisms behind informal activity in high-income countries, building a model where agents of different abilities choose whether or not they want to become workers, managers of a firm in the formal sector or managers of a firm in the informal sector. The trade-off between the latter two exists due to the probability of getting caught and being punished. While she considers the effect of institutions on informal activity, government policies are considered to be exogenous and using a real model she only considers taxes as policy.

In terms of modelling strategy, we focus on two properties of informal activity: tax evasion and cash intensiveness. To capture these features, we use a search-based monetary model combining elements from recent advances in the field such as Lagos and Wright (2005), Rocheteau and Wright (2005) and Aruoba and Chugh (2007).[3]

[3]In this class of models, a medium of exchange is "essential" for trade in decentralized exchange. In

Our work is also related to a large literature on the macroeconomic effects of institutions. Hall and Jones (1999), one of the seminal papers in this literature, show that differences in output per worker across countries can be largely explained by differences in social infrastructure which include government policies and institutions.

The paper is organized as follows. In Section 2, we provide a detailed description of the data used in our cross-country exercise and the facts that emerge. In Section 3, we present our model, show the equilibrium for given policies and the Ramsey equilibrium where policies are also endogenous. In Section 4 we present our quantitative results. Section 5 concludes. An appendix available from the author provides more details on the data and the derivations.

2 Data and Facts

In this section we establish the facts we seek to explain using the model we develop in subsequent sections. To that end, we put together a data set that covers 127 countries. The data set includes measures of institutions, informal activity, government policies and economic indicators for 1996-2004 or a subset as dictated by data restrictions. Looking ahead to our model, since our model does not have any short-run fluctuations, we want to focus on a point in time and we take averages over this short interval to prevent any idiosyncratic events or business cycles to affect our results. Details about the data, including list of countries and detailed sources as well as some alternative measures we used, are available in the Appendix.

2.1 Data Sources

Our main measure of institutions is Rule of Law as reported in the World Bank Governance Matters IV, Kaufmann, Kraay and Mastruzzi (2005), averaged over the period 1996-2004. This is a widely-used measure that is computed combining data from many a number of different sources. We also use Control of Corruption from the same source, as well as some measures from the World Economic Forum (WEF) Competitiveness Report and the Heritage Foundation's (HF) Index of Economic Freedom as alternatives.[4] All of these measures are highly correlated among themselves (as the first row of panel (a) of Table 1 shows) and our conclusions are unchanged using any of the alternative measures.

our model the decentralized market is narrowly defined as the informal sector but these papers take a more general view of the decentralized market.

[4]The alternative institutions measures are Irregular Payments (WEF), Property Rights (HF) and Freedom from Corruption (HF). Some of these measures are available for a smaller sample of countries.

There are a number of alternative estimates of the size of the informal sector that differ in terms of their methodology.[5] Since the size of the informal sector is latent by its nature, all of these methods use some observed data along with some identifying assumptions to provide an estimate. For example, the currency demand approach starts with the assumption that transactions in the informal sector uses cash and any "excess" money holdings over and above what a standard money demand regression indicates should be a sign of informal activity. The physical input method starts with the premise that any production, formal or informal, should use some inputs such as electricity, and as such one should be able to infer the size of the informal sector by comparing the GDP imputed using these inputs and the measured GDP. Schneider (2004) uses a statistical method called the DYMIMIC (dynamic multiple indicators multiple causes) where some structural equations provide causal relationships between two sets of variables and the size of the informal sector: those identified as causes of informal activity and those identified as being affected by informal activity. For example, these equations assume that burden of taxation and burden of regulation are among the causes, while various monetary and labor market variables are among those affected by informal activity. Since the DYMIMIC method provides only a relative measures across countries, Schneider (2004) combines his relative measures with absolute measures from the currency demand approach for some selected countries to compute absolute measures for all countries. In addition to this quantitative measure, we use some qualitative measures from the WEF Global Competitiveness Report as alternative measures.[6] All of these measures are highly correlated among themselves (as the first row of panel (b) of Table 1 shows).

Turning to government policies, tax rates requires some care. Computing tax rates that are conformable with assumptions in macroeconomic models is a difficult task. For example, in such models, the labor income tax creates a wedge between the real wage of a worker and his marginal product. According to this definition, in addition to the taxes that the worker pays, social security taxes that an employer pays should also be included in a measure of labor income tax, even though it does not affect the workers take-home pay. Mendoza, Razin and Tesar (1994) measure consumption, labor and capital income taxes using detailed government revenue accounts for the OECD countries. We use two recent studies that extend their methodology to more countries and/or extend the sample. In particular Carey and Tchilinguirian (2000) provide updated measures for the OECD countries and IMF

[5] Schneider (2004) provides a summary of some of the major methods along with a detailed bibliography.

[6] The alternative measure of the size of the informal sector are Unreported Profits and Wages Informal Sector and Tax Evasion. These measures are available for a smaller set of countries.

World Economic Outlook (2003) provide measures for a small set of non-OECD countries. Combining these two sources we get only 33 countries.[7] We also consider tax revenues as a fraction of GDP as an alternative measure.

We obtain a number of macroeconomic indicators from the IMF's International Financial Statistics (IFS) database and Penn World Tables (PWT 6.2). These include inflation, output relative to the United States and output per worker relative to the United States. Finally, we need a measure of government spending and for this we use the World Bank's World Development Indicators which provides government spending as a percentage of GDP. This measure is fairly broad and includes both consumption and investment activities of the government. Whenever this measure is not available, we use the share of government consumption in output from PWT which is a narrower measure.

One issue that needs to be addressed is whether or not official estimates of GDP include any activity that we would label informal. While certain details could differ from country to country, all illegal activities (e.g. drug sales) and most household production activities are excluded from official measures.[8] In our empirical analysis, we make the assumption that the macroeconomic data that we observe reflect only formal activity and do not include any information, either as explicit measurements or as adjustments, about the informal sector. While making different assumptions would certainly alter some of the details, we need to make this assumption since we do not have information about how the statistical agency of *each* country treat informal activity.

2.2 Facts

We focus on five facts regarding informal activity, government policies and institutions that is obtained using the data as described above. Table 1 shows all correlations mentioned below.

Fact 1: Institutions and inflation are negatively correlated.

Figure 1 shows the relationship between rule of law, our main measure of institutions and inflation with a correlation coefficient of -0.32 in the full sample and -0.43 in a restricted sample where we eliminate 9 countries with more than 20% inflation. Alternative measures for institutions yield a correlation between -0.29 and -0.43.

[7] As is common practice, we combine the labor income taxes (τ^h) and consumption taxes (τ^c) to create a measure of total taxes (τ) using the formula $(1-\tau) = \left(1-\tau^h\right) / \left(1+\tau^c\right)$.

[8] Bureau of Economic Analysis (2001) explain the treatment of hidden and informal activities in U.S. national accounts.

Fact 2: Institutions and taxes are positively correlated.

Figure 2 shows that both tax rates (top panel) and tax revenues as a percentage of GDP (bottom panel) are positively correlated with institutions, with correlation coefficients of 0.59 and 0.69, respectively. When we use alternative measures of institutions, the correlation coefficient is between 0.46 and 0.74. These calculations use only 33 and 37 countries, respectively, due to data limitations.

Fact 3: Institutions and the size of the informal sector are negatively correlated.

Figure 3 shows the strong negative relationship between institutions and the size of the informal sector with a correlation coefficient of -0.72. Looking at alternative pairs of measures for both, we find correlation coefficients between -0.61 and -0.83.

Fact 4: Inflation and the size of the informal sector are positively correlated.

Figure 4 shows that inflation and the size of the informal sector are mildly positively correlated, with a correlation coefficient of 0.26. The bottom panel restricts the sample to those countries with an inflation rate less than 20%. We compute the same correlation using the alternative measures for the size of the informal sector and the correlations range from 0.30 to 0.46.

Fact 5: Tax rates and the size of the informal sector are negatively correlated.

Figure 5 shows that both tax rates and tax revenues are negatively correlated with the size of the informal sector for the countries we have tax data for. The correlations are -0.50 and -0.55, respectively and the alternative measures of informal activity yield correlations ranging from -0.22 to -0.56.

2.3 Discussion

The objective of this paper is to explain the relationships in Facts 1, 2 and 3 as *causal* relationships using a structural model. In this model the exogenous variation will come from institutions as well as (labor) productivity and government expenditures as a percentage of GDP. In order to explore the plausibility of this exercise, in Table 2 we report results from some simple regressions where we investigate the determinants of inflation, taxes and size of the informal sector.[9] In panel (a) we show that rule of law is an important determinant of

[9] One can argue that the regressions here suffer from endogeneity and institutions should be instrumented. We find it useful to explore the relationships in these regressions since in our model we assume institutions are exogenous.

government policies, even in the presence of the other two exogenous variables. For taxes this is especially important as one could argue the relationship captured in Figure 2 and Fact 2 is simply due to countries with better institutions having larger governments. While the latter statement is correct, column (4) in Table 2 shows that even after controlling for level of government expenditures, institutions are important for understanding cross-country differences in taxes.

Panel (b) of Table 2 uses the size of the informal sector as the dependent variable. Two important conclusions emerge. First, columns (5) and (7) interpret Facts 4 and 5 as causal relationships, as a number of previous papers have assumed. That is, exogenous government policies cause differences in informal activity. However comparing columns (5) to (6) and (7) to (8) reveals that any relationship between these variables are simply due to variations in institutions. In other words, once we control for the countries' institutions, neither the level of inflation nor taxes are important in explaining informal sector activity. Thus the positive relationship reported in Fact 4 is simply due to exogenous changes in institutions affecting inflation and size of the informal sector in the same direction. To further emphasize this point, Figure 6 plots inflation and the size of the informal sector for three selected countries over the period 1989-2008: Australia, Greece, and the United States. Since institutions are persistent and vary very slowly over time, we can argue that during this period of about 20 years the changes in institutions for each country is negligible. Figure 6 shows strong negative correlations between inflation and size of the informal sector for these countries, instead of the positive correlation shown in Fact 4. Similarly, the negative relationship reported in Fact 5 is due to exogenous changes in institutions affecting taxes and size of the informal sector in the opposite direction. The second result from panel (b) of Table 2 can be seen in column (9). Among the three exogenous variables we use in our analysis, institutions is the only one which have strong explanatory power for the size of the informal sector.

To reiterate our objective, we describe a model of a country where households and the government choose their actions optimally, taking as given the institutions of the country, along with its labor productivity and government expenditures as a fraction of GDP. In particular the households choose, among other things, how much informal activity to do taking as given government's choices of policies and the three exogenous variables. Understanding their impact on the household's incentives, the government chooses its policies. Our goal is to explain the cross country correlations we presented in Facts 1, 2 and 3 as causal relationships where the exogenous variation comes from the three exogenous variables.

3 Model

The model is based on the structure in Rocheteau and Wright (2005), who in turn build on the setup in Lagos and Wright (2005). Time is discrete and continues forever. As it is not central to the question at hand in this paper, we abstract from any aggregate uncertainty. The economy is a closed one with no interaction with the rest of the world. The economy is populated by infinitely-lived households with measure $\Lambda + 1$, where $\Lambda > 1$. In every period, a formal market meets, followed by an informal market. In the formal market (FM) all households have identical desires and abilities, they supply labor to a neoclassical firm, pay labor income taxes to the government denoted by τ, consume and adjust their portfolio of assets. In this market labor and goods markets are frictionless and everyone acts as price-takers. Transactions can be completed without a need for a medium of exchange. This assumption, that money is not necessary for consuming in the FM, means inflation does not have a direct impact on FM consumption and it will be key in understanding some of our quantitative results below.

In the informal market (IM), measure 1 of households would like to purchase goods and measure Λ of households are able to produce goods. We label these households as buyers and sellers, respectively. The buyers and sellers are randomly matched in the IM where it is possible for some households to be unmatched. Once a buyer-seller pair successfully matches, they bargain over the terms of trade and the buyer pays d units of money for q units of the good. This transaction occurs outside the purview of the government and therefore the proceeds are not taxed. We assume that buyers in this market are anonymous and therefore contracts are not enforceable. As a result the sellers demand a *quid pro quo* and the buyers bring money into the IM to pay for their purchases. After the IM is complete, the buyers consume the goods they purchased in the two markets where we assume the goods are Edgeworth substitutes.

The buyers participate in the IM at no cost. The sellers, on the other hand, face possible audits from the government. Specifically, with probability ζ, the seller will be audited and if found evading taxes the government imposes a utility cost of $\mathcal{P}$.[10] The sellers choose whether or not they want to enter the IM understanding the audit structure. In equilibrium, due to free entry, the marginal seller will be indifferent between entering and not entering, taking

[10] One can consider a number of alternative ways of punishment. One way would be to impose a cost in terms of goods, instead of utility as we do here. In this case, however, the government can in principle use the proceeds to pay for its expenditures. This would raise the possibility to use audits to raise revenue which we choose to avoid.

into account the ex-ante cost of entering, which is $\zeta\mathcal{P}$ and the actions of other sellers which determine the probability of him finding a buyer. We denote $\kappa \equiv \zeta\mathcal{P}$ and in what follows, we formulate sellers problem with κ denoting the (certain) cost of entering the IM for a particular period. The measure of sellers that pay this cost and enter the IM is denoted by n where $n \in (0, \Lambda)$.[11] We interpret κ as the difficulty of evading or avoiding taxes, or quite generally as the rule of law, which is our preferred measure of institutions. The two components of κ, probability of a tax audit and the punishment for evading taxes, can easily be linked to the institutions of the country.

The government's objective is to finance a constant amount of government expenditures, denoted by $\mathcal{G}$, using revenues from income taxes in the FM, seigniorage and a one-period nominal bond. The government conducts all its activities in the FM and its budget constraint is given by

$$M_{t+1} + B_{t+1} + \tau p_t w_t H_t = M_t + R_{t-1} B_t + p_t \mathcal{G} \tag{1}$$

where M and B denote the money and bond stocks of the government, τ is the labor income tax rate, w is the wage rate, H is the aggregate labor supply, p is the price level and R_t is the nominal return of the bond issued in period t. We assume that bonds are book entries with no tangible proof that can be carried in to the IM. This assumption guarantees that money is the only possible asset that can be used as a medium of exchange in the IM.

Since all buyers participate in the IM, the number of successful matches are given by the matching function $\mu(n)$ with $0 \leq \mu(n) \leq \min\{n, 1\}$. Accordingly, the probability that a buyer can find a seller is given by $\alpha_b \equiv \mu(n)$ and the probability that a seller can find a buyer is given by $\alpha_s \equiv \mu(n)/n$ where α_b and α_s are taken as given by the agents. Households have utility function $u(q, x)$ where x denotes the quantity of FM consumption. We make standard assumptions on the utility function $u_q, u_x > 0$ and $u_{qq}, u_{xx} < 0$ and as mentioned above we assume $u_{qx} < 0$, which makes q and x Edgeworth substitutes.[12] The sellers operate a constant-returns-to-scale (CRS) production function in the IM given by $q = Se$ where e denotes the sellers effort in the IM and S is labor productivity which is common across markets. In the FM, a neoclassical firm operates the same CRS production function $Y = SH$ where H is the labor they hire in a competitive market and pays pre-tax real wages $w = S$.

[11] As a technical point, in our quantitative work, we pick a Λ where $n < \Lambda$ always holds. The determination of n depends on a free-entry condition for which this restriction is key.

[12] The latter is necessary for technical reasons. As Aruoba and Wright (2003) show for the model in Lagos and Wright (2005), which immediately applies to the model here as well, without such an assumption a dichotomy would prevail where the IM and FM variables do not interact. From a more substantive point of view, this assumption makes it clear that the goods sold in the two markets are similar goods.

Households have linear disutility in the FM and IM markets.[13] Given the linear disutility in e, we can express the utility cost of production for a seller in the IM as $c(q) = q/S$.

In what follows, we first describe the optimization problems of the buyers and the sellers in the two markets and arrive at the equilibrium which takes government policies τ and R as given. We then turn to the Ramsey problem in order to endogenize the decisions of the government.

3.1 Formal Market

We use superscripts for variables to denote the type of the agent it belongs to, where B, P and N denote buyers and participating sellers and non-participating sellers, respectively. Using $W^B(.)$ to denote the value of entering the FM and $V^B(.)$ the value of entering the IM, a buyer that enters the FM faces the problem

$$W^B(\tilde{m}_t, \tilde{b}_t) = \max_{x_t^B, h_t^B, m_t^B, b_{t+1}^B} \left\{ -Ah_t^B + V^B(m_t^B, x_t^B, b_{t+1}^B) \right\} \tag{2}$$

subject to

$$P_t x_t^B = P_t S(1-\tau) h_t^B + \tilde{m}_t - m_t^B + R_{t-1}\tilde{b}_t - b_{t+1}^B \tag{3}$$

$$m_t^B \geq 0 \tag{4}$$

where he chooses purchases of the FM good, his labor supply and his money and bond holdings. He experiences a disutility Ah_t^B where $A > 0$ is a parameter. He then continues to the next IM with his purchases and his money holdings. The first order conditions of this problem are given by

$$\frac{A}{S(1-\tau)} = V_x^B(m_t^B, x_t^B, b_{t+1}^B) \tag{5}$$

$$-\chi_t + V_m^B(m_t^B, b_{t+1}^B) \leq 0, \ = 0 \text{ if } m_t^B > 0 \tag{6}$$

$$\chi_t = V_b^B(m_t^B, x_t^B, b_{t+1}^B) \tag{7}$$

[13] The assumption that the utility function in the FM features some linearity, in our case in the disutility of labor, is key for tractability of our model. This issue is discussed in detail in Lagos and Wright (2005).

where χ_t denotes the shadow value of money or simply the multiplier on (3) and is given by

$$\chi_t = \frac{A}{P_t (1-\tau) S} \tag{8}$$

We also observe that the value function is linear in its arguments with slopes given by

$$W_m^B(\tilde{m}_t, \tilde{b}_t) = \chi_t \tag{9}$$

$$W_b^B(\tilde{m}_t, \tilde{b}_t) = \chi_t R_{t-1} \tag{10}$$

As Lagos and Wright (2005) argue in detail, (6) show that the money demand of buyers does not depend on their money holdings as they entered the present FM, $\tilde{m}_t$, and if $V_m^B(.)$ is strictly monotonic, then it can be uniquely determined. This is simply a result of the linearity of the disutility of labor.

The problem of a seller who enters the FM with $\tilde{m}_t$ units of money is

$$W^S(\tilde{m}_t, \tilde{b}_t) = \max \left\{ \begin{array}{l} \max\limits_{x_t^P, h_t^P, m_t^P, b_{t+1}^P} u(0, x_t^P) - A h_t^P + V^S(m_t^P, b_{t+1}^P) - \kappa, \\ \max\limits_{x_t^N, h_t^N, m_t^N, b_{t+1}^N} u(0, x_t^N) - A h_t^N + \beta W^S(m_t^N, b_{t+1}^N) \end{array} \right\} \tag{11}$$

where they choose between participating in the following IM and continuing to the FM next period and both problems are subject to

$$P_t x_t^i = P_t S (1-\tau) h_t^i + \tilde{m}_t - m_t^i + R_{t-1} \tilde{b}_t - b_{t+1}^i \tag{12}$$

$$m_t^i \geq 0 \tag{13}$$

for $i = P, N$. As discussed above κ is our measure of institutions in this model and represents the expected punishment for evading taxes. Note that the value function for the sellers is also linear in its arguments with the slopes given by (9) and (10).

The first order conditions for a seller who chooses to participate are

$$u_x\left(0, x_t^P\right) = \frac{A}{S(1-\tau)} \tag{14}$$

$$-\chi_t + V_m^P(m_t^P) \leq 0, \ = 0 \text{ if } m_t^P > 0 \tag{15}$$

$$\chi_t = V_b^B(m_t^P, b_{t+1}^P) \tag{16}$$

where χ_t is as defined in (8) and a non-participant seller's first order conditions are

$$\begin{aligned} u_x\left(0, x_t^N\right) &= \frac{A}{S\left(1-\tau\right)} && (17) \\ -\chi_t + \beta\chi_{t+1} &\leq 0, \ = 0 \text{ if } m_t^N > 0 && (18) \\ \chi_t &= \beta R_t \chi_{t+1} && (19) \end{aligned}$$

where we used (9) and (10).

We assume that there is free entry to the IM by sellers (after paying the cost κ) and this implies the free-entry condition

$$u(0, x_t^P) - Ah_t^P + V^S(m_t^P, b_{t+1}^P) - \kappa = u(0, x_t^N) - Ah_t^N + \beta W^S(m_t^N, b_{t+1}^N) \tag{20}$$

We need to obtain expressions for the IM value functions and envelope conditions to characterize the optimal choices for households, which we turn to next.

3.2 Informal Market

The value function for a buyer entering the IM is given by

$$\begin{aligned} V^B(m_t^B, x_t^B, b_{t+1}^B) &= \alpha_b\left[u(q_t^B, x_t^B) + \beta W^B(m_t^B - d_t^B, b_{t+1}^B)\right] \\ &\quad + (1-\alpha_b)\left[u\left(0, x_t^B\right) + \beta W^B(m_t^B, b_{t+1}^B)\right] \\ &= \alpha_b\left[u(q_t^B, x_t^B) - u\left(0, x_t^B\right) - \beta d_t^B \chi_{t+1}\right] \\ &\quad + u\left(0, x_t^B\right) + \beta W^B(m_t^B, b_{t+1}^B) \end{aligned} \tag{21}$$

where $\left(q_t^B, d_t^B\right)$ denotes the terms of trade the buyer faces and we used the linearity of the FM value function from (9). The first term shows that in the event the buyer is able to match with a seller, he purchases q_t^B units of the IM good, enjoys the utility of consuming this good together with the goods he bought in the FM and exits the market with d_t^B less money. The second term shows that if he cannot meet a seller he simply consumes his FM goods and proceeds to the next FM.

Similarly, the value function for a participating seller entering the IM is

$$\begin{aligned} V^S(m_t^P, b_{t+1}^P) &= \alpha_s\left[-c(q_t^S, S) + \beta W^S\left(m_t^S + d_t^S, b_{t+1}^P\right)\right] + (1-\alpha_s)\beta W^S\left(m_t^S, b_{t+1}^P\right) \\ &= \alpha_s\left[-c(q_t^S, S) + \beta d_t^S \chi_{t+1}\right] + \beta W^s\left(m_t^S, b_{t+1}^P\right) \end{aligned} \tag{22}$$

where $\left(q_t^S, d_t^S\right)$ denote the terms of trade the seller faces and linearity of the FM value function simplifies the expression. The first term shows the payoff to the seller when he meets a buyer, in which case he incurs a utility cost but acquires more money to spend in the next FM and the second term shows that if he is not able to meet a buyer, he moves on to the next FM.

The terms of trade in the IM are determined via proportional bargaining where the buyer receives θ of the surplus and the seller receives $1-\theta$ of it. This bargaining protocol has a number of virtues over, say generalized Nash bargaining which are described in detail in Aruoba, Rocheteau and Waller (2007).[14] The outcome of the bargaining will be $d = m_t^B$, that is the buyer spends all his money and that q_t solves

$$\frac{u(q_t, x_t^B) - u\left(0, x_t^B\right) - \beta m_t^B \chi_{t+1}}{-c(q_t, S) + \beta m_t^B \chi_{t+1}} = \frac{\theta}{1-\theta} \tag{23}$$

where the numerator on the left hand side is the surplus of the buyer as shown in (21) and the denominator is the surplus of the seller from (22). This expression simplifies to

$$\beta m_t^B \chi_{t+1} = g\left(q, x_t^B, S\right) \tag{24}$$

where $g(.)$ is a combination of some primitive utility functions

$$g\left(q_t, x_t^B, S\right) \equiv \theta c(q_t, S) + (1-\theta)\left[u(q_t, x_t^B) - u\left(0, x_t^B\right)\right] \tag{25}$$

With the IM problem laid out, we are now ready to derive the relevant envelope conditions. For the buyers we get

$$V_x^B(m_t^B, x_t^b, b_{t+1}^B) = \alpha_b u_x(q_t^B, x_t^B) + (1-\alpha_b)\, u_x\left(0, x_t^B\right) \tag{26}$$

$$V_m^B(m_t^B, x_t^b, b_{t+1}^B) = \alpha_b \left[\beta\chi_{t+1} \frac{u_q(q_t^B, x_t^B)}{g_q\left(q_t, x_t^B, S\right)} - \beta\chi_{t+1}\right] + \beta\chi_{t+1} \tag{27}$$

$$V_b^B(m_t^B, x_t^b, b_{t+1}^B) = \beta W_b^B(m_t^B, b_{t+1}) = \beta\chi_{t+1} R_t \tag{28}$$

[14] The key advantage of using proportional bargaining over Nash bargaining is that the former has strong monotonicity as one of its properties, which means the payoff of the buyer strictly increases as he brings more money in to the FM. In our Ramsey problem, as Aruoba and Chugh (2007) show in a related problem, with θ sufficiently away from unity, optimal policy becomes the Friedman rule since the Ramsey planner tries to fix the inefficiency caused by the non-monotonicity of the Nash solution. In contrast, with proportional bargaining, the Friedman rule is never optimal for any θ. Given that our quantitative exercises feature positive interest rates, using proportional bargaining is a better alternative.

where in the last expression we used (10). For participating sellers we get

$$V_m^S(m_t^P, b_{t+1}^P) = \beta\chi_{t+1} \quad (29)$$

$$V_b^S(m_t^P, b_{t+1}^P) = \beta W_b^S(m_t^P, b_{t+1}^P) = \beta\chi_{t+1}R_t \quad (30)$$

3.3 Household Optimality

Putting together everything we obtained so far, we can summarize our results with the following proposition.

Proposition 1 *Optimality for the households entails the following:*

- *Participating or nonparticipating sellers will choose not to hold any money and buyers will hold money, i.e.* $m_t^B > m_t^P = m_t^N = 0$. *We denote the money holdings of buyers with* m_t.
- *All households will hold the same quantity of bonds, which we denote by* b_t.
- *Participating and non-participating sellers choose the same level of consumption in the FM which we denote by* x_t^S.
- *Given the heterogeneity in the experiences of households in the previous IM, there will be 4 types of households in a given FM: matched/unmatched buyers and matched/unmatched sellers. These households will have different levels of money holdings as they enter the FM and this will be reflected in their labor supply.*
- *Free-entry condition is given by*

$$\alpha_s\left[-c(q_t) + g\left(q_t, x_t^B, S\right)\right] = \kappa$$

where q_t *follows from the bargaining problem.*

Proof. See Appendix A.1. ■

3.4 Equilibrium

Combining everything obtained so far, using M_t and B_t to denote aggregate money and bond holdings and defining $\pi_{t+1} \equiv p_{t+1}/p_t$ and $\mathcal{M}_t \equiv M_t/p_t$ and $\mathcal{B}_t \equiv B_{t+1}/p_t$, we can define a monetary equilibrium.

Proposition 2 *Given $R_t \geq 1$, a monetary equilibrium is a list of sequences $\left\{x_t^B, x_t^S, H_t, \mathcal{B}_t, \mathcal{M}_t, n_t, q_t, \pi_t\right\}$ that satisfy*

$$u_x(0, x_t^S) = \frac{A}{S(1-\tau)} \tag{31}$$

$$\mu(n_t)\, u_x(q_t, x_t^B) + [1 - \mu(n_t)]\, u_x\left(0, x_t^B\right) = \frac{A}{S(1-\tau)} \tag{32}$$

$$1 = \frac{\beta R_t}{\pi_{t+1}} \tag{33}$$

$$1 = \frac{\beta}{\pi_{t+1}} \left\{ \mu(n_t) \left[\frac{u_q(q_t, x_t^B)}{g_q\left(q_t, x_t^B, S\right)} - 1 \right] + 1 \right\} \tag{34}$$

$$\frac{A\beta \mathcal{M}_t}{S(1-\tau)\pi_{t+1}} = g\left(q_t, x_t^B, S\right) \tag{35}$$

$$\mu(n_t)\left[-c(q_t) + g\left(q_t, x_t^B, S\right)\right] = \kappa n_t \tag{36}$$

$$\mathcal{M}_t + \mathcal{B}_t + \tau S H_t = \frac{\mathcal{M}_{t-1} + R_{t-1}\mathcal{B}_{t-1}}{\pi_t} + \mathcal{G} \tag{37}$$

$$S H_t = x_t^B + \Lambda x_t^S + \mathcal{G} \tag{38}$$

Proof. See the Appendix. ■

3.5 Ramsey Problem

Having defined the equilibrium, which takes the policies of the government (R, τ) as given, we now turn to endogenizing these policies. We do so using the basic idea in Ramsey's (1927) original work, as further developed by Lucas and Stokey (1983) and Chari, Christiano and Kehoe (1991). The treatment of this problem is similar to that in Aruoba and Chugh (2007).

We consider the problem of a benevolent planner, the Ramsey planner, who seeks to pick the least distorting policies (R, τ) in order to finance the given government expenditures $\mathcal{G}$. We assume that the Ramsey planner is able to commit to these policies. Mechanically, the Ramsey problem then is to find policies that maximize social welfare in the resulting equilibrium. The proposition below summarizes the Ramsey problem which is stated in Lucas and Stokey's (1983) primal form.

Proposition 3 *The Ramsey planner's problem is to choose allocations $\left\{x_t^B, x_t^S, q_t, n_t, H_t\right\}$*

given by the objective function

$$\sum_{t=0}^{\infty}\beta^{t}\left\{\mu\left(n_{t}\right)\left[u\left(q_{t},x_{t}^{B}\right)-c\left(q_{t},S\right)\right]-n_{t}\kappa+\left[1-\mu\left(n_{t}\right)\right]u\left(0,x_{t}^{B}\right)+\Lambda u\left(0,x_{t}^{S}\right)-AH_{t}\right\} \tag{39}$$

maximized subject to the Present-Value Implemetability Constraint (PVIC)

$$\sum_{t=0}^{\infty}\beta^{t}\left\{u_{x}(0,x_{t}^{S})\left(x_{t}^{B}+\Lambda x_{t}^{S}\right)-AH_{t}+\mu\left(n_{t}\right)g\left(q_{t},x_{t}^{B},S\right)\left[\frac{u_{q}(q_{t},x_{t}^{B})}{g_{q}\left(q_{t},x_{t}^{B},S\right)}-1\right]\right\}=\mathcal{A}_{0}\ \textit{(multiplier }\xi\textit{)} \tag{40}$$

where $\mathcal{A}_{0}\equiv u_{x}(0,x_{0}^{S})\left[R_{-1}\mathcal{B}_{-1}/\pi_{0}+\mathcal{M}_{-1}/\pi_{0}\right]$*, the resource constraint (RC)*

$$SH_{t}=x_{t}^{B}+\Lambda x_{t}^{S}+\mathcal{G}\ \textit{(multiplier }\nu\textit{)} \tag{41}$$

uniform-tax condition (UT)

$$\mu\left(n_{t}\right)u_{x}(q_{t},x_{t}^{B})+\left[1-\mu\left(n_{t}\right)\right]u_{x}\left(0,x_{t}^{B}\right)=u_{x}\left(0,x_{t}^{S}\right)\ \ \textit{(multiplier }\lambda\textit{)} \tag{42}$$

the free-entry condition (FE)

$$\mu\left(n_{t}\right)\left[-c(q_{t},S)+g\left(q_{t},x_{t}^{B},S\right)\right]=\kappa n_{t}\ \textit{(multiplier }\eta\textit{)} \tag{43}$$

and the zero-lower-bound condition (ZLB) to ensure the existence of monetary equilibrium in the form of

$$\mu\left(n_{t}\right)\left[u_{q}(q_{t},x_{t}^{B})-g_{q}\left(q_{t},x_{t}^{B},S\right)\right]\geq0\ \textit{(multiplier }\iota\textit{)} \tag{44}$$

given $\mathcal{B}_{-1}$ *and* $\mathcal{M}_{-1}$*.*

This problem yields allocations $\left\{x_{t}^{B},x_{t}^{S},q_{t},n_{t},H_{t}\right\}$ *that are associated with the optimal policies which in turn can be obtained using*

$$\tau_{t} = 1-\frac{A}{Su_{x}(0,x_{t}^{S})} \tag{45}$$

$$R_{t} = \mu\left(n_{t}\right)\left[\frac{u_{q}(q_{t},x_{t}^{B})}{g_{q}\left(q_{t},x_{t}^{B},S\right)}-1\right]+1 \tag{46}$$

Proof. See Appendix A.3 ■

The PVIC is a compact way of summarizing the equilibrium conditions that the Ramsey planner is subject to. Typically, the PVIC and the RC fully summarize the said conditions

but in this problem we need three more conditions. First, as is standard in any monetary version of the problem, we need to make sure that the interest rate implied by the choices of the Ramsey planner is non-negative, which is necessary for the existence of monetary equilibrium. This leads to the ZLB constraint. Second, the choice of the Ramsey planner as to the number of participating buyers should be consistent with the free-entry condition, which is guaranteed by FE. Finally, the Ramsey planner is forced to impose a uniform income tax on the buyers and sellers, and due to the differences in marginal utility of consumption created by the non-separability of the utility function, this necessitates imposing UT.

Solving this optimization problem, imposing steady state and, to ease notation, using the following shorthands

$$u^M \equiv u\left(q, x^B\right), \; u^N \equiv u\left(0, x^B\right), \; u^S \equiv u\left(0, x^S\right) \tag{47}$$

and dropping all arguments of remaining functions we obtain the following result.

Proposition 4 *The solution to the Ramsey problem that characterizes the optimal allocations and policies* $\left(q, x^S, x^B, n, H, \xi, \iota, \eta, \lambda, \tau, R\right)$ *is given by*

$$\mu(n)\, u_x^M + [1-\mu(n)]\, u_x^N + \xi u_x^S - \frac{(1+\xi)\, A}{S} + \xi\mu(n)\left[g_x\left(\frac{u_q^M}{g_q} - 1\right) + \frac{g}{g_q^2}\left(u_{qx}^M g_q - u_q^M g_{qx}\right)\right] \tag{48}$$

$$+\lambda\left\{\mu(n)\, u_{xx}^M + [1-\mu(n)]\, u_{xx}^N\right\} + \eta\mu(n)\, g_x + \iota\mu(n)\left(u_{qx}^M - g_{qx}\right) = 0$$

$$(1+\xi)\left(u_x^S - \frac{A}{S}\right) + u_{xx}^S\left(\xi x^S - \frac{\lambda}{\Lambda}\right) = 0 \tag{49}$$

$$(1+\xi)\, u_q^M - (1+\eta)\, c_q + (\eta-\xi)\, g_q + \xi\frac{g}{g_q^2}\left(u_{qq}^M g_q - u_q^M g_{qq}\right) + \lambda u_{xq}^M + \iota\left(u_{qq}^M - g_{qq}\right) = 0 \tag{50}$$

$$u^M - u^N - (1+\eta)\, c + g\left[\xi\left(\frac{u_q^M}{g_q} - 1\right) + \eta\right] + \lambda\left(u_x^M - u_x^N\right) + \iota\left(u_q^M - g_q\right) - \frac{\kappa(1+\eta)}{\mu'(n)} = 0 \tag{51}$$

$$u_x^S\left(x^B+\Lambda x^S\right)-AH+\mu\left(n\right)g\left(\frac{u_q^M}{g_q}-1\right)=\frac{\left(1-\beta\right)}{\beta}\left(u_x^S\mathcal{B}_{-1}+g\right) \tag{52}$$

$$SH=x^B+\Lambda x^S+\mathcal{G} \tag{53}$$

$$\mu\left(n\right)u_x^M+\left[1-\mu\left(n\right)\right]u_x^N=u_x^S \tag{54}$$

$$\mu\left(n\right)\left(g-c\right)-\kappa n=0 \tag{55}$$

$$\iota\mu\left(n\right)\left(u_q^M-g_q\right)=0 \tag{56}$$

along with (45) and (46), given $\mathcal{B}_{-1}$.

Proof. See Appendix A.4. ■

4 Results

We have two major goals in this section. First, we show that when government policies (inflation and taxes) are exogenous, as some earlier studies reviewed in the Introduction assume, we cannot explain the five facts discussed in Section 2. We do this numerically, calibrating the model to the U.S. economy, since the model does not yield unambiguous analytical results and we impose some discipline using appropriate calibration targets. Second, we demonstrate that the full model, where both policies and the size of the informal sector are endogenous, can match the said facts qualitatively and quantitatively. In order to do so, we once again calibrate the model the U.S. economy and vary the three exogenous variables, institutions, labor productivity and government expenditures as a fraction of GDP, across countries.

One of the key objects we compute is the size of the informal sector relative to the formal sector. This measure, which we denote by $\mathcal{R}$, is computed the same way the measure in Schenider (2004) is computed but, naturally, our measure embodies the structure of our model. It is defined as

$$\mathcal{R}\equiv\frac{\mu\left(n\right)\mathcal{M}}{Y} \tag{57}$$

where $\mu\left(n\right)$ is the measure of matches in the IM, $\mathcal{M}$ is the real quantity of money spent in each of these trades and Y is the output in the FM. We also define total output of a country $\mathcal{Y}$, to be the sum of the outputs in the IM and the FM expressed in FM prices, which is

given by

$$\mathcal{Y} \equiv \mu(n)\,\mathcal{M} + Y \tag{58}$$

We assume total government spending, which we denoted by $\mathcal{G}$ above, is a fixed fraction of total FM output

$$\mathcal{G} = GY \tag{59}$$

where G is a country-specific parameter.

4.1 Functional Forms and Fixed Parameters

The underlying assumption in our quantitative exercises is that every country in the world is populated by people with identical preferences. Assuming otherwise would imply that at least a part of the cross-country differences in informal activity and policies are due to different preferences. We pick the United States as our benchmark country for calibration.

We assume that buyers and sellers in the IM are matched via the urn-ball matching function which describes the number of matches as

$$\mu(n) = n\left[1 - \exp\left(-\frac{1}{n}\right)\right] \tag{60}$$

and α_b and α_s can be easily computed. This matching function, which is used in labor-search models, has microfoundations where each buyer "applies" to a seller with equal probability and the probability of a given seller not finding a match is $\exp(-1/n)$ and has the standard constant returns to scale property.[15]

We assume that households have constant-relative-risk-aversion utility over a composite good Q

$$U(q,x) = \begin{cases} \dfrac{Q(q,x)^{1-\sigma}}{1-\sigma} & \text{if } \sigma \neq 1 \\ \log\left[Q(q,x)\right] & \text{if } \sigma = 1 \end{cases} \tag{61}$$

where Q is given by the constant-elasticity-of-substitution function

$$Q(q,x) = \begin{cases} \left\{\gamma\left[(q+b)^{\varepsilon} - b^{\varepsilon}\right] + x^{\varepsilon}\right\}^{1/\varepsilon} & \text{if } \varepsilon \neq 0 \\ \left(\dfrac{q+b}{b}\right)^{\gamma} x & \text{if } \varepsilon = 0 \end{cases} \tag{62}$$

In this specification $b > 0$ is a small number to make sure $U(0,x)$ is well-defined, γ determines

[15] Rogerson, Shimer and Wright (2005), page 974 provides more details.

the relative weights of IM and FM goods and ε determines the elasticity of substitution.[16] Note that in order to preserve Edgeworth-substitutes property of the utility function, we need $\varepsilon > 1 - \sigma$. [17] Also note that with $\varepsilon = 0$ and $\sigma = 1$ we get $U(q, X) = \gamma \log(q + b) - \gamma \log b + \log(X)$ which is as closely we can nest the original setup in Rocheteau and Wright (2005).

At this point we fix some parameters based on U.S. data for the period 1998-2004. We set $\beta = 0.956$ based on the real return for Aaa-rated corporate bonds in the U.S. We set G to 21% of GDP. We fix $\theta = 0.5$ (egalitarian bargaining where the surplus of the match in the IM is split equally between buyers and sellers), $\varepsilon = 0.5$, $\sigma = 1$ and $b = 0.0001$. We use $\Lambda = 4$, which is large enough that in all our experiments $n < \Lambda$ is satisfied. After fixing these parameters, two parameters remain to be calibrated: γ and A. We describe in turn our two calibrations and Table 3 reports summaries of the results.

4.2 Exogenous Policies

In this section, we demonstrate the properties of our model when policies are exogenous. We use the mapping summarized in (31)-(38) where the exogenous variables $(\tau, \pi, \kappa, S, G)$ are mapped in to the endogenous variables $\left(q, n, x^S, x^B, H, \mathcal{B}, \mathcal{M}, R, \mathcal{R}\right)$. From the outset, it should be clear that this version of the model will be silent about Facts 1 and 2 since both government policies and institutions are exogenous. Therefore we focus on Facts 3, 4 and 5.

We calibrate the model to observations from the U.S., where, in addition to those explained above, we set $\pi = 2.35\%$ and $\tau = 0.27$. We fix $\kappa = 0.3$ whose level doesn't affect the results, and calibrate the remaining exogenous variable S and parameters γ and A to match the following targets

$$Y = 1, \ H = 0.3 \tag{63}$$

$$\mathcal{R} = 0.086 \tag{64}$$

where (63) show normalizations that help pin down A and S and (64) uses Scheneider's (2004) measure for the size of the informal sector in the US to pin down γ.[18]

[16] When $b = 0$, the threat points of the buyers may become undefined when, for example, $\varepsilon = 0$. For the relevant part of the domain, when $q > 0$, this utility function is virtually identical to the standard one where $b = 0$.

[17] To see this, note that $u_q = \gamma Q^{1-\varepsilon-\sigma} q^{\varepsilon-1}$ and $u_{qx} = (1 - \varepsilon - \sigma)\gamma q^{\varepsilon-1} Q^{1-2\varepsilon-\sigma} x^{\varepsilon-1}$ which is negative if $1 - \varepsilon - \sigma < 0$.

[18] The exact number we use for $\mathcal{R}$ is not central for our results. Many qualitative and quantitative

Figure 7 shows how the size of the IM changes when we change each of the exogenous variables. The red star in each of the panels represents the U.S. calibration. The first panel shows that an exogenous increase in κ will reduce informal activity, in line with Fact 3. Everything equal, if the expected punishment from an audit increases, the IM will be less attractive for the sellers and fewer of them choose to enter it, reducing the size of the market. The second panel shows that when inflation increases the size of the IM decreases, which contradicts Fact 4. In this model, and arguably in many similar models with exogenous policies, an increase in inflation would increase the cost of holding money and since the IM is cash-intensive, buyers choose to bring less money to the IM. As a result, number of sellers who participate in the IM as well as the quantity traded (not shown) fall, making the IM smaller. Similarly, the last panel shows that when the income tax rate increases the size of the IM increases, contradicting Fact 5. The channel through which this happens is identical to the one for inflation, where higher income taxes make the IM more attractive for both the buyers and sellers.

To sum up, this version of the model where policies are exogenous is able to explain Fact 3 but it is silent about Facts 1 and 2 and it is fundamentally at odds with Facts 4 and 5.

4.3 Endogenous Policies and Informal Activity

Having shown that considering government policies as exogenous cannot fully explain the facts we laid out in Section 2, we now turn to calibrating and testing the full model. We use the mapping summarized in (48)-(56) where the exogenous variables (κ, S, G) are mapped in to the endogenous variables $\left(q, n, x^S, x^B, H, \mathcal{M}, R, \mathcal{R}, \tau, \pi, \text{multipliers}\right)$.

Our first step is calibrating this model to the U.S. with the following targets

$$Y = 1, \ H = 0.3 \tag{65}$$

$$\mathcal{R} = 0.086 \tag{66}$$

$$\pi = 2.35\% \tag{67}$$

where (65) once again are normalizations that pin down the level of labor productivity S^{US} and the parameter A. (66) imposes the size of the informal sector as Schneider (2004) measures for the U.S. and (67) uses the measured inflation rate for the U.S. in our sample. The latter two targets help determine the utility parameter γ and the level of institutions

measures agree that the informal sector in the US is small.

for the U.S., κ^{US}.[19]

In order to first qualitatively show that this model is able to capture the facts, we trace the effects of changing κ, holding everything else constant, in essence taking a partial derivative. This is important because our main hypothesis is that the facts we summarized in Section 2 are caused by changes in institutions. The results are reported in Figure 8. As the rule of law increases, the sellers find it less profitable to enter the IM and the measure of participating sellers (not shown) declines. Since expected consumption in the IM for a buyer falls, his consumption in the FM increases, increasing measured output (not shown). As buyers are moving away from IM consumption, the government reduces inflation since there is no need to "tax" money holdings as much as before and in order to make up for the lost revenue, it increases FM income taxes. The three panels in this figure show that this model is able to account for Facts 1, 2 and 3 by exogenous variations in institutions since inflation goes down, taxes go up and informal activity goes down as institutions improve. Moreover, since inflation and informal activity react in the same direction to a change in institutions, and taxes and informal activity react in opposite directions, Facts 4 and 5 can also be qualitatively accounted for.

Next we conduct a cross-country calibration exercise where we vary institutions and other exogenous variables across countries and investigate whether (a) our model can generate similar quantitative outcomes for government policies and size of the informal sector to those observed in the data and (b) our model can deliver similar correlations when we focus on the five facts summarized in Section 2. In order to parameterize the model for each country, we fix all deep parameters, including γ and A, at their calibrated values for the U.S. We need to set the values for three exogenous variables for each country: rule of law (κ^i), share of government expenditures in output (G^i) and labor productivity (S^i) where the i superscript denotes a country-specific value.

Our measure of output per worker relative to the U.S. (RLP^i) from Penn World Tables provide a way to calibrate S^i using

$$S^i = S^{US} \times RLP^i \tag{68}$$

We have direct measures of G^i from our dataset. Finally, since the Rule of Law measure we use from Governance Matters is in an arbitrary scale, we use a simple linear transformation

[19] Note that the income tax rate τ is not targeted in this calibration and comes out to be 22.2%. This number is slightly smaller than the one in our data, 27%, but sufficiently close not to be of much concern.

that maps this measure in to the interval $[0.06, 0.46]$ to create κ^i. While this transformation is arbitrary, it ensures that κ is positive and the value calibrated for the US, 0.38, is comfortably in this range.

We solve for the Ramsey equilibrium using the common parameters and calibrated exogenous variables and out of the 127 countries in our sample we are able to find solutions for 76 of them. For the remaining 51 countries, our numerical procedure converges to the nonmonetary equilibrium where the IM is shut down. In other words, we cannot find a monetary equilibrium, one where a nontrivial measure of sellers participate and buyers hold money. While there may be several reasons for this that vary from country to country, the most likely explanation is that for these countries the level of government expenditures is above the maximum level that can be raised as measured from the Laffer curve. As the government is trying to increase taxes to raise the necessary revenue, it is causing people to flee to the IM. In response, the government increases inflation to such a high level that shuts down the IM. We will turn to the characteristics of these 51 countries and what that means for our exercise below.

Figure 9 plots the three key variables obtained from the model versus their data counterparts: inflation, income tax rate and the size of the informal sector. Each panel is set up such that clusters below (above) the 45 degree line indicate that our model produces smaller (larger) numbers than those in the data. Focusing first on the correlations, we see that with the possible exception of inflation, which shows a correlation of 0.42, all three of the variables display similar cross-country variation in the model and in the data, given the limited exogenous variation we had in our quantitative exercise. Even though the level of the income tax rates was not targeted we see that our model delivers remarkably similar numbers to those in the data, albeit slightly smaller. Our measure of informal activity is also strongly correlated with all of the three alternatives in the data and with a correlation coefficient of around 0.8 (not reported), it is even more strongly correlated with the World Economic Forum measure of informal activity. Overall, we conclude that our quantitative exercise is successful in capturing the essence of cross-country differences in these three variables.

Comparing levels, however, we see that our model generates on average too much inflation, to little income taxes and too small informal sectors, relative to the data. This is the direct result of inflation having no direct impact on FM activity, which followed from the assumption that transactions in the FM did not require a medium of exchange. In reality for many countries this is not a good approximation. Formal sector purchases also use money

to some (varying) extent in many countries. This is especially important in countries where many consumers are hand-to-mouth consumers with no ability to save and most transactions are cash-based. Taking this additional use for money creates a direct channel for inflation to affect formal sector outcomes and, therefore, overall welfare. In our model, then, the Ramsey planner does not realize the cost of inflation on welfare through the FM and as a result chooses a level of inflation that is higher than in the data. Since inflation is too high (relative to data), informal activity is discouraged too much in the model. This also means the income tax rate is too low since sufficient revenue is raised through seigniorage. We can fix this problem by, for example, splitting the FM into two parts: one where money is essential, just as it is in the IM, and one where it is not. We believe doing so, and using a measure of velocity of money in our calibrations for each country, would bring the levels of these three key variables closer to those in the data without affecting the cross-country variations and correlations.

The qualitative test of our hypothesis that cross-country differences in institutions can explain differences in policies and extent of informal activity observed in the data is provided in Table 4. The table reports the correlations that correspond to the three key facts we documented in Section 2 for three sets of countries where our model was able to deliver a solution: all countries (76 out of a possible 127), all the OECD countries in our sample (all 28) and all countries that are classified as "free" in 2003 by Freedom House[20] (52 out of a possible 58).[21] For each set of countries, we report the correlations from the data and those obtained from our model. The results show that for Fact 1, the model delivers a somewhat stronger relationship than what is in the data, possibly indicating that there are other reasons (other than the three exogenous variables we consider) why inflation rates differ across countries.[22] Nevertheless these results show that institutions are one of the key determinants of inflation for these countries. Turning to tax rates and size of the informal sector, the correlations from the model are quite similar to those from the data, especially for the latter. The match between the data and the model for all three facts are especially close for the OECD countries.

[20] These are countries whose ratings are 1.0, 1.5, 2.0 or 2.5. Details are provided in the appendix. Using the Index of Democracy produced by the Economist Intelligence Unit, our methodology is successful for all 24 countries classified as full democracies and the success rate for flawed democracies, hybrid regimes and authoritarian regimes are 28 of 44, 6 of 24 and 11 of 33, respectively.

[21] The correlations are re-computed for each set and therefore are not identical to those reported in Table 1.

[22] This is also evident in Table 2 where the R^2 of the simple regression in column (2) that explains inflation using the three exogenous variables is only 0.12.

Finally, we turn to the 51 countries for which our model was not able to deliver a solution and the consequences for our hypothesis. To provide further details about these 51 countries, Figure 10 shows the empirical distributions of four measures for all countries (blue bars) and for the 51 countries that we were "unsuccessful" in finding a solution (red bars): rule of law, output relative to the US, data quality as reported in Penn World Tables and the measure of Freedom by Freedom House. In each panel values to the right are better values. We see that these 51 countries are very poor (the richest country in this list has 17.9% of the output of the US), have poor institutions (the country with the best institutions in this list has only slightly better than the median of the full sample), have bad data quality[23] and are ranked lower on the Freedom scale. A flip side of this, which may be important to emphasize, for countries that have at least 18% of the output of the U.S., or those with institutions above the median, or those with data quality of A or B or those labeled as "Free" by Freedom House, our model delivers a solution and quantitatively is successful in matching the key facts.

We find this result a very useful test of our modeling strategy. As we emphasized, we assumed, among other things, that government policies are chosen by a benevolent planner that seeks to maximize social welfare. For countries towards the left of the scale in each of the four panels, it is not reasonable to expect this to be the case. For example, countries that are ranked low in the Freedom ranking generally have authoritarian regimes or otherwise flawed democracies. But for countries to the right this can be a reasonable approximation. We consider it a success that our methodology works for virtually *all* countries to the right of the scale in terms of these measures and does not work for *some* of the countries to the left. For the latter countries many other considerations, including the political-economy ones we summarized in the Introduction, may be much more important.

5 Conclusion

We present five key facts using a data set of 127 countries regarding informal activity, government policies and institutions. We argue that in order to explain all these facts, we need a model where both private actions and government policies are jointly (and optimally) determined, taking as given, among other things, the institutions of the country. Our numerical exercise shows that our model is indeed able to explain the cross-country variations

[23] Penn World Tables advises caution for using the data for countries that receive grades C and D.

in informal activity and policies.

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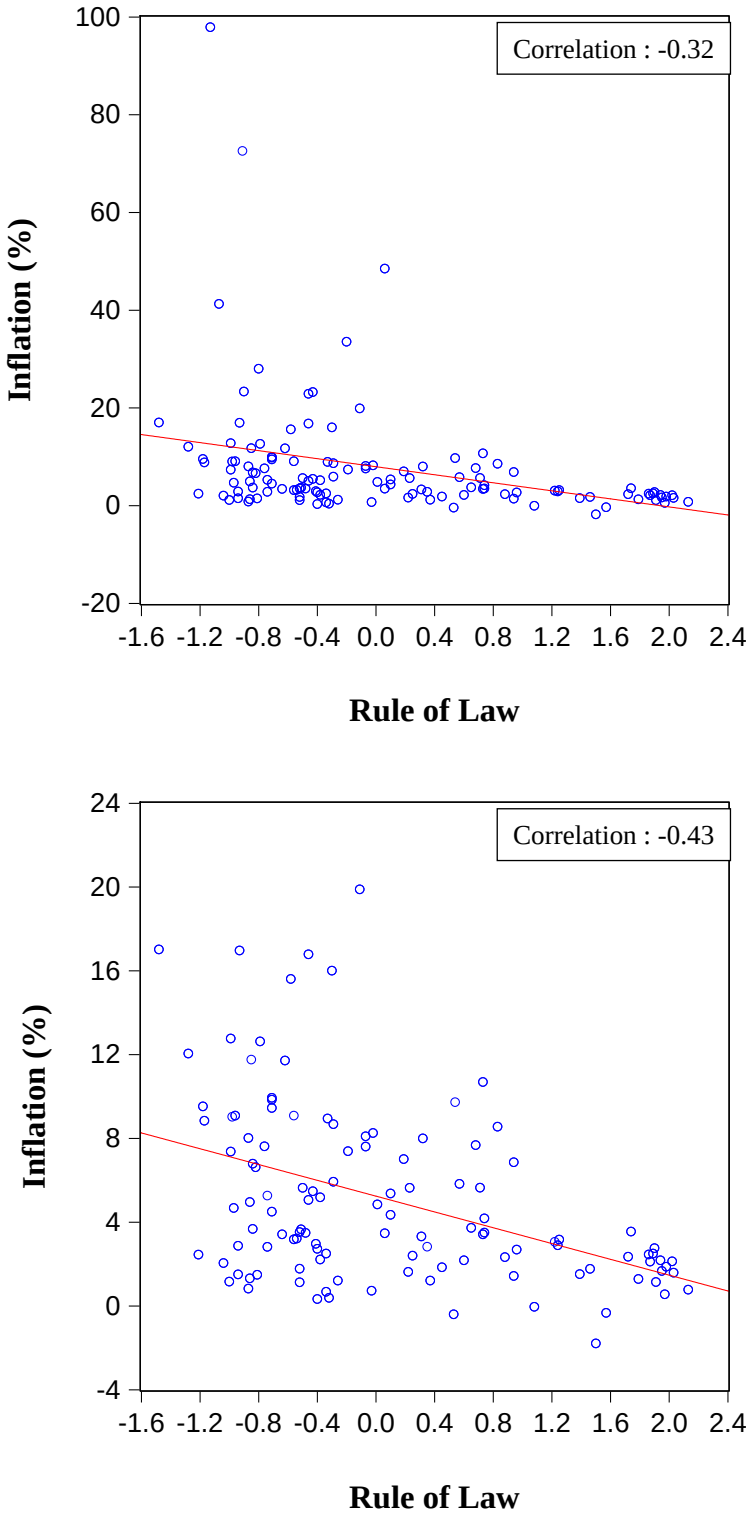
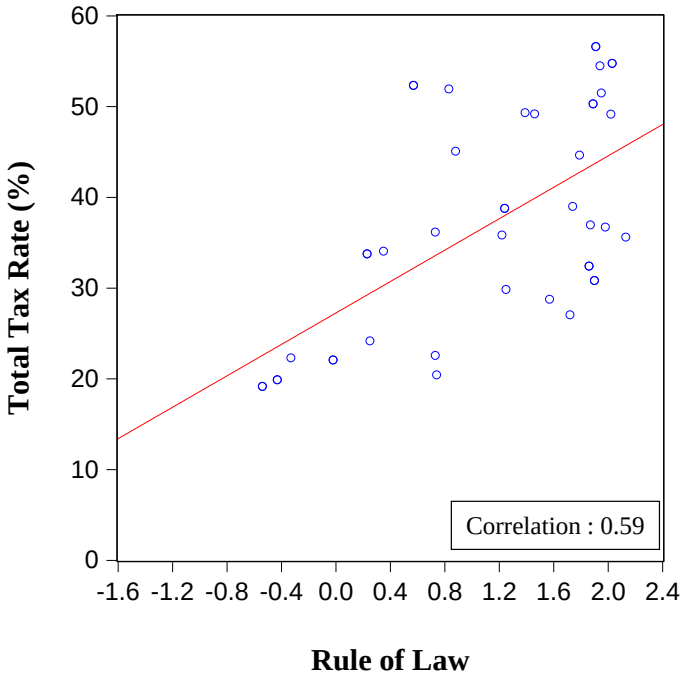
Figure 1: Fact 1 - Institutions and Inflation



Notes: The second panel restricts the sample to countries with less than 20% annual inflation.

Figure 2: Fact 2 - Institutions and Taxes



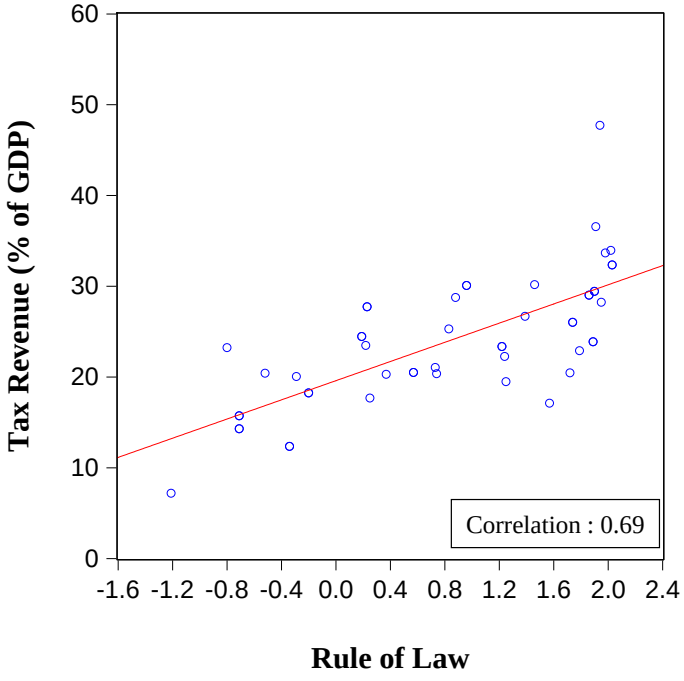


Figure 3: Fact 3 - Institutions and Size of Informal Sector

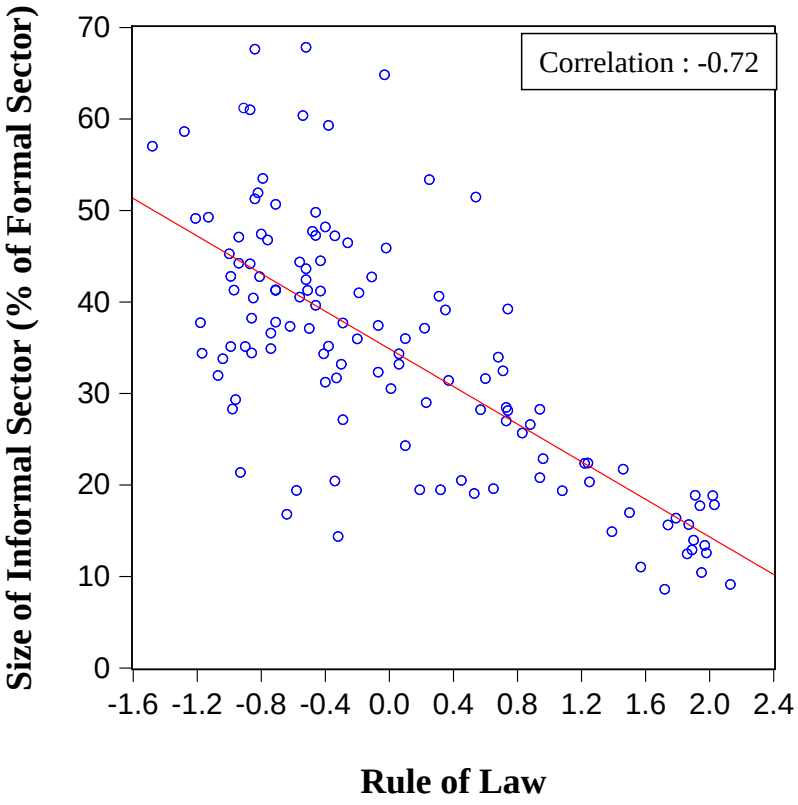
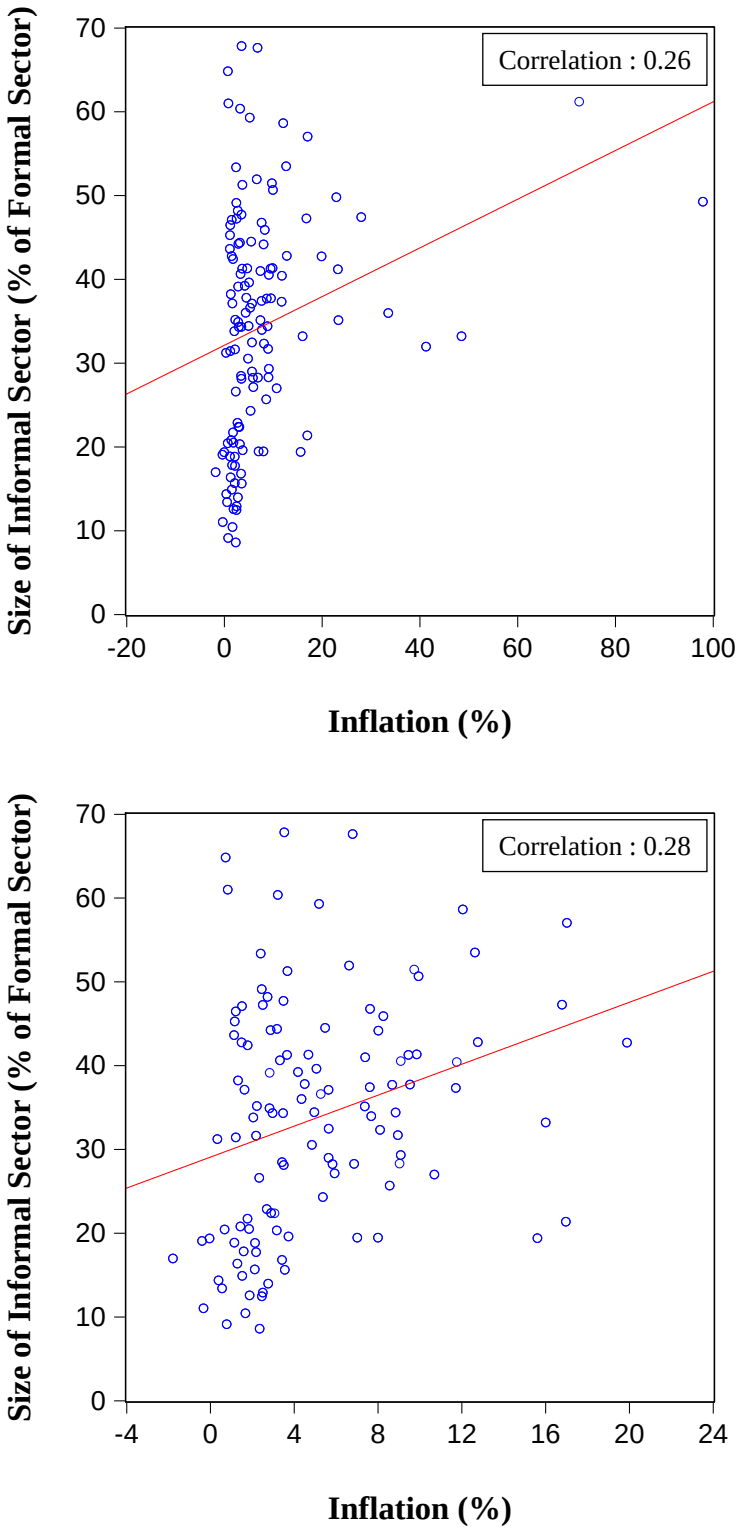


Figure 4: Fact 4 - Inflation and Size of Informal Sector



Notes: The second panel restricts the sample to countries with less than 20% annual inflation.

Figure 5: Fact 5 - Taxes and Size of Informal Sector

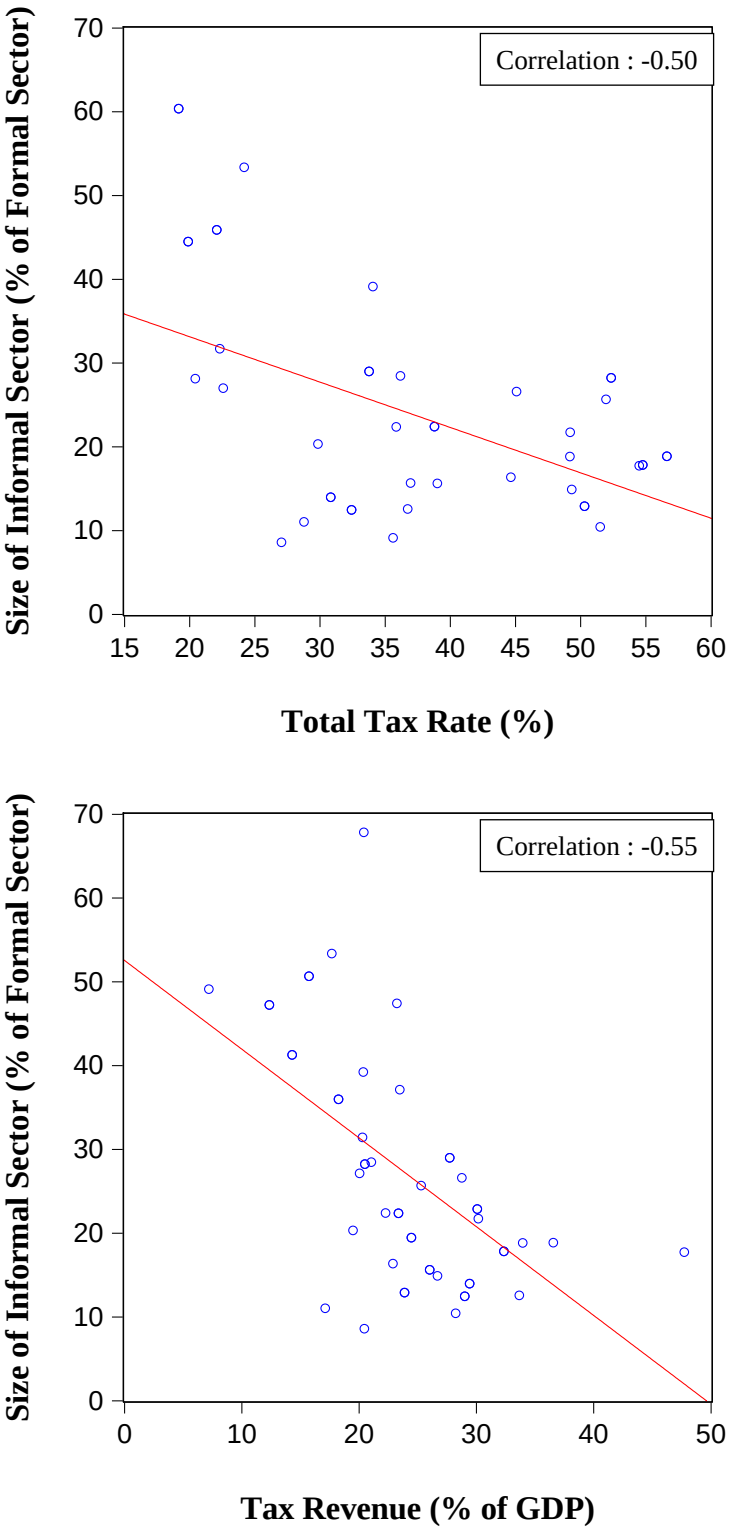


Figure 6: Inflation and Size of Informal Sector - 3 Selected Countries

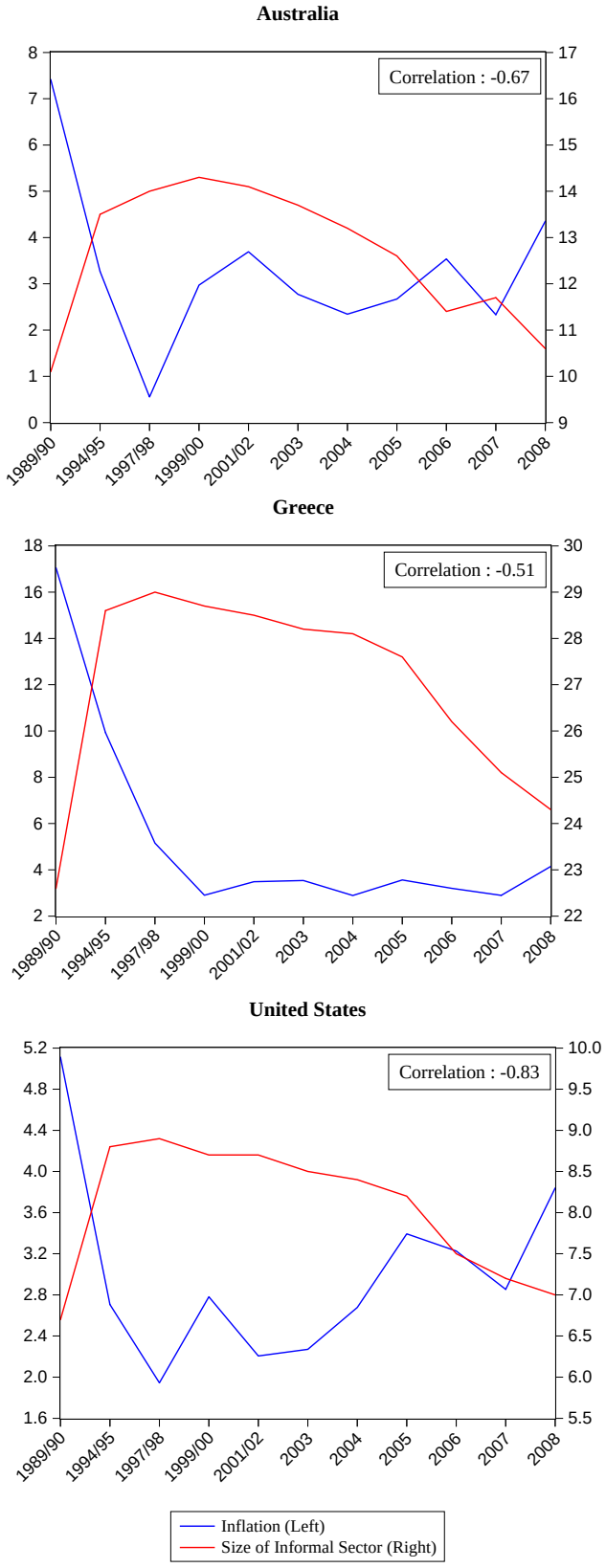
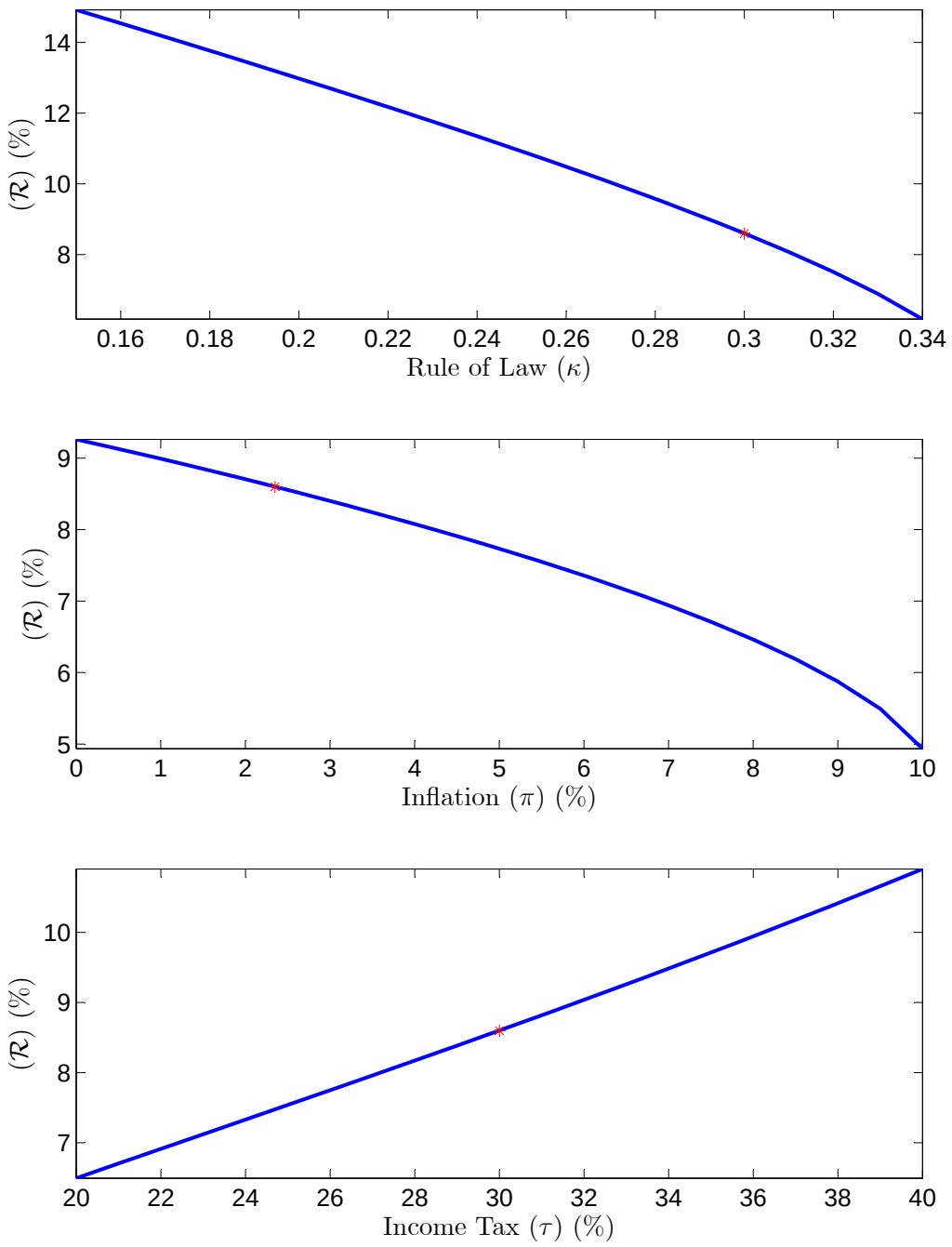
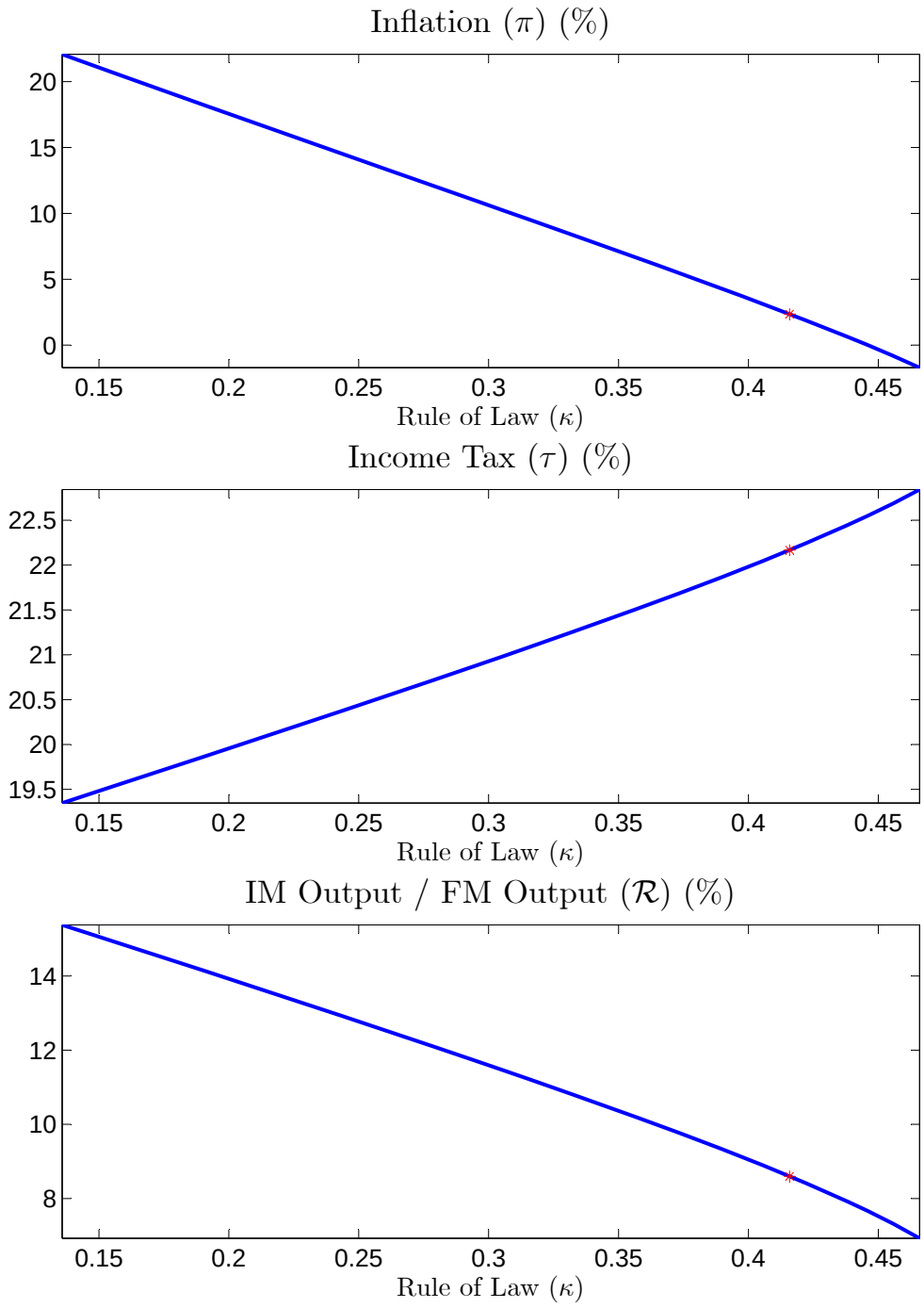


Figure 7: Exogenous Policies and Institutions - Size of Informal Sector



Notes: The red stars in each panel correspond to the calibrated value for the United States.

Figure 8: Exogenous Institutions, Endogenous Policies



Notes: The red stars in each panel correspond to the calibrated value for the United States.

Figure 9: Cross Country Exercise - Key Variables, Data vs. Model

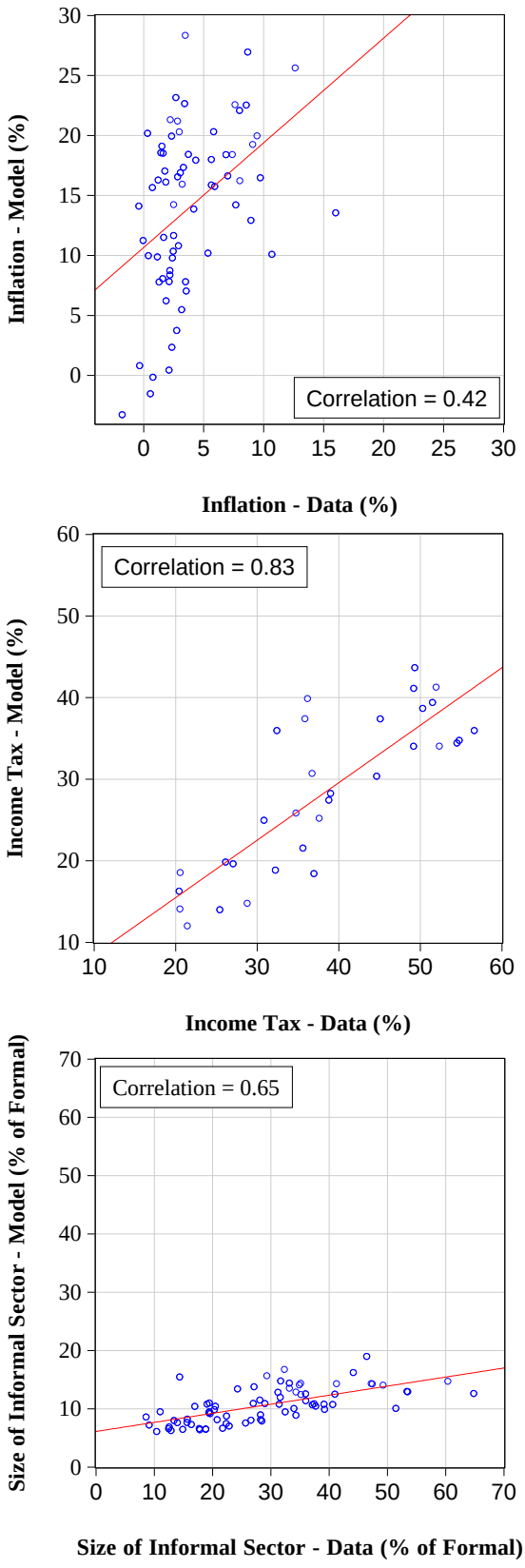
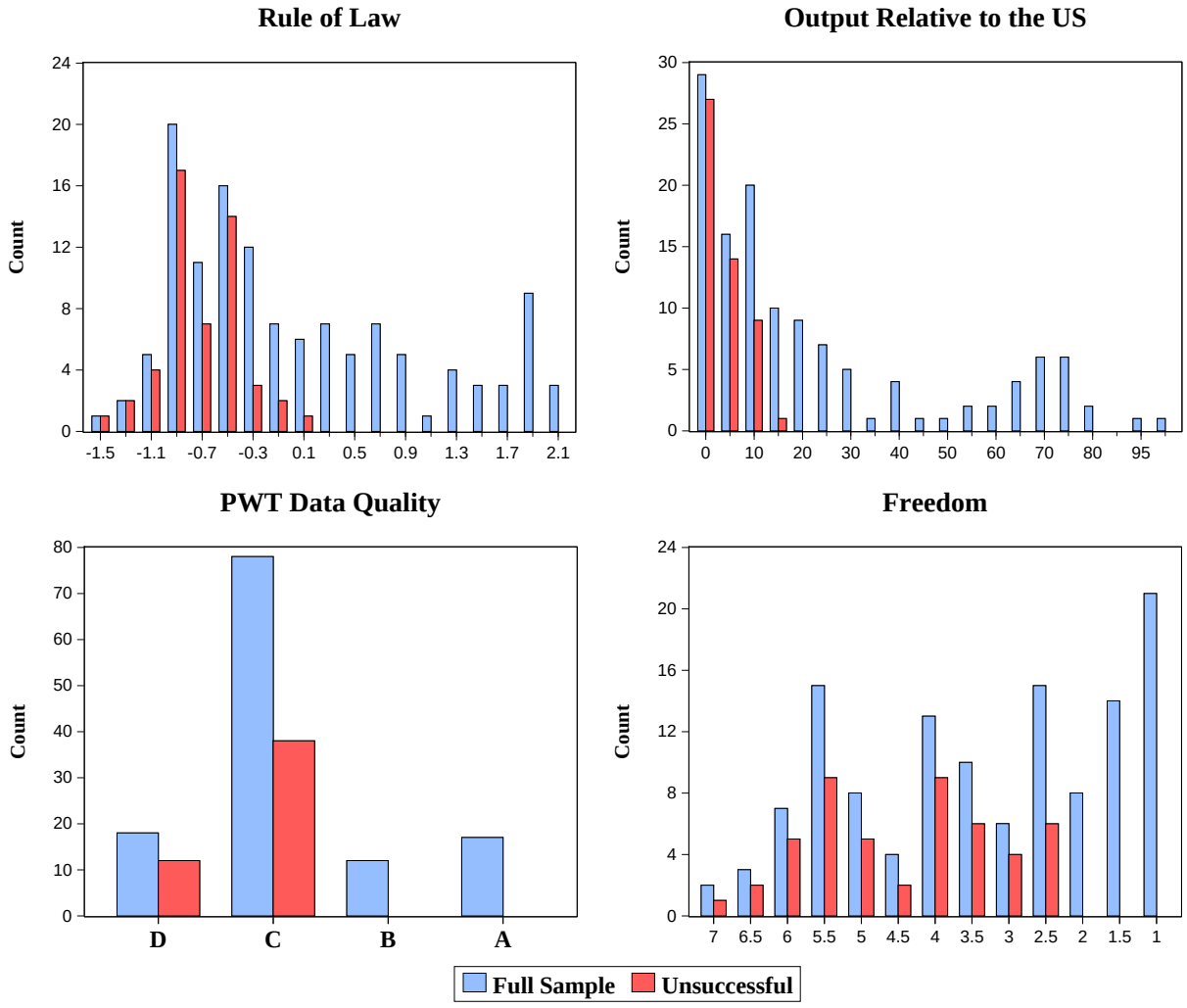


Figure 10: Cross Country Exercise - Characteristics of Countries



Notes: This figure shows the empirical distribution of four variables across all countries (blue bars) and across those countries that the cross-country exercise was unsuccessful. There are 19, 20, 4 and 13 equally-spaced bins each panel, respectively. Each variable is measured such that values to the right are better.

Table 1: Simple Correlations between Institutions, Government Policy and the Size of the Informal Sector

(a) Facts 1, 2 and 3

Correlations of ...				
	Rule of Law (GM)	Irregular Payments (WEF)	Property Rights (WEF)	Freedom from Corruption (HF)
... with Rule of Law	-	0.87	0.86	0.90
Fact 1				
... with Inflation	**-0.32**	-0.43	-0.39	-0.29
Fact 2				
... with Total Tax Rate	**0.59**	0.52	0.52	0.54
... with Tax Revenue	**0.69**	0.66	0.46	0.74
Fact 3				
... with Size (Schneider)	**-0.72**	-0.61	-0.66	-0.63
... with Size (WEF)	-0.83	-0.74	-0.69	-0.76
... with Unrep. Wages Profits	-0.72	-0.74	-0.67	-0.76
... with Tax Evasion	-0.83	-0.79	-0.77	-0.82

(b) Facts 4 and 5

Correlations of ...				
	Size (Schneider)	Size (WEF)	Unreported Wages / Profits	Tax Evasion
... with Size (Schneider)	-	0.77	0.63	0.64
Fact 4				
... with Inflation	**0.26**	0.46	0.30	0.43
Fact 5				
... with Total Tax Rate	**-0.50**	-0.53	-0.48	-0.22
... with Tax Revenue	**-0.55**	-0.56	-0.51	-0.35

Notes: The numbers in boldface correspond to the "headline" correlations used in Figures 1 through 5.

Table 2: Determinants of Government Policies and Size of Informal Sector

(a) Government Policies

	Dependent Variable : Inflation		**Dependent Variable : Tax**	
	(1)	(2)	(3)	(4)
Constant	11.54 (*)	3.98	5.37	7.97
Rule of Law	-	-7.42 (*)	-	6.13 (*)
Productivity	-0.09 (*)	0.12	0.10 (*)	-0.06
Government Exp.	-0.05	0.02	0.88 (*)	0.86 (*)
R^2	0.05	0.12	0.72	0.76
N	127	127	33	33

(b) Size of Informal Sector

Dependent Variable : Size of Informal Sector					
	(5)	(6)	(7)	(8)	(9)
Constant	32.14 (*)	34.53 (*)	43.97 (*)	38.91 (*)	38.47 (*)
Inflation	0.29 (*)	0.04	-	-	-
Taxes	-	-	-0.54 (*)	0.05	-
Rule of Law	-	-10.10 (*)	-	-14.54 (*)	-7.09 (*)
Productivity	-	-	-	-	-0.12
Government Exp.	-	-	-	-	-0.01
R^2	0.07	0.52	0.25	0.80	0.53
N	127	127	33	33	127

Table 3: Results from Calibration

	Exogenous Policies	**Endogenous Policies**
Calibrated Parameters		
A	13.90	15.98
γ	0.27	0.31
Calibrated / Fixed Exogenous Variables		
τ	27.05%	-
π	2.35%	-
κ	0.3 (*)	0.38
S	3.33	3.33
G	0.21	0.21
$\mathcal{B}$	-	0.41
Calibration Targets		
$\mathcal{R}$	8.6%	8.6%
Y	1	1
H	0.33	0.33
π	-	2.35%

Notes: (*) Arbitrarily fixed.

Table 4: Facts : Data vs. Model

	All Countries (76 of 127)		**OECD Countries** (28 of 28)		**"Free" Countries** (52 of 58)	
	Data	**Model**	**Data**	**Model**	**Data**	**Model**
Fact 1 (κ vs π)	-0.41	-0.68	-0.55	-0.61	-0.47	-0.63
Fact 2 (κ vs τ)	0.52	0.43	0.31	0.23	0.52	0.39
Fact 3 (κ vs $\mathcal{R}$)	-0.75	-0.87	-0.82	-0.81	-0.79	-0.83

Notes: For this table, the correlations for each fact is recomputed for the relevant sample of countries and are not identical to the values reported in Figures 1 through 5. "Free" refers to the countries label as such by Freedom House.