KOÇ UNIVERSITY-TÜSİAD ECONOMIC RESEARCH FORUM
WORKING PAPER SERIES

LEARNING WHILE SETTING PRECEDENTS

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Working Paper 1810
April 2018

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Learning While Setting Precedents

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February 24, 2018

Abstract

A decision maker (DM) must address a series of problems over time. Each period, a random case arises and the DM must make a yes-or-no decision, which we call a ruling. She is uncertain about the correct ruling until she conducts a costly investigation. A ruling establishes a precedent, which may be costly to violate in the future. We compare the DM's incentive to acquire information, the evolution of standards and the social welfare under two institutions: nonbinding precedent and binding precedent. Under nonbinding precedent, the DM is not required to follow previous rulings, but under binding precedent, she must follow previous rulings where applicable. We find that, compared to nonbinding precedent, the incentive for information acquisition is stronger under binding precedent in earlier periods when few precedents exist, but as more precedents are established over time, the incentive for information acquisition becomes weaker under binding precedent. Even though erroneous rulings may be perpetuated under binding precedent, social welfare can be higher because of the more intensive investigation conducted early on.

JEL Classification: D02, D23, D83, K4.

Keywords: precedent; binding precedent; information acquisition; transparency.

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In our progress towards political happiness my station is new; and, if I may use the expression, I walk on untrodden ground. There is scarcely any part of my conduct wch. [sic] *may not hereafter be drawn into precedent. Under such a view of the duties inherent to my arduous office, I could not but feel a diffidence in myself on the one hand; and an anxiety for the Community that every new arrangement should be made in the best possible manner on the other.*

George Washington, 9 January 1790

1 Introduction

Decision makers within organizations typically face and must address a series of problems under uncertainty over time. For example, an organization's executive could be tasked with deciding whether to accept or reject a series of project proposals, and government agencies such as the FDA evaluate proposals for approval or rejection. For reasons of consistency, fairness, predictability and public confidence, a cost is often associated with treating similar cases differently. Thus, a yes-or-no decision (which we call a ruling) regarding a current case may affect future decisions by creating a *precedent*. Acquiring the information needed to make the correct ruling is costly, but these decisions may have far-reaching consequences through two channels. First, the information acquired may reveal some general principle applicable not just to the case at hand but to future cases as well. Second, as the ruling on the current case becomes a precedent, the decision maker may incur some cost in making a different ruling on a future case that has similar characteristics, and this cost associated with overturning a precedent may prevent the decision maker from making certain rulings, even when it becomes clear that an erroneous ruling was made on a previous case.

This kind of dynamic problem solving has a broad range of applications, including judge-made law, which provides a natural application for our study because of the central role of precedents in common law systems. For example, the U.S. Bankruptcy Code allows firms to restructure their debt under Chapter 11 by confirming a reorganization plan. This can be achieved if all classes of creditors and interest holders vote to accept the plan. Without the consent of all classes, confirmation is possible only if the reorganization plan is judged to be "fair and equitable" to each class that voted against it. However, the definition of "fair and equitable" within the Bankruptcy Code is not entirely clear and the standard has therefore been set gradually over time by the courts on a case-by-case basis.

Another reason that judge-made law provides a good application for our study is *variation* within the legal system in the effects that precedents can have on future cases that are similar. United States law, for example, has two kinds of precedents: persuasive and binding. Under a persuasive precedent, a judge is not required to follow previous rulings but can use the information acquired in the ruling of a previous case. Under binding precedent, in contrast, the principle of *stare decisis* requires that a judge must follow previous rulings when they apply.[1] Since our study's primary goal is to understand how the dynamic consequences of precedents affect the decision maker's incentive to acquire information and in turn the overall qualities of the decisions, the institutions of common law system provide a fitting framework to anchor our analysis.

First employing a simple three-period model and then an infinite-horizon model, we analyze the DM's incentives. Under both models, we find that in early periods, when few precedents have been established, the DM's incentives to acquire information are stronger if precedents are binding. But as more precedents are established over time, the incentive to acquire information becomes weaker under binding precedents. Our results suggest that decision makers spend more time and resources deliberating on cases when an organization is in an early stage of development and less when its rules and standards have matured. Furthermore, this contrast is especially pronounced when precedents are binding.

To see why, note that the cost of making a wrong decision is higher under binding precedent since the DM has to follow precedents in the future when they are binding even if the previous rulings that established these precedents turn out to be erroneous. Because of the long-term repercussions of an early erroneous ruling when precedents are binding, a DM who faces few precedents is more inclined to acquire information to avoid making mistakes. (This point is illustrated by the "diffidence" and "anxiety" expressed by the first U.S. President George Washington in a letter to historian Catharine Macaulay.) As more precedents are established over time, however, the value of information becomes lower under binding precedent since the DM, now bound by previous rulings, may not be able to use the newly acquired information, and is

[1] In the U.S. legal system, a lower court is bound by a precedent set by a higher court in its region, but cases decided by lower courts or by peer or higher courts from other jurisdictions are only persuasive and not binding. We note that both kinds of precedents exist in the legal system, but modeling the legal hierarchy is beyond the scope of our study. For models of judicial learning and precedent that incorporate the legal hierarchy, see, for example, Bueno De Mesquita and Stephenson [2002] and Callander and Clark [2017].

therefore discouraged from acquiring information under binding precedent.[2] An interesting parallel is the "two-phase" process discussed in March and Simon [1958] (p. 208): when a new organizational unit is created to develop a new program, there is "a spurt of innovative, program-developing activity" which automatically diminishes as the program is elaborated and the unit is "bound and hampered by traditions and precedents."

The appropriate social welfare criterion for our model is not obvious since the DM is the only agent explicitly modeled. Provided the rest of society does not bear the cost of information acquisition but cares about the rulings, the payoffs derived from the ruling decisions may constitute a reasonable measure of the social welfare.[3] For example, judicial decision-making can be viewed as a principal-agent problem in which society delegates important decisions to courts. Since the court's rulings have broad implications that affect the society at large, it would be misleading to use the DM's payoff alone to measure the social welfare. When using payoffs from rulings as our welfare measure, we find that the social welfare can be higher under binding precedent than under nonbinding precedent. This is because the social benefit coming from the more intensive investigation conducted by the DM early on can outweigh losses attributable to the persistent mistakes in ruling potentially arising under binding precedent. Our result shows that increasing the cost of violating precedent can provide an effective way to strengthen a DM's incentive to gather information and improve the quality of rulings.

Although our measure of social welfare excludes the cost of investigation, this cost nonetheless indirectly affects it through the effects on incentives. When the cost of investigation is either very high or very low, investigations undertaken and ruling decisions are identical under nonbinding and binding precedent, resulting in the same welfare. In the intermediate range, when the cost is relatively low, the gain from more intensive early investigation under binding precedent dominates, resulting in higher social welfare. However, when the cost is relatively high, this gain does not compensate for the losses resulting from perpetuating erroneous rulings. As a result, the social welfare is lower under binding precedent in this case.

[2]Even though the lack of information acquisition seems consistent with the current U.S. President's behavior, we do not believe that our model provides a good explanation for it.

[3]The seminal paper by Tullock [1971] discusses the tension between the private cost of information acquisition and the public benefit of correct decisions, which results in a problem of under-investment in information acquisition, but it does not provide any solutions. For a recent synthesis of possible solutions to this problem, see Stephenson [2010].

Apart from institutions for which costs of violating precedent are heterogeneous, such as those in the legal system, other interpretations make our analysis applicable. Specifically, the cost of violating precedent may correspond to the degree of *transparency* in the decision making process. DMs whose actions are hidden face no penalty in treating similar problems differently. However, a DM whose actions are visible to the public can be held accountable for inconsistency or unfairness when overturning a precedent. In light of this, our findings demonstrate a social benefit associated with transparency in decision making that has not being pointed out in the literature. By making a violation of precedent public and thus punishable, transparency encourages more deliberate and careful initial decision making.[4] Indeed, some organizations choose to publish their decisions and refer to these decisions as precedents. For example, the U.S. Department of Justice Executive Office for Immigration Review publishes precedent decisions, and some Australian universities publish their transfer credit decisions in an online database.[5]

Related literature

The study most closely related to ours is Baker and Mezzetti [2012], which analyzes the process by which a long-lived court makes rulings under uncertainty when previous rulings become precedent. Unlike our study, they do not explicitly distinguish between nonbinding precedent and binding precedent and, in effect, analyze only nonbinding precedent. The focus of Baker and Mezzetti [2012] is to show how analogical reasoning arises endogenously as optimal behavior of a DM when resources are limited. In contrast, the focus of our study is the comparison of a DM's incentives to acquire information under nonbinding versus binding precedent.

Viewed more broadly, our paper is related to the literature on judicial decision-making in a dynamic setting. Rasmusen [1994] shows that in a repeated games setting, equilibrium *stare decisis* can arise even if a judge's own preference goes against previous rulings because it is the only way to ensure that his own rulings are followed by future judges.[6] More recently, Cameron, Kornhauser, and Parameswaran [2017] consider a repeated game among the judges in a heterogenous bench and establish a possible

[4]Increasing the transparency of an agent's action, or of the consequence of an agent's action, can have subtle effects. See, for example, Prat [2005], Fox [2007], Levy [2007], Fox and Van Weelden [2012].

[5]See https://www.uscis.gov/laws/precedent-decisions and http://www.uq.edu.au/uqabroad/uq-credit-precedent-database.

[6]Kornhauser [1989] investigates other reasons why a court might adopt *stare decisis*.

equilibrium partial *stare decisis* in which judges adhere to a common rule for clear-cut cases, but adopt their preferred rules for contestable cases. Daughety and Reinganum [1999] and Talley [1999] show that inefficient informational cascade can arise in equilibrium when courts have private information and learn from the decisions of previous courts. Levy [2005] looks at a model in which judges are driven by career concerns, that is, they care about their reputation of being able to correctly apply law, and not about social efficiency. In the model of Levy (2005), binding precedent increases the probability that a ruling contradicting precedent will be subject to reversal, which is costly for the judge and therefore causes her to behave inefficiently. Gennaioli and Shleifer [2007] provide a model in which the law evolves when judges distinguish cases from precedents by adding new material dimensions while at the same time endorsing the existing precedent. In Anderlini, Felli, and Riboni [2014], *stare decisis* is a tool for commitment which provides a benefit by alleviating time-inconsistent preferences of the court.

A primary insight obtained through our study is that although mistakes may perpetuate when overruling a precedent is costly, it is precisely this *ex post* inefficiency that motivates the decision maker to acquire more information so as to avoid making the mistakes in the first place. Our paper is therefore connected to a strand of literature showing how *ex post* inefficient rules provide incentives for agents to gather costly information, leading to better decision-making overall. Li [2001] shows that it can be optimal for a group to commit to a "conservative" decision rule that biases against the alternative favored by the group's common prior because it alleviates the problem of free-riding in information acquisition among the group's members. Szalay [2005] investigates how the set of feasible actions affects an agent's decision regarding how much information to acquire. He finds that it may be optimal for the principal to give only extreme options to the agent to motivate him to collect more information.[7]

The remainder of this paper is organized as follows. In the next section, we present our model. In section 3, we discuss a three-period model to illustrate some of the intuition before analyzing the infinite-horizon model in section 4. Then, in section 5, we compare the social welfare under nonbinding and binding precedents. In section 6, we discuss various extensions of our model and conclude. Appendix A contains the proofs and Appendix B presents our formal results on partial learning.

[7]In contrast, in Aghion and Tirole [1997], Baker, Gibbons, and Murphy [1999] and Armstrong and Vickers [2010], to motivate the agent to exert more effort to acquire information, the principal allows a larger set of options, including some that are undesirable for the principal.

2 Model

A decision maker (DM) in an organization regulates a set of activities by permitting or banning them. In each period, a new case arises which must be decided by the DM. The DM prefers to permit activities that she regards as beneficial and ban activities which she regards as harmful.

As in Baker and Mezzetti [2012], we model a case by $x \in [0, 1]$. For example, the DM could be regulating the duration of non-competition agreements allowed in employment contracts, and x might correspond to the duration of the non-compete clause in a given contract. The DM has a threshold value θ such that she regards case x as socially beneficial and would like to permit it if and only if $x \leq \theta$. The preference parameter θ is unknown initially, and we assume that θ is distributed according to a continuous cumulative distribution function F with support $[\underline{\theta}, \overline{\theta}]$ where $0 \leq \underline{\theta} < \overline{\theta} \leq 1$.

Denote the case at time t by x_t. We assume that the cases are independent across periods and each has a continuous cumulative distribution function G.[8] The precedent at time t is captured by two numbers L_t and R_t where L_t is the highest case that was ever permitted and R_t is the lowest case that was ever banned by time t. Assume that $L_1 = 0$ and $R_1 = 1$, that is, the initial precedent is consistent with the DM's preferences and does not impose any mistake in ruling.

In period t, after case x_t is realized, the DM chooses whether to conduct an investigation or not before deciding whether to permit or ban the case. For expositional simplicity, we assume that the DM permits the case when indifferent between permitting and banning, and conducts an investigation when indifferent between investigating and not investigating. Suppose that an investigation allows the DM to learn the value of θ at a fixed cost $z > 0$.[9] The stark form of the learning process assumed here is for

[8]Having endogenous arrival of cases is interesting but is beyond the scope of our paper. Fox and Vanberg [2014] analyze a two-period model in which the case heard in the second period is endogenously determined by the ruling in the first period. In their model, the judges can issue *narrow* rulings which stick to the facts at hand, or *broad* rulings that go beyond. They provide conditions under which broad rulings are optimal because they increase the informational value of future cases. Another paper that incorporates endogenous arrival of cases is Parameswaran [2018]. He analyzes an infinite-horizon model in which judicial learning takes place when a firm experiments in a legally ambiguous region, and shows that broad ruling tends to inhibit efficient learning.

[9]We assume that the DM learns about her preference parameter θ through investigation. Alternatively, we can assume that the DM learns about her preferences in terms of the *consequences* of cases, but does not know the consequence of a particular case unless she investigates. To illustrate, let $c(x)$ denote the consequence of a case x and assume that $c(x) = x + \gamma$. The DM would like to permit case x if $c(x)$ is below some threshold $\bar{c}$ and would like to ban it otherwise. Suppose that the DM knows $\bar{c}$ and observe x, but γ is unknown until the DM investigates. This alternative model is equivalent to

tractability: it simplifies the analysis significantly since either the DM is fully informed or her belief is the same as the prior. If learning is partial, then we have to keep track of the DM's belief in addition to the precedent as a state variable, which complicates the analysis significantly. We revisit this issue and extend our analysis to partial learning in section 6.

Let $s = ((L, R), x)$. For expositional convenience, we refer to s as the state even though it does not include the information about θ. Let $S = [0, 1]^3$ denote the set of possible states.

Denote the ruling at time t by $r_t \in \{0, 1\}$, where $r_t = 0$ if the case is banned and $r_t = 1$ if the case is permitted. After the DM makes her ruling, the precedent changes to L_{t+1} and R_{t+1}. If x_t was permitted, then $L_{t+1} = \max\{L_t, x_t\}$ and $R_{t+1} = R_t$; if x_t was banned, then $L_{t+1} = L_t$ and $R_{t+1} = \min\{R_t, x_t\}$. Formally, the transition of the precedent is captured by the function $\pi : S \times \{0, 1\} \to [0, 1]^2$ where

$$\pi(s_t, r_t) = \begin{cases} (L_t, \min\{R_t, x_t\}) & \text{if } r_t = 0 \\ (\max\{L_t, x_t\}, R_t) & \text{if } r_t = 1. \end{cases} \tag{1}$$

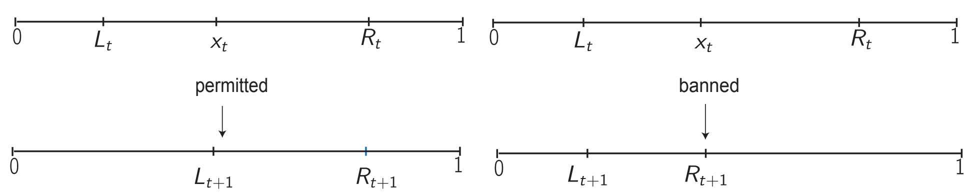

Figure 1: Evolution of precedents.

We consider two institutions: *nonbinding precedent* and *binding precedent*. Under nonbinding precedent, the DM is free to make any ruling; under binding precedent, the DM must permit x_t in period t if $x_t \le \min\{L_t, R_t\}$ and must ban x_t if $x_t \ge \max\{L_t, R_t\}$. To understand this assumption, note that if $x_t \le \min\{L_t, R_t\}$, then there must be some case higher than x_t that was permitted in the past and there is no case lower than x_t that was banned in the past, so the only ruling that is consistent with precedent is to permit x_t. Similarly, if $x_t \ge \max\{L_t, R_t\}$, then there must be some case lower than x_t that was banned in the past and there is no case higher than x_t that was permitted in the past, so the only ruling that is consistent with precedent is to ban x_t. Note that

ours.

under binding precedent, after the DM learns the value of θ, she may be bound to make certain rulings even if she knows them to be erroneous. If we allow the DM to overturn precedents once they are found to be erroneous, then the precedents effectively become nonbinding.[10]

More generally, we say that a ruling regarding x *violates* precedent (L, R) if $x \leq \min\{L, R\}$ and the DM bans x or if $x \geq \max\{L, R\}$ and the DM permits x. We can think of the cost of violating precedent to be infinite when it is binding and zero when it is nonbinding. We focus on these two extremes to highlight the difference in the incentives that the DM faces.[11]

Note that, $L_t < R_t$ on the equilibrium path under binding precedent. But it is possible that $R_t < L_t$ off the equilibrium path; in this case, for $x_t \in (R_t, L_t)$, the DM can either permit or ban x_t even under binding precedent. This is because if the DM permits x_t, the ruling is supported by precedent since there is a higher case that has been permitted before, and if the DM bans x_t, the ruling is still supported by precedent since there is a lower case that has been banned before.[12]

The payoff of the DM from the ruling r_t on case x_t in period t is given by

$$u(r_t; x_t, \theta) = \begin{cases} 0 & \text{if } x_t \leq \theta \text{ and } r_t = 1, \text{ or } x_t \geq \theta \text{ and } r_t = 0, \\ -\ell(x_t, \theta) & \text{otherwise,} \end{cases}$$

where $\ell(x_t, \theta) > 0$ for $x_t \neq \theta$ is the cost of making a mistake, that is, permitting a case when it is above θ or banning a case when it is below θ. Assume that $\ell(x, \theta)$ is continuous in x and θ for $x \neq \theta$, strictly increasing in x and strictly decreasing in θ if $x > \theta$ and strictly decreasing in x and strictly increasing in θ if $x < \theta$. (We allow there to be a discontinuity at $x = \theta$ to reflect a fixed cost of making a mistake in ruling.) For example, if $\ell(x, \theta) = f(|x - \theta|)$ where $f(y) : \mathbb{R}_+ \to \mathbb{R}_+$ is continuous for $y > 0$, strictly increasing, and $f(0) = 0$, then these assumptions are satisfied.

The dynamic payoff of the DM is the sum of her discounted payoffs from the rulings

[10] Indeed, legal scholars recognize that it is "an essential feature of *any* coherent doctrine of *stare decisis*" that any overruling should not be solely based on the belief that a prior ruling is erroneous (Nelson [2001]).

[11] If we allow the DM to choose in each period whether to make her ruling binding, the analysis would be the same as the case of nonbinding precedent since the DM would not choose to make her ruling a binding precedent if she did not conduct an investigation.

[12] Another way to formalize how binding precedent affects the decision problem is to assume that the set of feasible actions depends on the precedent. Specifically, under binding precedent (L_t, R_t), if $x \leq L_t$, then the only feasible ruling r_t is 1, and if $x \geq R_t$, then the only feasible ruling r_t is 0. Under these assumptions on feasible actions, $L_t < R_t$ always holds.

made in each period net of the cost of violating a precedent and net of the discounted investigation cost if the DM carries out one. The discount factor is denoted by $\delta \in (0,1)$.

Before we analyze the infinite-horizon model, we discuss a three-period model to illustrate some of the intuition.

3 Three-period model

Suppose there are three periods. We characterize the optimal investigation and ruling policies for each period using backward induction. We then compare the information acquisition incentives under the two institutions.

3.1 Nonbinding precedent

In any period, if the DM has investigated in a previous period, then θ is known and she permits or bans case x according to θ. If the DM has not investigated in a previous period, then her belief about θ is the same as the prior. If $x < \underline{\theta}$, then the DM strictly prefers to permit the case since $x < \theta$ regardless of what the realization of θ is. Likewise, if $x > \underline{\theta}$, then the DM strictly prefers to ban the case. For any $x \in [\underline{\theta}, \bar{\theta}]$, the difference in expected payoff between banning the case and permitting the case is given by

$$E_\theta[u(0;x,\theta) - u(1;x,\theta)] = \int_x^{\bar{\theta}} -\ell(x,\theta)dF(\theta) - \int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta).$$

Given the assumptions on $\ell(x,\theta)$, it follows that $E_\theta[u(0;x,\theta) - u(1;x,\theta)]$ is continuous and increasing in x. Since $E_\theta[u(0;x,\theta) - u(1;x,\theta)] < 0$ if $x = \underline{\theta}$ and $E_\theta[u(0;x,\theta) - u(1;x,\theta)] > 0$ if $x = \bar{\theta}$, there exists $\tilde{x} \in (\underline{\theta}, \bar{\theta})$ such that

$$\int_{\underline{\theta}}^{\tilde{x}} \ell(\tilde{x},\theta)dF(\theta) = \int_{\tilde{x}}^{\bar{\theta}} \ell(\tilde{x},\theta)dF(\theta), \tag{2}$$

that is, $\tilde{x}$ is the case such that the uninformed DM is indifferent between permitting and banning. Note that $E_\theta[u(0;x,\theta) - u(1;x,\theta)] < 0$ for $x < \tilde{x}$ and $E_\theta[u(0;x,\theta) - u(1;x,\theta)] > 0$ for $x > \tilde{x}$ which gives us the following result.

Lemma 1. *Under nonbinding precedent, in any period t, if the DM is uninformed, she permits x_t if $x_t \leq \hat{x}$ and bans x_t if $x_t > \hat{x}$.*

This characterization of the ruling decision of an uninformed DM under nonbinding precedent holds not just for the three-period model but for any time horizon.

Now we analyze the DM's investigation decisions. We say that *case x triggers an investigation* if the DM conducts an investigation when the case is x. If DM decides to investigate in period t, her payoff is $-z$ in period t and 0 in all future periods. The following lemma says that when the investigation cost is sufficiently low, the uninformed DM investigates with positive probability in each period; the cases that trigger an investigation in period t forms an interval; and the interval of investigation is larger in an earlier period. Intuitively, for the cases that fall in the middle, it is less clear to the DM whether she should permit it or ban it and the expected cost of making a mistake is higher. Hence, the value of investigation for these cases is higher. Thus if case x triggers an investigation in period t and case $x' > x$ triggers an investigation in period t, then any case in $[x, x']$ triggers an investigation in period t. Moreover, since the DM can use the information she acquires in an earlier period for later periods, the value of investigation is higher in an earlier period, resulting in more investigation in an earlier period.

Lemma 2. *In the three-period model under nonbinding precedent, the set of cases that trigger an investigation in period t, denoted by X_t^N, is an interval (possibly empty). Moreover, $X_3^N \subseteq X_2^N \subseteq X_1^N$.*

3.2 Binding precedent

We first show that in each period t, the cases that trigger an investigation form a (possibly degenerate) interval under binding precedent as well. Moreover, if the DM's hands are tied regarding case x_t, that is, if $x_t \leq L_t$ or if $x_t \geq R_t$, then x_t does not trigger an investigation.

Lemma 3. *Under binding precedent, for any precedent (L_t, R_t), the set of cases that trigger an investigation in period t, denoted by $X_t^B(L_t, R_t)$, is an interval (possibly empty). Moreover, $X_t^B(L_t, R_t) \subseteq (L_t, R_t)$.*

This characterization of the investigation decision of an uninformed DM under binding precedent holds not just for the three-period model but for any time horizon.

In the next proposition, we compare the investigation decisions under binding precedent and nonbinding precedent.

Proposition 1. *In the three-period model, the uninformed DM investigates more under binding precedent than under nonbinding precedent in period* 1*, and investigates less under binding precedent than under nonbinding precedent in periods* 2 *and* 3*. Specifically,*

(i) $X_1^N \subseteq X_1^B(L_1, R_1)$.

(ii) If $[\underline{\theta}, \bar{\theta}] \subseteq [L_2, R_2]$*, then* $X_2^N = X_2^B(L_2, R_2)$*; otherwise* $X_2^N \subseteq X_2^B(L_2, R_2)$.

(iii) $X_3^B(L_3, R_3) = (L_3, R_3) \cap X_3^N$.

To understand this result, first consider the last period. The reason for the DM to investigate less in period 3 under binding precedent is that an investigation has no value if $x_3 \leq L_3$ or if $x_3 \geq R_3$ since the DM must permit any $x_3 \leq L_3$ and must ban any $x_3 \geq R_3$ no matter what the investigation outcome is; moreover, since period 3 is the last period, the information about θ has no value for the future either. For $x_3 \in (L_3, R_3)$, the DM faces the same incentives under binding and nonbinding precedent and therefore the same set of cases trigger an investigation.

If the precedent in period 2 satisfies $[\underline{\theta}, \bar{\theta}] \subseteq [L_2, R_2]$, then investigation avoids mistakes in ruling in the current period as well as the future period even under binding precedent. In this case, the DM faces the same incentives under binding and nonbinding precedent and therefore i the same set of cases trigger an investigation. However, if the precedent in period 2 does not satisfy $[\underline{\theta}, \bar{\theta}] \subseteq [L_2, R_2]$, then even if $x_2 \in (L_2, R_2)$ and the DM investigates, mistakes in ruling can still happen in period 3 under binding precedent if $\theta \notin [L_2, R_2]$ since the DM is bound to follow the precedent. In this case, the value of investigation is lower under binding precedent than under nonbinding precedent and therefore the DM investigates less under binding precedent.

Since the precedent in period 1 satisfies $[\underline{\theta}, \bar{\theta}] \subseteq [L_1, R_1]$, investigation avoids mistakes in ruling in the current period as well as in future periods even under binding precedent. However, for $x_1 \in (\underline{\theta}, \bar{\theta})$, if the DM makes a ruling without an investigation when x_1 is realized, then she changes the precedent in a way such that $[\underline{\theta}, \bar{\theta}] \not\subseteq [L_2, R_2]$. As discussed in the previous paragraph, the binding precedent arising from this ruling potentially results in mistakes in the future and diminishes the DM's incentive to investigate future periods, which in turn lowers the DM's dynamic payoff. Hence, the

DM's payoff from *not* investigating in period 1 is lower under binding precedent which gives her a stronger incentive to investigate early on.

4 Infinite-horizon model

We now consider the infinite-horizon model. We start by defining optimal policies under the two institutions.

Nonbinding precedent

With nonbinding precedent, the payoff-relevant state in any period is the realized case $x \in [0,1]$ and the information about θ.

If θ is known at the time when the relevant decisions are made, then it is optimal not to investigate; to permit x if $x \leq \theta$; and the DM's payoff is 0.

If θ is unknown at the time when the relevant decisions are made, a policy for the DM is a pair of functions $\sigma_N = (\mu_N, \rho_N)$, where $\mu_N : [0,1] \to \{0,1\}$ is an *investigation policy* and $\rho_N : [0,1] \to \{0,1\}$ is an *uninformed ruling policy*, with $\mu_N(x) = 1$ if and only if an investigation is made when the case is x and $\rho_N(x) = 1$ if and only if case x is permitted.

For each policy $\sigma_N = (\mu_N, \rho_N)$, let $V_N(\cdot; \sigma_N)$ be the associated value function, that is, $V_N(x; \sigma_N)$ represents the dynamic payoff of the DM when she is uninformed, faces case x in the current period, and follows the policy σ_N. In what follows, we suppress the dependence of the dynamic payoffs on σ_N for notational convenience. Given any dynamic payoff V_N, let EV_N^* denote its expected value, that is, $EV_N^* = \int_0^1 V_N(x')dG(x')$.

The policy $\sigma_N^* = (\mu_N^*, \rho_N^*)$ is optimal if σ_N^* and the associated value function V_N^* satisfy the following conditions:

(N1) The uninformed ruling policy satisfies $\rho_N^*(x) = 1$ if and only if for any case x

$$\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) \geq \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta).$$

(N2) Given V_N^* and the uninformed ruling policy ρ_N^*, the investigation policy for the uninformed DM satisfies $\mu_N^*(x) = 1$ if and only if for any case x

$$-z \geq \rho_N^*(x) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) + (1 - \rho_N^*(x)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_N^*.$$

(N3) Given σ_N^*, for any case x, the dynamic payoff satisfies

$$\begin{aligned} V_N^*(x) &= -z\mu_N^*(x) + (1-\mu_N^*(x)) \Bigg[\rho_N^*(x) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) \\ &+ (1-\rho_N^*(x)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta E V_N^* \Bigg]. \end{aligned}$$

Condition (N1) says that the DM chooses the ruling that minimizes the expected cost of making a mistake in the current period. The ruling decision depends only on the current period payoff because the ruling does not affect the DM's continuation payoff under nonbinding precedent. Condition (N2) says that when uninformed, a case triggers an investigation if and only if the DM's dynamic payoff from investigating is higher than her expected dynamic payoff from not investigating. If a case triggers an investigation, then θ becomes known and no mistake in ruling will be made in the current period as well as in the future. In this case, the dynamic payoff of the DM is negative of the cost of investigation. If a case does not trigger an investigation, then the DM's dynamic payoff is the sum of the expected cost of making a mistake in the current period and the continuation payoff. This payoff is the value function given in condition (N3).

Binding precedent

With binding precedent, the payoff-relevant state in any period is the precedent pair (L, R), the realized case x, and the information about θ.[13]

If θ is known at the time when the relevant decisions are made, then it is optimal not to investigate. Moreover, the optimal ruling policy is as follows. If $\theta \in (L, R)$, then it is optimal to permit x iff $x \leq \theta$. If $\theta \leq L$, then it is optimal to permit x iff $x \leq L$. If $\theta \geq R$, then it is optimal to permit x iff $x < R$.

Let $C(L, R)$ denote the expected dynamic payoff of the DM when the precedent is (L, R), conditional on θ being known when decisions regarding the cases are made where the expectation is taken over θ before it is revealed and over all future cases x. Note that $C(L, R)$ has two components, one is when future cases fall below L and the other is when future cases fall above R. We denote the first component by $c(L)$ and

[13] For expositional simplicity, we consider precedents with $L < R$ in our analysis of binding precedent. Under binding precedent, this must happen on the equilibrium path and the analysis is without loss of generality.

the second by $c(R)$. Formally,

$$c(L) = \begin{cases} 0 & \text{if } L \leq \underline{\theta}, \\ \frac{1}{1-\delta} \int_{\underline{\theta}}^{L} \int_{\theta}^{L} -\ell(x,\theta) dG(x) dF(\theta) & \text{if } L > \underline{\theta}, \end{cases}$$

$$c(R) = \begin{cases} 0 & \text{if } R \leq \bar{\theta}, \\ \frac{1}{1-\delta} \int_{R}^{\bar{\theta}} \int_{R}^{\theta} -\ell(x,\theta) dG(x) dF(\theta) & \text{if } R < \bar{\theta}, \end{cases}$$

and

$$C(L, R) = c(L) + c(R).$$

To see how we derive $c(L)$ and $c(R)$, note that if $\theta < L$ and $x \in (\theta, L]$, then the DM incurs a cost of $-\ell(x,\theta)$ since she has to permit x, but if $\theta \geq L$, then the DM incurs no cost if the case falls below L; similarly, if $\theta > R$ and $x \in [R, \theta)$, then the DM incurs a cost of $-\ell(x,\theta)$ since she has to ban x, but if $\theta \leq R$, then the DM incurs no cost if the case falls above R. Note that $c(L)$ is decreasing in L and $c(R)$ is increasing in R, and therefore $C(L, R)$ is decreasing in L and increasing in R. Intuitively, $C(L, R)$ captures the expected cost in ruling mistakes due to the binding power of the precedent L and R, and therefore, it is higher when the precedent is tighter.

If θ is unknown at the time when the decisions regarding the cases are made, a policy for the DM is a pair of functions $\sigma_B = (\mu_B, \rho_B)$, where $\mu_B : S \to \{0, 1\}$ is an investigation policy and $\rho_B : S \to \{0, 1\}$ is an uninformed ruling policy, where $\mu_B(s) = 1$ if and only if an investigation is made when the state is s, and $\rho_B(s) = 1$ if and only if case x is permitted when the state is s.

Let $A(s)$ denote the DM's dynamic payoff if she investigates in state $s = ((L, R), x)$, not including the investigation cost. Formally, let $\mathcal{L}$ be the (possibly degenerate) interval $[\underline{\theta}, \max\{L, \underline{\theta}\}]$ and $\mathcal{R}$ be the (possibly degenerate) interval $[\min\{R, \bar{\theta}\}, \bar{\theta}]$, we have

$$A(s) = \mathbf{1}_{\mathcal{L}}(x) \int_{\underline{\theta}}^{x} -\ell(x,\theta) dF(\theta) + \mathbf{1}_{\mathcal{R}}(x) \int_{x}^{\bar{\theta}} -\ell(x,\theta) dF(\theta) + \delta C(L, R).$$

For each policy $\sigma_B = (\mu_B, \rho_B)$, let $V_B(\cdot; \sigma_B)$ denote the associated value function, that is, $V_B(s; \sigma_B)$ represents the dynamic payoff of the DM when the state is s, θ is unknown, and she follows the policy σ_B. In what follows, we suppress the dependence

V_B on σ_B for notational convenience. For notational convenience, let $EV_B(L,R) = \int_0^1 V_B(L,R,x')dG(x')$.

Recall that the transition of precedent is captured by the function π, defined in (1). The policy $\sigma_B^* = (\mu_B^*, \rho_B^*)$ is optimal if σ_B^* and the associated value function V_B^* satisfy the following conditions:

(B1) Given V_B^*, for any state s, the uninformed ruling policy satisfies $\rho_B^*(s) = 1$ if either $x \le L$ or $x \in (L,R)$ and

$$\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) + \delta EV_B^*(\pi(s,1)) \ge \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_B^*(\pi(s,0));$$

and $\rho_B^*(s) = 0$ if either $x \ge R$ or $x \in (L,R)$ and

$$\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) + \delta EV_B^*(\pi(s,1)) < \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_B^*(\pi(s,0)).$$

(B2) Given V_B^* and the uninformed ruling policy ρ_B^*, for any state s, the investigation policy for the uninformed DM satisfies $\mu_B^*(s) = 1$ if and only if

$$-z + A(s) \ge \rho_B^*(s) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta)$$
$$+(1-\rho_B^*(s)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV^*(\pi(s,\rho^*(s))).$$

(B3) Given σ^*, for any state s, the dynamic payoff satisfies

$$\begin{aligned} V_B^*(s) &= \mu_B^*(s)\left[-z + A(s)\right] \\ &+ (1-\mu_B^*(s)) \left[\rho_B^*(s) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) \right. \\ &+ \left. (1-\rho_B^*(s)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_B^*(\pi(s,\rho^*(s)))\right]. \end{aligned}$$

Under binding precedent, the ruling decision may change the precedent, which in turn may affect the continuation payoff. As such, condition (B1) says the ruling decision depends on both the current period payoff and the continuation payoff. In particular, the DM chooses the ruling that maximizes the sum of the current period payoff and the continuation payoff, taking into consideration how her ruling affects the precedent in the next period. Condition (B2) says that the DM chooses to investigate a case if and only if her dynamic payoff from investigating is higher than her expected dynamic payoff from not investigating.

If the DM investigates case x, then θ becomes known. When the precedents are binding, however, mistakes in ruling can still happen if $\theta < L$ or if $\theta > R$. In this case, the dynamic payoff $V_B^*(s)$ is the expected cost of making mistakes in the ruling, both in the current period and in future periods, minus the cost of investigation. If the DM does not investigate case x, then her dynamic payoff is the sum of the expected cost of making a mistake in the current period and the continuation payoff. Condition (B3) formalizes this.

Existence and uniqueness

We next show that the DM's value functions and optimal policies as defined in (N1-P3) and (B1-B3) exist and are unique.

Proposition 2. *Under either nonbinding precedent or binding precedent, the DM's optimal policy exists and is unique.*

To prove this, we first apply the Contraction Mapping Theorem to show that the value functions V_N^* and V_B^* exist and are unique. The optimality conditions (N1-N2) and (B1-B2) then uniquely determine the optimal policies σ_N^* and σ_B^*. We next turn to the characterization of the value function and optimal policy.

4.1 Nonbinding precedent

If the DM already investigated in a previous period, then she knows the value of θ and would permit or ban a case according to θ. The following result is analogous to Lemma 2 in the three-period model.

Lemma 4. *Under nonbinding precedent, the set of cases that trigger an investigation in any period is an interval.*

Let $X_N = \{x : \mu_N^*(x) = 1\}$, that is, X_N is the set of cases that trigger an investigation under nonbinding precedent. Recall $\hat{x}$ is the case such that the uninformed DM is indifferent between permitting and banning. Let

$$\hat{z} = \int_{\underline{\theta}}^{\hat{x}} \ell(x, \theta) dF(\theta) \tag{3}$$

denote the expected cost from making uninformed ruling on $\hat{x}$. If $X_N \neq \varnothing$, let $a_N = \inf\{x : \mu_N^*(x) = 1\}$ and $b_N = \sup\{x : \mu_N^*(x) = 1\}$.

We next show that if the DM faces a case such that there is no uncertainty about what the correct ruling is for that case (that is, if $x \leq \underline{\theta}$ or if $x \geq \bar{\theta}$), then there is no investigation. Intuitively, because of discounting, it is optimal to delay investigation until it is useful immediately even though the information from investigation is valuable for future rulings. In this case, the optimal ruling decision is to permit x if and only if $x \leq \hat{x}$ as shown in Lemma 1.

Let λ denote the expected loss if the DM makes a ruling without an investigation, that is,

$$\lambda = \int_{\underline{\theta}}^{\hat{x}} \int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta)dG(x) + \int_{\hat{x}}^{\bar{\theta}} \int_{x}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)dG(x).$$

Let $z^* = \hat{z} - \frac{\delta\lambda}{1-\delta}$. We also show that the uninformed DM investigates with positive probability if the investigation cost is below the threshold z^*. In that case, the uninformed DM permits any case below a_N and bans any case above b_N.

Proposition 3. *Under nonbinding precedent, any case $x \notin (\underline{\theta}, \bar{\theta})$ does not trigger an investigation. If $z > z^*$, then $X_N = \varnothing$, and the uninformed DM permits x if $x \leq \hat{x}$ and bans x otherwise. If $z \leq z^*$, then $X_N = [a_N, b_N] \neq \varnothing$ and the uninformed DM permits x if $x < a_N$ and bans x if $x > b_N$.*

Suppose $X_N \neq \varnothing$. Recall that $EV_N^* = \int_0^1 V_N^*(x')dG(x')$. Proposition 3 says that the optimal policies are characterized by a_N and b_N. To find a_N and b_N, note that

$$V_N^*(x) = \begin{cases} \delta EV_N^* & \text{if } x \leq \underline{\theta}, \text{ or if } x \geq \bar{\theta}, \\ \int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta) + \delta EV_N^* & \text{if } \underline{\theta} < x < a_N, \\ -z & \text{if } x \in [a_N, b_N], \\ \int_{x}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_N^* & \text{if } b_N < x < \bar{\theta}. \end{cases} \tag{4}$$

To see how we derive this, note that if $x \leq \underline{\theta}$ or if $x \geq \bar{\theta}$, then the DM does not investigate and makes no mistake in her ruling in the current period. In this case, her current-period payoff is 0 and her continuation payoff is δEV_N^*. If $\underline{\theta} < x < a_N$ or if $b_N < x < \bar{\theta}$, the DM does not investigate in the current period and incurs some cost of making a mistake in expectation. Since θ remains unknown, her continuation payoff is

δEV_N^*. If $x \in [a_N, b_N]$, then the DM investigates. Since she makes no mistake in her ruling both in the current period and in all future periods, her current period payoff is $-z$ and her continuation payoff is 0.

From (4), we have

$$\begin{aligned} EV_N^* = & -z[G(b_N) - G(a_N)] + \delta EV_N^* \left[G(a_N) + 1 - G(b_N)\right] \\ & + \int_{\underline{\theta}}^{a_N} \int_{\underline{\theta}}^{x} -\ell(x,\theta) dF(\theta) dG(x) + \int_{b_N}^{\bar{\theta}} \int_{x}^{\bar{\theta}} -\ell(x,\theta) dF(\theta) dG(x). \end{aligned}$$

For any a, b such that $\underline{\theta} < a \leq b < \bar{\theta}$, let $h(a,b) = \int_{\underline{\theta}}^{a} \int_{\underline{\theta}}^{x} -\ell(x,\theta) dF(\theta) dG(x) + \int_{b}^{\bar{\theta}} \int_{x}^{\bar{\theta}} -\ell(x,\theta) dF(\theta) dG(x)$. That is, $h(a,b)$ is the expected cost of making a mistake before the realization of the case when the DM's investigation interval is $[a,b]$. Then

$$EV_N^* = \frac{h(a_N, b_N) - z[G(b_N) - G(a_N)]}{1 - \delta[G(a_N) + 1 - G(b_N)]}. \tag{5}$$

Since the DM is indifferent between investigating and not investigating when $x = a_N$ or when $x = b_N$, we have

$$-z = \int_{\underline{\theta}}^{a_N} -\ell(a_N, \theta) dF(\theta) + \delta EV_N^* = \int_{b_N}^{\bar{\theta}} -\ell(b_N, \theta) dF(\theta) + \delta EV_N^*. \tag{6}$$

We can solve for EV_N^*, a_N, b_N from equations (5) and (6) which gives us the characterization of the optimal policies. Plugging these in (4), we can solve for the value function $V_N^*(x)$.

4.2 Binding precedent

We now consider binding precedent. We first establish that the value function V_B^* is decreasing in L and increasing in R; and the optimal investigation policy μ^* is also decreasing in L and increasing in R. This result says that as the precedent gets tighter, the DM investigates less and her payoff also becomes lower.

In what follows, let S^e denote the set of possible precedents that can arise on the equilibrium path under binding precedent, that is, $S^e = \{(L,R) \in [0,1]^2 : L < R\}$.

Proposition 4. *Suppose the precedent $(\hat{L}, \hat{R})$ is tighter than (L,R), that is, $L \leq \hat{L} < \hat{R} \leq R$. Under binding precedent, for any case $x \in [0,1]$, if it triggers an investigation under precedent $(\hat{L}, \hat{R})$, then it also triggers investigation under precedent (L,R), that* *is, $\mu^*_B(L,R,x)$ is decreasing in L and increasing in R. Moreover, the value function $V^*_B(L,R,x)$ is decreasing in L and increasing in R, and $EV^*_B(L,R)$ is continuous in L and R for any $(L,R) \in S^e$.*

We next show that as in the three-period model, the set of cases that the DM investigates is an interval, and moreover, if the DM's hands are tied regarding a case, then that case does not trigger an investigation.

Proposition 5. *Under binding precedent, for any precedent $(L,R) \in S^e$ (i) the set of cases that trigger an investigation is an interval; (ii) if $x \le L$ or if $x \ge R$, then x does not trigger an investigation.*

Let $X_B(L,R) = \{x : \mu^*_B(L,R,x) = 1\}$ denote the investigation interval under (L,R). For any $(L,R) \in S^e$ such that $X_B(L,R) \neq \varnothing$, let $a(L,R) = X_B(L,R)$ and $b(L,R) = \sup X_B(L,R)$.

We next show that, analogous to Proposition 1 in the three-period model, the DM investigates more under binding precedent than under nonbinding precedent early on but investigates less under binding precedent in later periods. We formalize the first part of this statement by comparing the investigation intervals in the first period. Formalizing the second part is trickier since the investigation interval under binding precedent in later periods depend on the realized path of the cases. We show that *eventually* there is less investigation under binding precedent by characterizing a *limit investigation interval under binding precedent.*

Before we give a formal definition, we discuss the idea. Suppose that given the the initial precedent, the set of cases that trigger an investigation is nonempty (if it is empty, then no investigation will be carried out in any period). For notational simplicity, let $a_1 = a(L_1,R_1)$ and $b_1 = b(L_1,R_1)$. Recall that the the initial precedent is consistent with the DM's preference, that is, $L_1 < \underline{\theta}$ and $R_1 > \bar{\theta}$. Hence, we have $L_1 < a_1 \le b_1 < R_1$, and x_1 triggers an investigation if and only if $x_1 \in [a_1,b_1]$.[14]

If $x_1 \in [a_1,b_1]$, then it triggers an investigation immediately. If $x_1 \notin [a_1,b_1]$, then the DM makes a ruling without any investigation and changes the precedent to $(L_2,R_2) = (x_1,R_1)$ if she permits the case and to $(L_2,R_2) = (L_1,x_1)$ if she bans the case. Note that the resulting new precedent satisfies $L_2 < a_1$ and $b_1 < R_2$. Monotonicity of μ^*_B in L and R as established in Proposition 4 implies that the investigation interval

[14]In Lemma A.2, we show that if $L < a(L,R) \le b(L,R) < R$, then the investigation interval under precedent (L,R) is closed.

in period 2, if nonempty, satisfies $a(L_2, R_2) \geq a_1$ and $b(L_2, R_2) \leq b_1$. Therefore we have $L_2 < a(L_2, R_2) \leq b(L_2, R_2) < R_2$ and the DM investigates x_2 if and only if $x \in [a(L_2, R_2), b(L_2, R_2)]$. An iteration of this argument shows that on any realized equilibrium path, given the precedent (L, R), the investigation interval (if nonempty) satisfies $L < a(L, R) \leq b(L, R) < R$. The investigation intervals either converge to $\varnothing$ or to some nonempty set $[\hat{a}, \hat{b}]$ such that if the precedent is $(L, R) = (\hat{a}, \hat{b})$, then $a(L, R) = \hat{a}$ and $b(L, R) = \hat{b}$.

We now define a limit investigation interval under binding precedent, denoted by X_B^∞. If there is no investigation under the initial precedent, that is, if $X_B(L_1, R_1) = \varnothing$, then there is no investigation in any other period by Proposition 4. In this case, $X_B^\infty = \varnothing$. If the investigation interval $X_B(L_1, R_1)$ under the initial precedent is nonempty, then construct a sequence $\{a_n, b_n, L_n, R_n\}$ as follows. Given L_n and R_n, if $X_B(L_n, R_n) \neq \varnothing$, then let $a_n = a(L_n, R_n)$, $b_n = b(L_n, R_n)$ and pick L_{n+1} and R_{n+1} such that $L_n < L_{n+1} < a(L_n, R_n)$ and $b(L_n, R_n) < R_{n+1} < R_n$. If $X_B(L_n, R_n) = \varnothing$, then let $a_n = b_n = \frac{L_n + R_n}{2}$, $a_{n+1} = a_n$, $b_{n+1} = b_n$, $L_{n+1} = L_n$, $R_{n+1} = R_n$ and $X_B^\infty = \varnothing$. Note that a_n is increasing and b_n is decreasing. Since a monotone and bounded sequence converges, $\lim a_n$ and $\lim b_n$ are well defined. If $X_B(L_n, R_n) \neq \varnothing$ for all n, then let $X_B^\infty = (\lim a_n, \lim b_n)$. Note that a limit investigation interval under binding precedent may depend on the particular sequence $\{L_n, R_n\}$ we pick.

Recall that X_N is the set of cases that trigger an investigation under nonbinding precedent.

Proposition 6. *The DM investigates more under binding precedent than under non-binding precedent early on but investigates less under binding precedent in later periods. Specifically, for any limit investigation interval X_B^∞ under binding precedent, we have $X_B^\infty \subseteq X_N \subseteq X_B(L_1, R_1)$.*

5 Social welfare

Since binding precedent places constraints on what the DM can do in terms of her rulings, her payoff is clearly higher under nonbinding precedent than under binding precedent.[15] However, since the decisions may affect the society at large, as in the case

[15]If the DM is given the choice between the two institutions, then she would prefer the nonbinding precedent. A third alternative is for the DM to be given the right to decide on a case-by-case basis whether to make her ruling binding. Since the DM would not make the ruling on any case binding

of court rulings, the DM's payoff is not a good measure of social welfare in the presence of this externality. If the rest of the society does not bear the cost of information acquisition but cares about the rulings, then a reasonable measure of social welfare may be simply the payoffs coming from the ruling decisions. Formally, we define a social welfare function $V_N^S(x)$ under nonbinding precedent and a social welfare function $V_B^S(x, L, R)$ under binding precedent as follows.

Under nonbinding precedent, the optimal policy that the DM chooses is given by (μ_N^*, ρ_p^*). If $\mu_N^*(x) = 1$, then the current ruling as well as all future rulings are correct, and therefore $V_N^S(x) = 0$. If $\mu_N^*(x) = 0$, then the social welfare consists of the expected social cost from the potential mistake in ruling today as well the discounted continuation payoff EV_p^S. In this case,

$$V_N^S(x) \quad = \quad \rho^*(x) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) + (1-\rho^*(x)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_N^S.$$

Similarly, under binding precedent, if $\mu_B^*(x) = 1$, then $V_B^S(s) = A(s)$, and if $\mu_B^*(x) = 0$, then

$$\begin{aligned} V_B^S(s) \quad &= \quad \rho^*(s) \int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta) \\ &+ \quad (1-\rho^*(s)) \int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta) + \delta EV_B^S(\pi(s,\rho^*(s))). \end{aligned}$$

Let the expected social welfare under nonbinding precedent be $EV_N^S = \int_0^1 V_N^S(x')dG(x')$ and the expected social welfare under binding precedent be $EV_B^S = \int_0^1 V_B^S(L_1, R_1, x')dG(x')$. We next compare the social welfare under the two institutions.

5.1 Welfare comparison

We first discuss the special cases of $\delta = 0$ and $\delta = 1$ since welfare comparison is straightforward in these cases.

Consider first $\delta = 0$. Since the DM cares about only the current decision, the equilibrium investigation interval is the same whether precedent is binding or nonbind-

without learning θ first, the equilibrium outcome is the same as that under the institution of nonbinding precedent.

ing. Even though the equilibrium investigation decisions are the same under the two institutions, the rulings may be different after the DM learns θ. Under nonbinding precedent, she makes the correct ruling regarding x in all future periods, but does not necessarily do so under binding precedent if $\theta \notin (L, R)$. Hence, social welfare is higher under nonbinding precedent when $\delta = 0$.

One interpretation of $\delta = 0$ is that it represents a situation in which a new DM is appointed in each period and each DM cares about only her own decision. If the discount factor that enters the social welfare function is not 0, then social welfare is higher under nonbinding precedent. Intuitively, with short-sighted DMs, making the precedent binding does not provide stronger incentive for information acquisition and only results in distortions in rulings.

Next consider $\delta = 1$. In this case, the DM investigates all $x \in [\underline{\theta}, \overline{\theta}]$ in the first period and the equilibrium policies are the same under binding and nonbinding precedent.[16] It follows that the social welfare coincides under the two institutions.

In these special cases, the stronger incentive for the DM to acquire information early on is absent, and nonbinding precedent yields (weakly) higher social welfare than binding precedent. But the opposite comparision is possible too, as illustrated in the following example.

Example 1. *Suppose that θ is uniformly distributed on $[0.2, 0.8]$, x is uniformly distributed on $[0, 1]$, $\delta = 0.95$, $z = 0.1$ and $\ell(x, \theta) = |x - \theta|$.*

In this example, more cases trigger investigation in the first period under binding precedent than under nonbinding precedent, but as the precedents are established over time, the set of cases that trigger an investigation eventually becomes empty under binding precedent.

To provide a sharper characterization of which institution is more conducive to higher social welfare, we next consider a simple and tractable discrete framework.

5.2 Discrete framework

In this subsection, suppose x_t is uniformly distributed on $\{x_L, x_H\}$ and θ is uniformly distributed on $\{\theta_L, \theta_M, \theta_H\}$ with $\theta_L < x_L < \theta_M < x_H < \theta_H$. Assume that under

[16] To see this, first note that it is straightforward to show that the investigation interval under nonbinding precedent is $[\underline{\theta}, \overline{\theta}]$ as δ goes to 1. Also note that the investigation interval under binding precedent contains the investigation interval under nonbinding precedent in the first period for any δ by Proposition 6. Since no case outside $[\underline{\theta}, \overline{\theta}]$ is ever investigated, it follows that the investigation interval under binding precedent in the first period is also $[\underline{\theta}, \overline{\theta}]$.

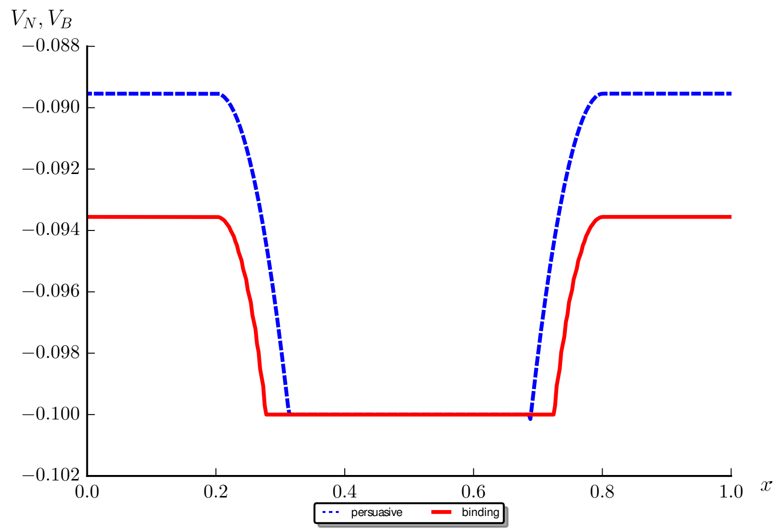

Figure 2: Comparison of the DM's dynamic payoffs

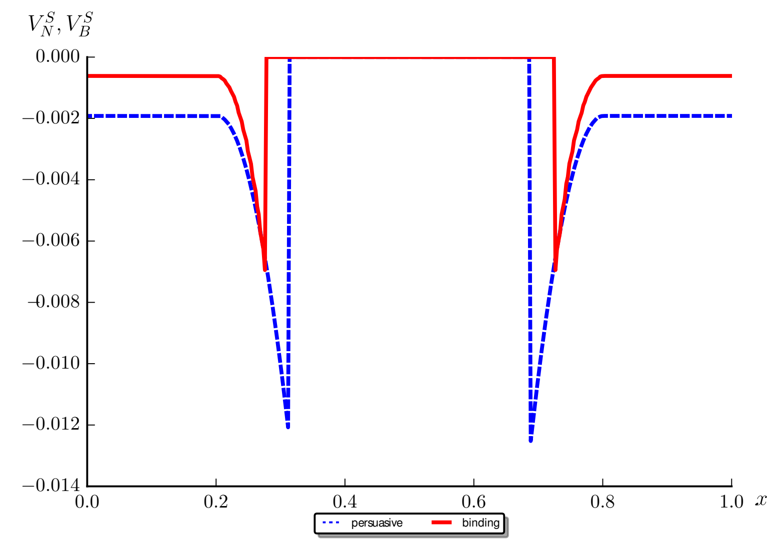

Figure 3: Comparison of social welfare

the prior, it is optimal to permit x_L and ban x_H. Let $c_L = \ell(x_L, \theta_L)$ and $c_H = \ell(x_H, \theta_H)$ and assume, without loss of generality, that $c_L \leq c_H$. As before, the initial precedent L_1 and R_1 are consistent with the DM's preferences, that is, $L_1 \leq \theta_L$ and $R_1 \geq \theta_H$.

The following lemma is an analog of Proposition 3, which characterizes the optimal policies under nonbinding precedent for the continuous framework. It also says that more cases trigger an investigation when the cost of investigation is lower.

Lemma 5. *There exists* $\underline{z}_N$ *and* $\bar{z}_N$ *such that under nonbinding precedent, if* $z \le \underline{z}_N$*, both* x_L *and* x_H *trigger investigation; if* $\underline{z}_N < z \le \bar{z}_N$*, only* x_H *triggers investigation; if* $z > \bar{z}_N$*, there is no investigation.*

A similar result holds for the binding precedent, in the sense that, in the first period, when the cost of investigation is low enough, both cases trigger investigation; when the cost of investigation is sufficiently high, there is no investigation; and in between, only x_H triggers investigation. The next result formalizes this observation, and establishes a comparison of the cost cutoffs relative to nonbinding precedent.

Lemma 6. *There exists* $\underline{z}_B$ *and* $\bar{z}_B$ *such that in the first period under binding precedent, if* $z \le \underline{z}_B$*, both* x_L *and* x_H *trigger investigation; if* $\underline{z}_B < z \le \bar{z}_B$*, only* x_H *triggers investigation; if* $z > \bar{z}_B$*, there is no investigation. Furthermore,* $\bar{z}_N = \bar{z}_B$ *and* $\underline{z}_N < \underline{z}_B$*.*

Note that when $z \le \underline{z}_N$, both cases trigger investigation immediately under either institution, and when $z > \bar{z}_B = \bar{z}_N$, there is no investigation under either institution. In these cases, the social welfare is the same under both institutions. By contrast, social welfare is different under the two institutions when $z \in (\underline{z}_N, \underline{z}_B]$, or when $z \in (\underline{z}_B, \bar{z}_N]$.

When $z \in (\underline{z}_N, \underline{z}_B]$, both cases trigger investigation immediately under binding precedent but only x_H triggers investigation under nonbinding precedent. As a result, the social welfare under binding precedent is higher than the social welfare under nonbinding precedent due to more information acquisition.

When $z \in (\underline{z}_B, \bar{z}_N]$, only x_H triggers investigation in the first period under both institutions. Consequently, if x_H is drawn in the first period, then the payoff is the same under both institutions. If x_L is drawn in the first period, however, the DM permits it without investigation, and the precedent becomes $L_2 = x_L$, $R_2 = R_1$. Under nonbinding precedent, she continues to permit x_L without investigation until x_H is drawn. At that point, she investigates and decides all cases correctly from then on. By contrast, under binding precedent, when x_H is drawn, depending on the parameter values, it may or it may not trigger investigation. If it does not trigger investigation, social welfare is lower under binding precedent due to less information acquisition. If it triggers investigation, social welfare is still lower under binding precedent because the DM cannot make full use of the information: if the investigation reveals that $\theta = \theta_L$, it is socially optimal to ban x_L but the DM has to follow the precedent and permit x_L. The following proposition summarizes these observations.

Proposition 7. *If the cost of information acquisition is sufficiently low or sufficiently high, then welfare is the same under two institutions; otherwise, it is different. Formally, (i) If $z \leq \underline{z}_N$ or $z > \bar{z}_N = \bar{z}_B$, then welfare is the same under nonbinding precedent and binding precedent. (ii) If $\underline{z}_N < z \leq \underline{z}_B$, then welfare is higher under binding precedent. (iii) If $\underline{z}_B < z \leq \bar{z}_N = \bar{z}_B$, then welfare is higher under nonbinding precedent.*

6 Conclusion

We have analyzed a simple model designed to capture an important consequence of the authority of the precedent, a common institution which can make violating previously established rulings costly. The main insight obtained is that this cost can serve to motivate decision makers to acquire information more intensively early on and therefore can be socially beneficial. There are many ways to enrich and extend the model; as concluding remarks, we discuss some promising directions.

Partial learning. The information structure we assumed so far is stark: an investigation reveals θ perfectly. We relax this assumption by considering partial learning. As in Baker and Mezzetti [2012], we now assume that an investigation reveals only whether it is optimal to permit or ban the case at hand, which is informative about the value of θ, but may not perfectly reveal it. With partial learning, the *informational state* is much richer than before,[17] thereby complicating the analysis significantly. For tractability, we continue to use the discrete framework introduced in the preceding section. In this setting, if the DM investigates when the current case is x_L and finds that it should be banned, she infers that $\theta = \theta_L$, but if she finds that it should be permitted, she infers that $\theta \in \{\theta_M, \theta_H\}$. Similarly, if the DM investigates when the current case is x_H and finds that it should be permitted, she infers that $\theta = \theta_H$, but if she finds that it should be permitted, she infers that $\theta \in \{\theta_L, \theta_M\}$. In Appendix B, we show that the analogs of Lemma 5, Lemma 6 and Proposition 7 hold in this partial learning setting, demonstrating the robustness of our results.

Different decision makers in different periods. We have considered a single long-lived DM and the only externality is the social implications of the rulings. If different DMs make decisions in different periods, then the model becomes a stochastic game,

[17] Recall that under perfect learning, the DM either knows θ perfectly or her belief about θ is the same as her prior.

and new complications arise. The smallest departure from our model would be a setting in which the DMs have the same preferences regarding the rulings (a common θ) and the information learned about θ from one DM's investigation is passed down to future DMs (for example, through written opinions), but the information acquisition cost is still private.[18] Since information acquisition is costly, the DMs have incentives to free-ride. But since this free-riding incentive is present under both nonbinding precedent and binding precedent, our main point that the current DM has a stronger incentive to acquire information under binding precedent than under nonbinding precedent in earlier periods but a weaker incentive to do so in later periods still holds. Even though this framework is a game rather than a single-agent decision problem, we can still formulate it in a way that allows us to apply contraction mapping theorem to establish existence of a unique equilibrium. Our numerical analysis shows that the main results are robust in this extension.[19]

A more significant departure is a setting in which the DMs have different preferences regarding outcomes, perhaps because of different ideological leanings. Then, under binding precedent, a DM may distort her decision in the direction that she favors so as to tie the hands of future DMs having different preferences than hers. This kind of inefficiency does not arise if precedents have only nonbinding influence.

Precedents as organizational memory In their seminal 1963 book, Cyert and March point out that "organizations have memories in the form of precedents, and individuals in the coalition are strongly motivated to accept the precedents as binding." And as observed by Levitt and March [1988], the details of the future path of an organization "depend significantly on the processes by which the memory is maintained and consulted." As such, how precedents shape the direction of an organization depends on the processes through which they are conserved and retrieved. This raises a number of interesting questions. For example, how does the technology of conservation and retrieval of precedents affect the performance of the organization? Is it optimal to have atrophy of organizational memory? Specifically, should more recent precedents have more authority?

Evolving preferences. We have assumed that θ is fixed over time, reflecting stable

[18]In section 5.1, we discussed the case of $\delta = 0$, which can be interpreted as modeling short-lived DMs who care only about their own decisions. Here we consider short-lived DMs who also care about future decisions made by other DMs.

[19]We omit the details here, but the uniqueness result and the numerical analysis are available upon request.

preferences. However, for certain matters, views on gay marriage for instance, social preferences may change over time. The rigidity of binding precedent may delay an organization's response to such changes in preferences. Our model does not capture the kind of inefficiency that arises in this case under binding precedent.

Varying degree of bindingness. We have compared two opposite cases: either it is costless to violate a precedent (the nonbinding case) or it is infinitely costly to do so (the binding case). In practice, it may be possible to violate precedent by incurring some cost. This cost may not be so large as to completely deter a DM from going against a previous ruling when new information makes it clear that doing so is socially beneficial.[20] These intermediate cases can be analyzed by extending our model to allow a richer cost structure associated with violating precedent, reflecting the varying degrees of bindingness that exist in different institutions. This richer structure permits the exploration of such institutional design questions as the optimal degree of bindingness.

Dynamic delegation. Implicitly in our model, the DM is delegated to make decisions regarding cases that arise over time. Our analysis of binding precedents shows that imposing history-dependent constraints on the set of feasible actions for the DM can improve social welfare. A broader question is what would be the optimal delegation mechanism in settings like ours. This opens up a new line of research since the literature on optimal delegation has mostly focused on static settings (see, for example, the seminal paper by Holmstrom [1984] and the recent works by Alonso and Matouschek [2008], Armstrong and Vickers [2010] and Amador and Bagwell [2013]). Recently, new models have been developed to study delegation in dynamic settings (see, for example, Li, Matouschek, and Powell [2017] and Lipnowski and Ramos [2017]). However, these papers consider a conflict of interest between the principal and the agent in terms of the difference in preferred actions but not in terms of costly information acquisition. How to use the dynamic link across decisions to design the optimal delegation mechanism to motivate information acquisition is an exciting question. We plan to explore this and the other interesting questions discussed above in future research.

[20] The conflict between *stare decisis* and the cost of judicial error has been recognized and studied by legal scholars. See, for example, Lash [2014].

Appendix A: Proofs

Proof of Lemma 2: Consider period 3 first. If $x_3 < \underline{\theta}$, then the DM permits x_3 without an investigation; if $x_3 > \bar{\theta}$, then the DM bans x_3 without an investigation. For $x_3 \in [\underline{\theta}, \hat{x})$, if the DM does not investigate, then she permits the case by Lemma 1 and her expected payoff is $\int_{\underline{\theta}}^{x_3} -\ell(x_3,\theta)dF(\theta)$, which is decreasing in x_3. For $x_3 \in (\hat{x}, \bar{\theta}]$, if the DM does not investigate, then she bans the case by Lemma 1 and her expected payoff is $\int_{x_3}^{\bar{\theta}} -\ell(x_3,\theta)dF(\theta)$, which is increasing in x_3.

Let $z^* = \int_{\underline{\theta}}^{\hat{x}} \ell(\hat{x},\theta)dF(\theta) > 0$. If $z > z^*$, then $X_3^N = \varnothing$. If $z \leq z^*$, then $X_3^N = [\underline{x}_3, \bar{x}_3]$ where $\underline{x}_3 < \hat{x}$ and $\bar{x}_3 > \hat{x}$ satisfy $\int_{\underline{\theta}}^{\underline{x}_3} \ell(\underline{x}_3,\theta)dF(\theta) = \int_{\bar{x}_3}^{\bar{\theta}} \ell(\bar{x}_3,\theta)dF(\theta) = z$.

Let EV_t^N be the expected continuation payoff of the uninformed DM in period t. We have

$$EV_3^N = \int_{\underline{\theta}}^{\underline{x}_3} \int_{\underline{\theta}}^{x} -\ell(x,\theta)dF(\theta)dG(x) + \int_{\bar{x}_3}^{\bar{\theta}} \int_{x}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)dG(x) - [G(\bar{x}_3) - G(\underline{x}_3)]z > -z.$$

Now consider period 2. For any $x_2 \in [0,1]$, the continuation payoff for the uninformed DM if she investigates is $-z$. For $x_2 \notin [\underline{\theta}, \overline{\theta}]$, since the DM's expected payoff is $\delta EV_3^N > -\delta z$ if she does not investigate and $-z$ if she investigates, it is optimal for her not to investigate.

Consider $x_2 \in [\underline{\theta}, \hat{x})$ and suppose the DM does not investigate. Since she permits such a case, her expected payoff in period 2 is $\int_{\underline{\theta}}^{x_2} -\ell(x_2,\theta)dF(\theta)$. Similarly, for $x_2 \in (\hat{x}, \overline{\theta}]$, if the DM does not investigate, she bans x_2 and her expected payoff in period 2 is $\int_{x_2}^{\bar{\theta}} -\ell(x_2,\theta)dF(\theta)$.

Now consider the DM's optimal investigation policy in period 2. For $x_2 \in [\underline{\theta}, \hat{x})$, it is optimal for the DM to investigate in period 2 iff

$$-z \geq \int_{\underline{\theta}}^{x_2} -\ell(x_2,\theta)dF(\theta) + \delta EV_3^N.$$

Similarly, for $x_2 \in (\hat{x}, \overline{\theta}]$, it is optimal for the DM to investigate in period 2 iff

$$-z \geq \int_{x_2}^{\bar{\theta}} -\ell(x_2,\theta)dF(\theta) + \delta EV_3^N.$$

Hence, if $z > z^* - \delta EV_3^N$, then $X_2^N = \varnothing$. If $z < z^* - \delta EV_3^N$, then $X_2^N = [\underline{x}_2, \bar{x}_2]$ where

$\underline{x}_2$ and $\bar{x}_2$ satisfy

$$-z = \int_{\underline{\theta}}^{\underline{x}_2} -\ell(\underline{x}_2, \theta) dF(\theta) + \delta EV_3^N = \int_{\bar{x}_2}^{\bar{\theta}} -\ell(\bar{x}_2, \theta) dF(\theta) + \delta EV_3^N.$$

Since $EV_3^N < 0$, we have $\underline{x}_2 \in (\underline{\theta}, \underline{x}_3)$ and $\bar{x}_2 \in (\bar{x}_3, \bar{\theta})$, which implies that $X_3^N \subseteq X_2^N$.

It also follows that

$$EV_2^N = \int_{\underline{\theta}}^{\underline{x}_2} \left[\int_{\underline{\theta}}^{x} -\ell(x, \theta) dF(\theta) + \delta EV_3^N \right] dG(x) + \int_{\bar{x}_2}^{\bar{\theta}} \left[\int_{x}^{\bar{\theta}} -\ell(x, \theta) dF(\theta) + \delta EV_3^N \right] dG(x) - [G(\bar{x}_2) - G(\underline{x}_2)] z.$$

Note that

$$EV_3^N = \max_{a,b \in [\underline{\theta}, \bar{\theta}], b > a} \int_{\underline{\theta}}^{a} \int_{\underline{\theta}}^{x} -\ell(x, \theta) dF(\theta) dG(x) + \int_{b}^{\bar{\theta}} \int_{x}^{\bar{\theta}} -\ell(x, \theta) dF(\theta) dG(x) - [G(b) - G(a)] z,$$

and $EV_3^N < 0$. It follows that $EV_2^N < EV_3^N < 0$.

Now consider period 1. By a similar argument as in period 2, if $z > z^* - \delta EV_2^N$, then $X_1^N = \varnothing$; if $z < z^* - \delta EV_2^N$, then $X_1^N = [\underline{x}_1, \bar{x}_1]$ where $\underline{x}_1$ and $\bar{x}_1$ satisfy

$$-z = \int_{\underline{\theta}}^{\underline{x}_1} -\ell(\underline{x}_1, \theta) dF(\theta) + \delta EV_2^N = \int_{\bar{x}_1}^{\bar{\theta}} -\ell(\bar{x}_1, \theta) dF(\theta) + \delta EV_2^N.$$

Since $EV_2^N < EV_3^N$, it follows that $\underline{x}_1 \in (\underline{\theta}, \underline{x}_2)$ and $\bar{x}_1 \in (\bar{x}_2, \bar{\theta})$ and therefore $X_2^N \subseteq X_1^N$. ∎

Proof of Lemma 3: Consider period 3 first. Recall that under nonbinding precedent, the investigation interval is X_3^N. Since under binding precedent, investigation has no value if $x_3 \leq L_3$ or if $x_3 \geq R_3$, it follows that x_3 triggers an investigation if and only if $x_3 \in X_3^N \cap (L_3, R_3)$. Hence, $X_3^B(L_3, R_3)$ is an interval and $X_3^B(L_3, R_3) \subseteq (L_3, R_3)$.

Let $k(L, R)$ denote the DM's expected payoff in period t under binding precedent when the precedents are (L, R) conditional on θ being known where the expectation is taken over θ before it is revealed and over all possible cases x. Formally

$$k(L, R) = \left[\int_{\mathcal{L}} \int_{\theta}^{L} -\ell(x, \theta) dG(x) dF(\theta) + \int_{\mathcal{R}} \int_{R}^{\theta} -\ell(x, \theta) dG(x) dF(\theta) \right]$$

where $\mathcal{L}$ is the (possibly degenerate) interval $[\underline{\theta}, \max\{L, \underline{\theta}\}]$ and $\mathcal{R}$ is the (possibly degenerate) interval $[\min\{R, \bar{\theta}\}, \bar{\theta}]$. That is, $k(L, R) = (1 - \delta)C(L, R)$ where $C(L, R)$ is defined on page 15. Note that $k(L, R)$ is decreasing in L and increasing in R; $k(L, R) \leq 0$ and $k(L, R) < 0$ if $L > \underline{\theta}$ or if $R < \bar{\theta}$.

To prove Lemma 3 for periods 1 and 2, we first establish Claim 1 below. Let $EV_t^B(L, R)$ denote the uninformed DM's expected equilibrium continuation payoff in period t under binding precedent given that the precedent in period t is (L, R).

Claim 1. *If $EV_t^B(L, R)$ is decreasing in L and increasing in R, then the set of cases that trigger an investigation in period $t - 1$ is an interval for any precedent in period $t - 1$.*

Proof: Suppose that $EV_t^B(L, R)$ is decreasing in L and increasing in R. Fix precedent in period $t - 1$ by (L_{t-1}, R_{t-1}). Suppose that x' and $x'' > x'$ are in $X_{t-1}^B(L_{t-1}, R_{t-1})$. We next show that any $\tilde{x} \in [x', x'']$ is also in $X_{t-1}^B(L_{t-1}, R_{t-1})$.

Let $g^p(L, R, x)$ be the DM's current-period payoff if she permits the case without investigation in state $s = (L, R, x)$ and $g^b(s)$ be her current-period payoff if she bans the case without investigation in state s. Note that for any (L, R), $g^p(L, R, x)$ is decreasing in x and $g^b(L, R, x)$ is increasing in x.

Suppose $\tilde{x} \in (L_{t-1}, R_{t-1})$. If the DM investigates, then her continuation payoff is $-z + \delta k(L_{t-1}, R_{t-1})$. Suppose, without loss of generality, that $\tilde{x} < \hat{x}$ so that it is optimal for the uninformed DM to permit $\tilde{x}$. Since $x' \in X_{t-1}^B(L_{t-1}, R_{t-1})$, we have $-z + \delta k(L_{t-1}, R_{t-1}) \geq g^p(L_{t-1}, R_{t-1}, x') + \delta EV_t^B(\max\{x', L_{t-1}\}, R_{t-1})$. Since g^p is decreasing in x, it follows that $g^p(L_{t-1}, R_{t-1}, x') > g^p(L_{t-1}, R_{t-1}, \tilde{x})$. Moreover, since $\hat{x} > \max\{x', L_{t-1}\}$ and EV_t^B is decreasing in L, we have $EV_t^b(\max\{x', L_{t-1}\}, R_{t-1}) > EV_t^B(\hat{x}, R_{t-1})$. Hence, we have $-z+\delta k(L_{t-1}, R_{t-1}) \geq g^p(L_{t-1}, R_{t-1}, \hat{x})+\delta EV_t^B(\hat{x}, R_{t-1})$, which implies that it is optimal for the DM to investigate when the case is $\tilde{x}$.

Suppose $\tilde{x} \leq L_{t-1}$. Then the DM has to permit $\tilde{x}$ regardless of whether she investigates or not. Hence, $\tilde{x}$ triggers an investigation iff $-z + \delta k(L_{t-1}, R_{t-1}) \geq \delta EV_t^B(L_{t-1}, R_{t-1})$. Since $x' \in X_{t-1}^B(L_{t-1}, R_{t-1})$ and $x' < \tilde{x}$, we have $-z+\delta k(L_{t-1}, R_{t-1}) \geq \delta EV_t^B(L_{t-1}, R_{t-1})$, implying that $\tilde{x} \in X_{t-1}^B(L_{t-1}, R_{t-1})$. A similar argument shows that $\tilde{x} \in X_{t-1}^B(L_{t-1}, R_{t-1})$ if $\tilde{x} \geq R_{t-1}$ as well. Hence, $X_{t-1}^B(L_{t-1}, R_{t-1})$ is an interval for any (L_{t-1}, R_{t-1}). ∎

We next show that $EV_3^B(L, R)$ is decreasing in L and increasing in R. Consider precedents (L_3, R_3) and $(\hat{L}_3, \hat{R}_3)$ such that $\hat{L}_3 \leq L_3$ and $\hat{R}_3 \geq R_3$. As shown before, under the precedent (L_3, R_3), it is optimal for the DM to investigate iff $x_3 \in (L_3, R_3) \cap X_3^N$. By following the same policy under precedent $(\hat{L}_3, \hat{R}_3)$, the DM receives the same payoff as under precedent (L_3, R_3). Hence, $EV_3^B(L_3, R_3) \leq EV_3^B(\hat{L}_3, \hat{R}_3)$, and it follows from Claim 1 that $X_2^B(L_2, R_2)$ is an interval. Consider $x_2 \notin (L_2, R_2)$. It triggers an investigation iff $-z + \delta k(L_2, R_2) \geq \delta EV_3^B(L_2, R_2)$. Since $EV_3^B(L_2, R_2) \geq -z + k(L_2, R_2)$, it follows that $\delta EV_3^B(L_2, R_2) > -z + \delta k(L_2, R_2)$ and therefore x_2 does not trigger an investigation. Hence, $X_2^B(L_2, R_2) \subseteq (L_2, R_2)$.

We next show that $EV_2^B(L, R)$ is decreasing in L and increasing in R. Consider precedents (L_2, R_2) and $(\hat{L}_2, \hat{R}_2)$ such that $\hat{L}_2 \leq L_2$ and $\hat{R}_2 \geq R_2$. We show that if the DM follows the same policy under precedent $(\hat{L}_2, \hat{R}_2)$ as the optimal policy under (L_2, R_2), then her continuation payoff is higher under precedent $(\hat{L}_2, \hat{R}_2)$ than under (L_2, R_2). Consider first $x_2 \in X_2^B(L_2, R_2)$. Then $x_2 \in (L_2, R_2)$, and in this case, the DM's continuation payoff is $-z$ under either (L_2, R_2) or $(\hat{L}_2, \hat{R}_2)$. Next consider $x_2 \notin X_2^B(L_2, R_2)$. Without loss of generality, suppose $x_2 \leq \hat{x}$, which implies that the uniformed DM permits x_2. In this case, the precedent in period 3 becomes $(\max\{x_2, L_2\}, R_2)$. If the DM follows the same policy under precedent $(\hat{L}_2, \hat{R}_2)$, then the precedent in period 3 becomes $(\max\{x_2, \hat{L}_2\}, \hat{R}_2)$. Since $EV_3^B(L, R)$ is decreasing in L and increasing in R, we have $EV_3^B(\max\{x_2, L_2\}, R_2) \leq EV_3^B(\max\{x_2, \hat{L}_2\}, \hat{R}_2)$. Since the DM's payoff in period 2 is the same under either (L_2, R_2) or $(\hat{L}_2, \hat{R}_2)$, it follows that her continuation payoff in period 2 is higher under precedent $(\hat{L}_2, \hat{R}_2)$ than under (L_2, R_2). A similar argument shows that the result holds for $x_2 > \hat{x}$. Hence, $EV_2^B(L, R)$ is decreasing in L and increasing in R. Claim 1 then implies that $X_1^B(L_1, R_1)$ is an interval. Since $L_1 = 0$, $R_1 = 1$ by assumption, we have $X_1^B(L_1, R_1) \subseteq (L_1, R_1)$. ∎

Proof of Proposition 1: Consider period 3 first. As shown in the proof of Lemma 3, under binding precedent, x_3 triggers an investigation iff $x_3 \in X_3^N \cap (L_3, R_3)$.

Now consider period 2. First suppose $[\underline{\theta}, \bar{\theta}] \subseteq [L_2, R_2]$. Then the incentive of the DM in period 2 is the same under binding precedent as under nonbinding precedent. In this case, $X_2^B(L_2, R_2) = X_2^N$. Next suppose $[\underline{\theta}, \bar{\theta}] \not\subseteq [L_2, R_2]$. Recall that if $X_t^N \neq \varnothing$, then $X_t^N = [\underline{x}_t, \bar{x}_t]$. We show below that under binding precedent, $\underline{x}_2$ does not trigger an investigation.

Recall that the DM is indifferent between investigating and not investigating when $x_2 = \underline{x}_2$ in period 2 under nonbinding precedent. That is, we have

$$- z = \int_{\underline{\theta}}^{\underline{x}_2} -\ell(\underline{x}_2, \theta) dF(\theta) + \delta k(\underline{x}_3, \bar{x}_3) - \delta z[G(\bar{x}_3) - G(\underline{x}_3)]. \quad \text{(A1)}$$

Consider binding precedent. If $\underline{x}_2 \notin (L_2, R_2)$, then $\underline{x}_2$ does not trigger an investigation, as shown in Lemma 3. Next consider $\underline{x}_2 \in (L_2, R_2)$. Without loss of generality, suppose that $\underline{x}_2 \leq \hat{x}$, which implies that the uninformed DM would permit $\underline{x}_2$, resulting in $L_3 = \underline{x}_2$ and $R_3 = R_2$. Since $X_3^B(L_3, R_3) = (L_3, R_3) \cap [\underline{x}_3, \bar{x}_3]$, the difference in the DM's continuation payoff between investigating and not investigating when the case is $\underline{x}_2$ is

$$\begin{aligned}&-z + \delta k(L_2, R_2)\\ &- \left[\int_{\underline{\theta}}^{\underline{x}_2} -\ell(\underline{x}_2, \theta) dF(\theta) + \delta k(\max\{\underline{x}_2, \underline{x}_3\}, \min\{R_2, \bar{x}_3\}) - \delta z[G(\min\{R_2, \bar{x}_3\}) - G(\max\{\underline{x}_2, \underline{x}_3\})] \right].\end{aligned}$$

Since $\max\{\underline{x}_2, \underline{x}_3\} = \underline{x}_3$, this is equal to

$$-z - \int_{\underline{\theta}}^{\underline{x}_2} -\ell(\underline{x}_2, \theta) dF(\theta) + \delta[k(L_2, R_2) - k(\underline{x}_3, \min\{R_2, \bar{x}_3\})] + \delta z[G(\min\{R_2, \bar{x}_3\}) - G(\underline{x}_3)].$$

Substituting for $-z$ from (A1), the difference in the DM's continuation payoff between investigating and not investigating when $x_2 = \underline{x}_2$ is

$$\delta[k(\underline{x}_3, \bar{x}_3) + k(L_2, R_2) - k(\underline{x}_3, \min\{R_2, \bar{x}_3\})] - \delta z[G(\bar{x}_3) - G(\min\{R_2, \bar{x}_3\})]$$

If $\bar{x}_3 \leq R_2$, then $k(\underline{x}_3, \bar{x}_3) + k(L_2, R_2) - k(\underline{x}_3, \min\{R_2, \bar{x}_3\}) = k(L_2, R_2) < 0$; if $\bar{x}_3 > R_2$, then $k(\underline{x}_3, \bar{x}_3) + k(L_2, R_2) - k(\underline{x}_3, \min\{R_2, \bar{x}_3\}) = k(L_2, \bar{x}_3) < 0$. Note also that $G(\bar{x}_3) - G(\min\{R_2, \bar{x}_3\}) \geq 0$. Hence, the DM's payoff is higher by not investigating when $x_2 = \underline{x}_2$ and therefore $\underline{x}_2$ does not trigger an investigation under binding precedent (L_2, R_2).

A similar argument establishes that under binding precedent, $\bar{x}_2$ does not trigger an investigation in period 2. Since both X_2^N and $X_2^B(L_2, R_2)$ are intervals, it follows that $X_2^B(L_2, R_2) \subseteq X_2^N$.

Now consider period 1. Recall that if $X_1^N \neq \varnothing$, then $X_1^N = [\underline{x}_1, \bar{x}_1]$. We show below that $\underline{x}_1$ triggers an investigation under binding precedent. Recall that the DM is indifferent between investigating and not investigating when $x_1 = \underline{x}_1$ under nonbinding precedent. That is,

$$
\begin{aligned}
-z &= \int_{\underline{\theta}}^{\underline{x}_1} -\ell(\underline{x}_1, \theta) dF(\theta) + \delta EV_2^N \\
&= \int_{\underline{\theta}}^{\underline{x}_1} -\ell(\underline{x}_1, \theta) dF(\theta) + \delta k(\underline{x}_2, \bar{x}_2) + \delta^2 [1 - G(\bar{x}_2) + G(\underline{x}_2)] k(\underline{x}_3, \bar{x}_3) \\
&- \delta z [G(\bar{x}_2) - G(\underline{x}_2) + \delta(G(\bar{x}_3) - G(\underline{x}_3))]
\end{aligned}
$$

Under binding precedent, if the DM investigates in period 1, her continuation payoff is $-z$; if the DM does not investigate, her continuation payoff is $\int_{\underline{\theta}}^{\underline{x}_1} -\ell(\underline{x}_1, \theta) dF(\theta) + \delta EV_2^B(\underline{x}_1, R_1)$. Note that $EV_2^B(\underline{x}_1, R_1) < V_2^N$ since the DM can follow the same policy under nonbinding precedent as the optimal policy under binding precedent and receive a higher payoff. Hence, $\underline{x}_1$ triggers an investigation in period 1 under binding precedent.

A similar argument establishes that under binding precedent, $\bar{x}_1$ triggers an investigation in period 1. Since both X_1^N and $X_1^B(L_1, R_1)$ are intervals, it follows that $X_1^N \subseteq X_1^B(L_1, R_1)$. ∎

Proof of Proposition 2: Let $\mathcal{F}$ denote the set of bounded measurable functions on S taking values in $\mathbb{R}$. For $f \in \mathcal{F}$, let $||f|| = \sup\{|f(s)| : s \in S\}$. An operator $Q : \mathcal{F} \to \mathcal{F}$ satisfies the contraction property for $|| \cdot ||$ if there is a $\beta \in (0, 1)$ such that for $f^1, f^2 \in \mathcal{F}$, we have $||Q(f^1) - Q(f^2)|| \leq \beta ||f^1 - f^2||$. For any operator Q that satisfies the contraction property, there is a unique $f \in V$ such that $Q(f) = f$.

We prove the proposition for binding precedent. A similar and less involved argument shows uniqueness under nonbinding precedent as well.

Let $g^p(s)$ be the DM's current-period payoff if she permits the case without investigation in state s and $g^b(s)$ be her current period payoff if she bans the case without investigation in state s. Formally,

$$
g^p(s) = \begin{cases} \int_{\underline{\theta}}^{\max\{x, \underline{\theta}\}} -\ell(x, \theta) dF(\theta) & \text{if } x < R, \\ -\infty & \text{if } x \geq R, \end{cases}
$$

$$
g^b(s) = \begin{cases} \int_{\min\{x, \bar{\theta}\}}^{\bar{\theta}} -\ell(x, \theta) dF(\theta) & \text{if } x > L, \\ -\infty & \text{if } x \leq L. \end{cases}
$$

For any $V \in \mathcal{F}$ and $(L, R) \in S^e$, let $EV(L, R) = \int_0^1 V(L, R, x') dG(x')$.

For $V \in \mathcal{F}$ and any $s \in S$, define

$$TV(s) = \max\{-z + A(s), g^p(s) + \delta EV(\max\{x, L\}, R), g^b(s) + \delta EV(L, \min\{x, R\})\}. \tag{A2}$$

Note that V_B^* as defined in (B3) satisfies $V_B^* = TV_B^*$.

Suppose that $V^1, V^2 \in \mathcal{F}$ and consider any $s \in S^e \times [0,1]$. Without loss of generality, suppose that $TV^1(s) \geq TV^2(s)$. There are three cases to consider.

(i) Suppose that $TV^1(s) = -z + A(s)$. Since $TV^1(s) \geq TV^2(s)$, we have $TV^2(s) = -z + A(s)$. It follows that $TV^1(s) - TV^2(s) = 0$.

(ii) Suppose that $TV^1(s) = g^p(s) + \delta EV^1(\max\{L, x\}, R)$. We have

$$\begin{aligned}
|TV^1(s) - TV^2(s)| &\leq g^p(s) + \delta EV^1(\max\{L, x\}, R) - g^p(s) - \delta EV^2(\max\{L, x\}, R) \\
&\leq \delta \int_0^1 \left[V^1(\max\{L, x\}, R, x') - V^2(\max\{L, x\}, R, x')\right] dG(x') \\
&\leq \delta \int_0^1 \left[|V^1(\max\{L, x\}, R, x') - V^2(\max\{L, x\}, R, x')|\right] dG(x')] \\
&\leq \delta ||V^1 - V^2||.
\end{aligned}$$

(iii) Suppose that $TV^1(s) = g^b(s) + \delta EV^1(L, \min\{x, R\})$. Then a similar argument as in case (ii) shows that $|TV^1(s) - TV^2(s)| \leq \delta ||V^1 - V^2||$.

Since either $|TV^1(s) - TV^2(s)| = 0$ or $|TV^1(s) - TV^2(s)| \leq \delta ||V^1 - V^2||$ for any $s \in S$ in all three cases, we have $||TV^1 - TV^2|| \leq \delta ||V^1 - V^2||$ and therefore T is a contraction. Since T is a contraction, it has a unique fixed point. Since V_B^* as defined in (B3) satisfies $TV_B^* = V_B^*$, there is a unique V_B^*. Once we solve for V_B^*, (B1) and (B2) determine the optimal policies ρ_B^* and μ_B^* uniquely. ∎

Proof of Lemma 4: Suppose x' and x'' trigger an investigation. Then, by (N1) and (N2), we have

$$-z \geq \max\left\{\int_{\underline{\theta}}^{\max\{x, \underline{\theta}\}} -\ell(x, \theta) dF(\theta), \int_{\min\{x, \bar{\theta}\}}^{\bar{\theta}} -\ell(x, \theta) dF(\theta)\right\} + \delta EV_N^*.$$

for $x \in \{x', x''\}$.

Suppose $\tilde{x} \in [x', x'']$. Since $\int_{\underline{\theta}}^{\max\{x,\underline{\theta}\}} -\ell(x,\theta)dF(\theta)$ is decreasing in x, we have

$$\int_{\underline{\theta}}^{\max\{\tilde{x},\underline{\theta}\}} -\ell(\tilde{x},\theta)dF(\theta) \leq \int_{\underline{\theta}}^{\max\{x',\underline{\theta}\}} -\ell(x',\theta)dF(\theta).$$

Since $\int_{\min\{x,\bar{\theta}\}}^{\bar{\theta}} -\ell(x,\theta)dF(\theta)$ is increasing in x, we have

$$\int_{\min\{\tilde{x},\bar{\theta}\}}^{\bar{\theta}} -\ell(\tilde{x},\theta)dF(\theta) \leq \int_{\min\{x'',\bar{\theta}\}}^{\bar{\theta}} -\ell(x'',\theta)dF(\theta).$$

It follows that

$$-z \geq \max\left\{\int_{\underline{\theta}}^{\max\{\tilde{x},\underline{\theta}\}} -\ell(\tilde{x},\theta)dF(\theta), \int_{\min\{\tilde{x},\bar{\theta}\}}^{\bar{\theta}} -\ell(\tilde{x},\theta)dF(\theta)\right\} + \delta EV_N^*$$

and therefore $\tilde{x}$ triggers an investigation. ∎

Proof of Proposition 3: We first show that if $z > z^*$, then $\rho_N^*(x) = 1$ iff $x \leq \hat{x}$, $\mu_N^*(x) = 0$ for any $x \in [0,1]$, and $EV_N^* = \frac{\lambda}{1-\delta}$. Condition $(N1)$ is clearly satisfied given the definition of $\hat{x}$. Condition $(N2)$ is satisfied since for any x, the payoff from not investigating is at least as high as $-\hat{z} + \delta EV_N^*$, which is greater than $-z$ if $z > \hat{z} - \delta EV_N^* = z^*$. And $(N3)$ is satisfied by plugging in ρ_N^*, μ_N^* and EV_N^*.

Note that if $z < z^*$, then $\mu_N^*(x) = 0$ for any $x \in [0,1]$ would violate $(N2)$. Hence, $X_N = \varnothing$ if and only if $z > z^*$.

We next show that $X_N \subset (\underline{\theta}, \bar{\theta})$. If $X_N = \varnothing$, then it is clearly true. We next show by contradiction that if $X_N \neq \varnothing$, then $\underline{\theta} < a_N < b_N < \bar{\theta}$. Suppose $a_N \leq \underline{\theta}$. Consider $x = a_N$. Since $a_N \leq \underline{\theta}$, the DM's dynamic payoff equals $-z$ if she investigates, and equals δEV_N^* if she does not investigate. Since $a_N = \inf\{x : \mu^*(x) = 1\}$, it follows that $-z \geq \delta EV_N^*$. Note that for any $x > \bar{\theta}$, the DM's dynamic payoff is $-z$ if she investigates, and δEV_N^* if she does not investigate. Hence, it must be the case that any case $x > \bar{\theta}$ triggers an investigation. It follows that $b_N = 1$. Moreover, since $a_N \leq \underline{\theta}$, the DM makes the correct decision for any case $x \leq a_N$. It follows that

$$EV_N^* = \int_0^{a_N} \delta EV_N^* dG(x) - z(1 - G(a_N)) = \delta G(a_N) EV_N^* - z(1 - G(a_N)).$$

Since $-z \geq \delta EV_N^*$, we have $EV_N^* > \delta EV_N^*$, but this is impossible since $EV_N^* < 0$. Hence, we have $a_N > \underline{\theta}$. A similar argument shows that $b_N < \bar{\theta}$.

We next prove that the uninformed DM permits $x < a_N$ and bans $x > b_N$. Note that the left-hand-side of ($N2$) achieves its minimum at $\hat{x}$, which implies that if $X_N \neq \varnothing$, then $\hat{x} \in X_N$, that is, $a_N \leq \hat{x} \leq b_N$. It follows from Lemma 1 that the uninformed DM permits $x < a_N$ and bans $x > b_N$. ■

Proof of Proposition 4: Recall from equation (A2) that

$$TV(s) = \max\{-z + A(s), g^p(s) + \delta EV(\max\{x, L\}, R), g^b(s) + \delta EV(L, \min\{x, R\})\}.$$

Let $KV(s) = 1$ if $TV(s) = -z + A(s)$ and $KV(s) = 0$ otherwise. Recall that $\mathcal{F}$ denote the set of bounded measurable functions on S taking values in $\mathbb{R}$. To prove the proposition, we first establish the following lemma.

Lemma A.1. *For $(L, R) \in S^e$, if $V \in \mathcal{F}$ satisfies the following properties: (i) V is decreasing in L and increasing in R, (ii) $EV(L, R) - EV(\hat{L}, \hat{R}) \leq C(L, R) - C(\hat{L}, \hat{R})$, and (iii) KV is decreasing in L and increasing in R, then TV also satisfies these properties, that is, (i) TV is decreasing in L and increasing in R, (ii) $ETV(L, R) - ETV(\hat{L}, \hat{R}) \leq C(L, R) - C(\hat{L}, \hat{R})$, and (iii) KTV is decreasing in L and increasing in R.*

Proof: We first show that if $V \in \mathcal{F}$ is decreasing in L and increasing in R, then TV is also increasing in L and decreasing in R. Fix $x \in [0, 1]$. If V is decreasing in L and increasing in R, then $EV(\max\{x, L\}, R)$ and $EV(L, \min\{x, R\})$ are decreasing in L and increasing in R. Note that $A(s)$ is decreasing in L and increasing in R, $g^p(s)$ is constant in L and increasing in R, $g^b(s)$ is constant in R and decreasing in L. Hence, $TV(s)$ is decreasing in L and increasing in R.

Let $\hat{s} = (\hat{L}, \hat{R}, x)$. We next show that if $V \in \mathcal{F}$ satisfies properties $(i), (ii)$, and (iii), then $ETV(L, R) - ETV(\hat{L}, \hat{R}) \leq C(L, R) - C(\hat{L}, \hat{R})$. Consider the following cases.

(a) Suppose $TV(\hat{s}) = -z + A(\hat{s})$. Then $KV(\hat{s}) = 1$. Since KV is decreasing in L and increasing in R, we have $KV(s) = 1$, which implies that $TV(s) = -z + A(s)$. Hence $TV(s) - TV(\hat{s}) = A(s) - A(\hat{s})$.

(b) Suppose $TV(\hat{s}) > -z + A(\hat{s})$. Without loss of generality, suppose that $TV(\hat{s}) = g^p(\hat{s}) + \delta EV(\max\{x, \hat{L}\}, \hat{R})$. Note that $KV(\hat{s}) = 0$ and $x < \hat{R}$. Suppose $KV(s) = 1$. Then $TV(s) = -z + A(s)$ and $TV(s) - TV(\hat{s}) < A(s) - A(\hat{s})$. Suppose $KV(s) = 0$

and $TV(s) = g^p(s) + \delta EV(\max\{x, L\}, R)$. Then

$$\begin{aligned} TV(s) - TV(\hat{s}) &= \delta \left[EV(\max\{x, L\}, R) - EV(\max\{x, \hat{L}\}, \hat{R}) \right] \\ &\leq \delta[EV(L, R) - EV(\hat{L}, \hat{R})] \leq \delta[C(L, R) - C(\hat{L}, \hat{R})]. \end{aligned}$$

Suppose $TV(s) = g^b(s) + \delta EV(L, \min\{x, R\})$. There are two cases to consider, either $x > \hat{L}$ or $x \leq \hat{L}$. First suppose $x > \hat{L}$. Then $g^b(\hat{s}) = g^b(s)$. Since $TV(\hat{s}) \geq g^b(\hat{s}) + \delta EV(\hat{L}, \min\{x, \hat{R}\})$, it follows that $TV(s) - TV(\hat{s}) \leq \delta EV(L, \min\{x, R\}) - \delta EV(\hat{L}, \min\{x, \hat{R}\}) \leq \delta[C(L, R) - C(\hat{L}, \hat{R})]$. Next suppose $x \leq \hat{L}$. Note that $TV(\hat{s}) = g^p(\hat{s}) + \delta EV(\max\{x, \hat{L}\}, \hat{R}) = g^p(\hat{s}) + \delta EV(\hat{L}, \hat{R})$ and $A(\hat{s}) = g^p(s) + \delta C(\hat{L}, \hat{R})$. Hence, $A(\hat{s}) - TV(\hat{s}) = \delta C(\hat{L}, \hat{R}) - \delta EV(\hat{L}, \hat{R})$. Note also that $TV(s) = g^b(s) + \delta EV(L, x) \leq g^b(s) + \delta EV(L, R)$ and $A(s) \geq g^b(s) + \delta C(L, R)$. Hence $A(s) - TV(s) > \delta C(L, R) - \delta EV(L, R)$. It follows that $A(s) - TV(s) - A(\hat{s}) + TV(\hat{s}) > \delta C(L, R) - \delta EV(L, R) - \delta C(\hat{L}, \hat{R}) + \delta EV(\hat{L}, \hat{R}) \geq 0$. Therefore $TV(s) - TV(\hat{s}) \leq A(s) - A(\hat{s})$. It follows that for all $x \in [0, 1]$, we have $TV(s) - TV(\hat{s}) \leq A(s) - A(\hat{s})$, and therefore $ETV(L, R) - ETV(\hat{L}, \hat{R}) \leq E[A(s) - A(\hat{s})] = C(L, R) - C(\hat{L}, \hat{R})$.

Lastly we show that if $V \in \mathcal{F}$ satisfies properties $(i), (ii)$, and (iii), then KTV is decreasing in L and increasing in R. Since $KTV(s) \in \{0, 1\}$ for any $s \in S$, it is sufficient to show that if $KTV(\hat{s}) = 1$, then $KTV(s) = 1$.

Suppose $KTV(\hat{s}) = 1$. Consider $x \in (\hat{L}, \hat{R})$ first. Then we have

$$-z + A(\hat{s}) \geq \max\{g^p(\hat{s}) + \delta ETV(x, \hat{R}), g^b(\hat{s}) + \delta ETV(\hat{L}, x)\}. \tag{A3}$$

Note that in this case, $A(\hat{s}) = \delta C(\hat{L}, \hat{R})$, $A(s) = \delta C(L, R)$, $g^p(\hat{s}) = g^p(s), g^b(\hat{s}) = g^b(s)$. As established earlier, if $V \in \mathcal{F}$ satisfies properties $(i), (ii)$, and (iii), then TV is decreasing in L and increasing in R and $C(L, R) - C(\hat{L}, \hat{R}) \geq ETV(L, R) - ETV(\hat{L}, \hat{R})$. Since $L < \hat{L} < x < \hat{R} < R$, we have $\max\{L, x\} = \max\{\hat{L}, x\} = x$ and $\min\{x, R\} = \min\{x, \hat{R}\} = x$. It follows that $ETV(\max\{L, x\}, R) - ETV(\max\{\hat{L}, x\}, \hat{R}) = ETV(x, R) - ETV(x, \hat{R})$ and $ETV(L, \min\{x, R\}) - ETV(\hat{L}, \min\{x, \hat{R}\}) = ETV(L, x) - ETV(\hat{L}, x)$. Since $C(x, R) - C(x, \hat{R}) \geq ETV(x, R) - ETV(x, \hat{R})$ and $C(L, R) - C(\hat{L}, \hat{R}) \geq C(x, R) - C(x, \hat{R})$, it follows that $C(L, R) - C(\hat{L}, \hat{R}) \geq ETV(\max\{L, x\}, R) - ETV(\max\{\hat{L}, x\}, \hat{R})$. Similarly, since $C(L, x) - C(\hat{L}, x) \geq ETV(L, x) - ETV(\hat{L}, x)$ and $C(L, R) - C(\hat{L}, \hat{R}) > C(L, x) - C(\hat{L}, x)$, it follows that and $C(L, R) - C(\hat{L}, \hat{R}) \geq ETV(L, \min\{x, R\}) -$

$ETV(\hat{L}, \min\{x, \hat{R}\})$. It then follows from (A3) that

$$-z + A(s) \geq \max\{g^p(s) + \delta ETV(\max\{L, x\}, R), g^b(s) + \delta ETV(L, \min\{x, R\})\}$$

and therefore $KTV(s) = 1$.

Next consider $x \notin (\hat{L}, \hat{R})$, and without loss of generality, suppose that $x \leq \hat{L}$. In this case, $A(\hat{s}) - \delta C(\hat{L}, \hat{R}) = g^p(\hat{s})$ and $A(s) - \delta C(L, R) \geq g^p(s)$. Specifically, $A(s) - \delta C(L, R) = g^p(s)$ if $x \leq L$ and $A(s) - \delta C(L, R) = 0$ if $L < x \leq \hat{L}$. Hence,

$$\begin{aligned}
&- z + A(s) - g^p(s) - \delta ETV(\max\{L, x\}, R) - [-z + A(\hat{s}) - g^p(\hat{s}) - \delta ETV(\max\{\hat{L}, x\}, \hat{R})] \\
&\geq \delta[C(L, R) - C(\hat{L}, \hat{R})] - \delta[ETV(x, R) - ETV(x, \hat{R})] \geq 0.
\end{aligned}$$

It follows that $-z + A(s) \geq g^p(s) + \delta ETV(\max\{L, x\}, R)$. We next show that $-z + A(s) \geq g^b(s) + \delta ETV(L, \min\{x, R\})$. If $x \leq L$, then clearly $-z + A(s) \geq g^b(s) + \delta ETV(L, \min\{x, R\})$. Suppose $L < x \leq \hat{L}$. Note that $-z + A(\hat{s}) \geq g^p(\hat{s}) + \delta ETV(\hat{L}, \hat{R})$ implies that $-z + \delta C(\hat{L}, \hat{R}) \geq \delta ETV(\hat{L}, \hat{R})$. Since $A(s) = \delta C(L, R) \geq g^b(s) + \delta C(L, R)$ and $C(L, R) - C(\hat{L}, \hat{R}) \geq ETV(L, R) - ETV(\hat{L}, \hat{R})$, it follows that $-z + A(s) \geq g^b(s) + \delta ETV(L, R) \geq g^b(s) + ETV(L, \min\{x, R\})$. Hence $KTV(s) = 1$. ∎

Since $V_B^* = TV_B^*$ and Lemma A.1 shows that the contraction mapping T preserves properties (i), (ii) and (iii), it follows that V_B^* is decreasing in L and increasing in R. It also follows that $EV_B^*(L, R) - EV_B^*(\hat{L}, \hat{R}) \leq C(L, R) - C(\hat{L}, \hat{R})$. Since $C(L, R)$ is continuous in L and R, we have $EV_B^*(L, R)$ is continuous in L and R. Since the optimal policy satisfies $\mu_B^*(s) = KV_B^*(s)$, it also follows from Lemma A.1 that μ_B^* is decreasing in L and increasing in R. ∎

Proof of Proposition 5: Fix $(L, R) \in S^e$. We first prove part (i). Note that $g^p(s)$ is decreasing in x and $g^b(s)$ is increasing in x. Since V_B^* is decreasing in L by Proposition 4, $V_B^*(\pi(s, P), x') = V_B^*((\max\{L, x\}, R), x')$ is decreasing in x. Similarly, since V_B^* is increasing in R, $V_B^*(\pi(s, B), x') = V_B^*((L, \min\{R, x\}), x')$ is increasing in x.

Suppose x_i $(i = 1, 2)$ triggers an investigation. Then we have

$$-z + A(L, R, x_i) \geq g^p(L, R, x_i) + \delta EV_B^*(\max\{L, x_i\}, R) \tag{A4}$$
$$-z + A(L, R, x_i) \geq g^b(L, R, x_i) + \delta EV_B^*(L, \min\{R, x_i\}). \tag{A5}$$

Recall that $\mathcal{L} = [\underline{\theta}, \max\{L, \underline{\theta}\}]$ and $\mathcal{R} = [\min\{R, \bar{\theta}\}, \bar{\theta}]$. Suppose $\tilde{x} \in [x_1, x_2]$. There are three cases to consider.

(a) Suppose $\mathbf{1}_{\mathcal{L}}(\tilde{x}) = \mathbf{1}_{\mathcal{R}}(\tilde{x}) = 0$. Then $A(L,R,\tilde{x}) \geq A(L,R,x_i)$ for $i = 1,2$. Since $\tilde{x} > x_1$, $g^p(s)$ is decreasing in x and $EV_B^*(\max\{L,x\},R)$ is decreasing in x, it follows from (A4) that

$$-z + A(L,R,\tilde{x}) \geq g^p(L,R,\tilde{x}) + \delta EV_B^*(\max\{L,\tilde{x}\},R).$$

Since $\tilde{x} < x_2$, $g^b(s)$ is increasing in x and $EV_B^*(L,\min\{R,x\})$ is increasing in x, it follows from (A5) that

$$-z + A(L,R,\tilde{x}) \geq g^p(L,R,\tilde{x}) + \delta EV_B^*(L,\min\{R,\tilde{x}\}).$$

Hence, it is optimal for the DM to investigate when $x = \tilde{x}$.

(b) Suppose $\mathbf{1}_{\mathcal{L}}(\tilde{x}) = 1$. Then $\mathbf{1}_{\mathcal{R}}(\tilde{x}) = 0$ and $\mathbf{1}_{\mathcal{R}}(x_1) = 0$. Moreover, we have $A(L,R,\tilde{x}) = g^p(L,R,\tilde{x}) + \delta C(L,R)$, $A(L,R,x_1) \geq g^p(L,R,x_1) + \delta C(L,R)$, $g^b(L,R,\tilde{x}) = -\infty$, and $g^b(L,R,x_1) = -\infty$. From (A4), we have

$$-z + \delta C(L,R) \geq \delta EV_B^*(\max\{L,x_1\},R).$$

Since $EV_B^*(\max\{L,x\},R)$ is decreasing in x and $\tilde{x} > x_1$, it follows that

$$-z + \delta C(L,R) \geq \delta EV_B^*(\max\{L,\tilde{x}\},R)$$

and therefore

$$-z + A(L,R,\tilde{x}) \geq g^p(L,R,\tilde{x}) + \delta EV_B^*(\max\{L,\tilde{x}\},R).$$

Since $g^b(L,R,\tilde{x}) = -\infty$, we also have

$$-z + A(L,R,\tilde{x}) > g^b(L,R,\tilde{x}) + \delta EV_B^*(L,\min\{R,\tilde{x}\}).$$

Hence, it is optimal for the DM to investigate when $x = \tilde{x}$.

(c) Suppose $\mathbf{1}_{\mathcal{R}}(\tilde{x}) = 1$. Then $\mathbf{1}_{\mathcal{L}}(\tilde{x}) = \mathbf{1}_{\mathcal{L}}(x_2) = 0$. Moreover, we have $A(L,R,\tilde{x}) = g^b(L,R,\tilde{x}) + \delta C(L,R)$, $A(L,R,x_2) \geq g^b(L,R,x_2) + \delta C(L,R)$, $g^p(L,R,\tilde{x}) = -\infty$, and $g^p(L,R,x_2) = -\infty$. A similar argument as in case (ii) shows that it is optimal for the DM to investigate when $x = \tilde{x}$.

We next prove part (ii) by contradiction. Consider a case $x \notin (L, R)$. Since the DM has to follow the precedent in the current period regarding x, the difference in her current period payoff between investigating and not investigating is $-z$. If she investigates, her continuation payoff is $\delta C(L, R)$; if she does not investigate, she does not change the precedent since $x \notin (L, R)$ and therefore her continuation payoff is $\delta EV_B^*(L, R)$.

Suppose x triggers an investigation. Then we have $-z + \delta C(L, R) \geq \delta EV_B^*(L, R)$. Note that neither side of the inequality depends on x, which implies that any $x \notin (L, R)$ triggers an investigation given precedent (L, R). It follows from part (i) of the proposition that any $x \in [0, 1]$ triggers an investigation given precedent (L, R). Hence, $EV_B^*(L, R) = -z + C(L, R)$, which contradicts $-z + \delta C(L, R) \geq \delta EV_B^*(L, R)$ since $-z < 0$ and $\delta < 1$. Hence, $x \notin (L, R)$ does not trigger an investigation. ∎

Lemma A.2. *If $L < a(L, R) \leq b(L, R) < R$, then $X_B(L, R)$ is closed, and the DM is indifferent between investigating and not investigating under precedent (L, R) if $x = a(L, R)$ or if $x = b(L, R)$.*

Proof: Suppose $L < a(L, R) \leq b(L, R) < R$ and consider $x \in (L, R)$. Since $x \in (L, R)$, the DM does not have to follow any precedent in her ruling of x. It follows that her expected current-period payoff in making a ruling regarding x without an investigation is continuous in x for $x \in (L, R)$ since $\ell(x, \theta)$ is continuous in x for any $x \neq \theta$. Since EV_B^* is also continuous by Proposition 4, the DM's dynamic payoff if she makes a ruling regarding x without an investigation is continuous in x for $x \in (L, R)$. Recall that the DM's dynamic payoff if she investigates when the case is x is $-z + A((L, R), x)$. Since $x \in (L, R)$, we have $A((L, R), x) = \delta C(L, R)$, which is constant in x. Hence, the DM's dynamic payoff if she investigates when the case is x is also continuous in x. It follows the DM is indifferent between investigating and not investigating when $x = a(L, R)$ and when $x = b(L, R)$. Recall that we assume that when the DM is indifferent between investigating and not investigating, she investigates. Hence, if $L < a(L, R) \leq b(L, R) < R$, then the set of cases that the uninformed DM investigates is the closed interval $[a(L, R), b(L, R)]$. ∎

Proof of Proposition 6: Recall that $a_1 = a(L_1, R_1)$ and $b_1 = b(L_1, R_1)$. We first show that $X_N \subseteq X_B(L_1, R_1)$. Recall that $X_N = [a_N, b_N]$ when $X_N \neq \varnothing$. We next show that $a_1 \leq a_N$. A similar argument shows that $b_1 \leq b_N$.

For $0 \le a < b \le 1$, let $h(a,b) = \int_{\underline{\theta}}^{a} \int_{\theta}^{a} -\ell(x,\theta) dG(x) dF(\theta) + \int_{b}^{\bar{\theta}} \int_{b}^{\theta} -\ell(x,\theta) dG(x) dF(\theta)$. Note that for any L, R, we have

$$
\begin{aligned}
EV_B^*(L,R) \le\ & h(a(L,R), b(L,R)) + \left[G(b(L,R)) - G(a(L,R))\right]\left[-z + \delta C(L,R)\right] \\
& + \delta EV_B^*(L,R)\left[G(a(L,R)) + 1 - G(b(L,R))\right],
\end{aligned}
$$

where the inequality comes from the property that $EV_B^*(L,R)$ is decreasing in L and increasing in R and that when the DM makes a decision without an investigation, the precedent either stays the same or gets tighter.

It follows that

$$
EV_B^*(L,R) \le \frac{h(a(L,R), b(L,R)) + \left[G(b(L,R)) - G(a(L,R))\right]\left[-z + \delta C(L,R)\right]}{1 - \delta\left[G(a(L,R)) + 1 - G(b(L,R))\right]}.
$$

Since $C(a(L,R), b(L,R)) \le 0$, we have $EV_B^*(L,R) \le \frac{h(a(L,R),b(L,R)) - z\left[G(b(L,R)) - G(a(L,R))\right]}{1 - \delta\left[G(a(L,R)) + 1 - G(b(L,R))\right]}$.

Consider any case $x \in (L_1, a_1)$. Since $x < a_1$, it does not trigger an investigation given precedent (L_1, R_1) and we have $\int_{\underline{\theta}}^{x} -\ell(x,\theta) dF(\theta) + \delta EV_B^*(x, R_1) > -z$. It follows that

$$
\int_{\underline{\theta}}^{x} -\ell(x,\theta) dF(\theta) > -z - \delta EV_B^*(x, R_1).
$$

Recall that under nonbinding precedent, a_N satisfies

$$
\int_{\underline{\theta}}^{a_N} -\ell(a_N,\theta) dF(\theta) = -z - \delta EV_N^*. \tag{A6}
$$

Since $EV_B^*(x, R_1) \le EV_N^*$, it follows that $\int_{\underline{\theta}}^{x} -\ell(x,\theta) dF(\theta) > \int_{\underline{\theta}}^{a_N} -\ell(a_N,\theta) dF(\theta)$ and therefore $x < a_N$. Since this is true for any $x < a_1$ we have $a_1 \le a_N$. A similar argument shows that $b_N \le b_1$.

Suppose that $X_B^\infty \neq \varnothing$. Let $X_B^\infty = (\hat{a}, \hat{b})$. We next show that $a_N < \hat{a} < \hat{b} < b_N$. Note that if the precedent satisfies $L = \hat{a}$ and $R = \hat{b}$, then x triggers an investigation if and only if $x \in (\hat{a}, \hat{b})$. Hence, we have

$$
EV_B^*(\hat{a},\hat{b}) = h(\hat{a},\hat{b}) + \left(-z + \delta C(\hat{a},\hat{b})\right)\left[G(\hat{b}) - G(\hat{a})\right] + \delta EV_B^*(\hat{a},\hat{b})\left[G(\hat{a}) + 1 - G(\hat{b})\right],
$$

which implies that

$$EV_B^*(\hat{a},\hat{b}) = \frac{h(\hat{a},\hat{b})}{1-\delta[G(\hat{a})+1-G(\hat{b})]} + \left(-z+\delta C(\hat{a},\hat{b})\right)\frac{G(\hat{b})-G(\hat{a})}{1-\delta[G(\hat{a})+1-G(\hat{b})]}. \quad \text{(A7)}$$

Moreover, we have

$$-z+\delta C(\hat{a},\hat{b}) = \int_{\underline{\theta}}^{\hat{a}} -\ell(\hat{a},\theta)dF(\theta) + \delta EV_B^*(\hat{a},\hat{b}) = \int_{\hat{b}}^{\bar{\theta}} -\ell(\hat{b},\theta)dF(\theta) + \delta EV_B^*(\hat{a},\hat{b}).$$

From the indifference condition above and (A7), we have

$$\begin{aligned}\int_{\underline{\theta}}^{\hat{a}} -\ell(\hat{a},\theta)dF(\theta) &= -z+\delta C(\hat{a},\hat{b}) - \delta EV_B^*(\hat{a},\hat{b}) \\ &= \frac{-(1-\delta)z+(1-\delta)\delta C(\hat{a},\hat{b})-\delta h(\hat{a},\hat{b})}{1-\delta[G(\hat{a})+1-G(\hat{b})]}.\end{aligned}$$

Since $(1-\delta)C(\hat{a},\hat{b}) = h(\hat{a},\hat{b})$, it follows that

$$\int_{\underline{\theta}}^{\hat{a}} -\ell(\hat{a},\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(\hat{a})+1-G(\hat{b})]}.$$

Similarly,

$$\int_{\hat{b}}^{\bar{\theta}} -\ell(\hat{b},\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(\hat{a})+1-G(\hat{b})]}.$$

From (5) and (A6), we have that under nonbinding precedent, a_N and b_N satisfy

$$\int_{\underline{\theta}}^{a_N} -\ell(a_N,\theta)dF(\theta) = \int_{b_N}^{\bar{\theta}} -\ell(b_N,\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(a_N)+1-G(b_N)]} - \frac{\delta h(a_N,b_N)}{1-\delta[G(a_N)+1-G(b_N)]}.$$

For $a \in [\underline{\theta},\bar{\theta}]$, let $\beta(a)$ be defined by $\int_{\underline{\theta}}^{a} -\ell(a_N,\theta)dF(\theta) = \int_{\beta(a)}^{\bar{\theta}} -\ell(b_N,\theta)dF(\theta)$. Also, let A equal the constant $-\frac{\delta h(a_N,b_N)}{1-\delta[G(a_N)+1-G(b_N)]} > 0$.

Note that a_N is the solution to

$$\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]} + A$$

and $\hat{a}$ is the solution to

$$\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) = \frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]}.$$

If $a = \underline{\theta}$, then $\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) = 0$. Moreover, since $-(1-\delta)z - \delta h(a_N, b_N) < 0$ and $\frac{-(1-\delta)z}{1-\delta[G(\underline{\theta})+1-G(\bar{\theta})]} + A < \frac{-(1-\delta)z-\delta h(a_N,b_N)}{1-\delta[G(\underline{\theta})+1-G(\bar{\theta})]}$, it follows that $\frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]} + A < 0$ for $a = \underline{\theta}$. Hence, for any $a \in [\underline{\theta}, a_N)$, we have

$$\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) > \frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]} + A.$$

Since $A > 0$, this implies that for any $a \in [\underline{\theta}, a_N)$, we have

$$\int_{\underline{\theta}}^{a} -\ell(a,\theta)dF(\theta) > \frac{-(1-\delta)z}{1-\delta[G(a)+1-G(\beta(a))]}.$$

It follows that $\hat{a} > a_N$. Since $\beta(a)$ is decreasing in a, it follows that $\hat{b} < b_N$. ∎

Proof of Lemma 5: We first show that if $z \leq \frac{c_L}{3(1-\delta)}$, then both x_L and x_H trigger an investigation. Note that if both cases trigger an investigation, then $EV_N^* = -z$. This is optimal if $-z \geq -\frac{1}{3}c_L + \delta EV_N^*$ and $-z \geq -\frac{1}{3}c_H + \delta EV_N^*$, which are satisfied when $z \leq \frac{c_L}{3(1-\delta)}$.

We next show that if $\frac{c_L}{3(1-\delta)} < z \leq \frac{(2-\delta)c_H+\delta c_L}{6(1-\delta)}$, then only x_H triggers an investigation. Note that if x_H but not x_L triggers an investigation, then $EV_N^* = \frac{1}{2}(-z) + \frac{1}{2}(-\frac{1}{3}c_L + \delta EV_p)$, implying that $EV_N^* = -\frac{z+\frac{1}{3}c_L}{2-\delta}$. Hence, it is optimal that only x_H triggers an investigation if $-z \geq -\frac{1}{3}c_H + \delta EV_N^*$ and x_L if $-z < -\frac{1}{3}c_L + \delta EV_N^*$, which are satisfied when $\frac{c_L}{3(1-\delta)} < z \leq \frac{(2-\delta)c_H+\delta c_L}{6(1-\delta)}$.

Finally, we show that if $z > \frac{(2-\delta)c_H+\delta c_L}{6(1-\delta)}$, then neither x_H nor x_L triggers an investigation. Note that if neither x_H or x_L triggers an investigation, then $EV_N^* = -\frac{c_L+c_H}{6(1-\delta)}$. Hence, it is optimal that neither cases trigger an investigation if $-z < -\frac{1}{3}c_H + \delta EV_N^*$ and $-z < -\frac{1}{3}c_L + \delta EV_N^*$, which are satisfied when $z > \frac{(2-\delta)c_H+\delta c_L}{6(1-\delta)}$.

Lemma 5 follows by letting $\underline{z}_N = \frac{c_L}{3(1-\delta)}$ and $\bar{z}_N = \frac{(2-\delta)c_H+\delta c_L}{6(1-\delta)}$. ∎

Proof of Lemma 6: Consider $x_1 = x_L$. If the DM investigates, her payoff is $-z$, and if she does not, her payoff is $-\frac{1}{3}c_L + \delta EV_B^*(x_L, 1)$. Hence, x_L triggers an investigation under the initial precedent if $-z \geq -\frac{1}{3}c_L + \delta EV_B^*(x_L, 1)$.

To find $EV_B^*(x_L, 1)$, note that under precedent $(x_L, 1)$, x_L does not trigger investigation, and x_H triggers investigation if and only if $-z - \frac{\delta}{6(1-\delta)}c_L \geq -\frac{1}{3}c_H - \frac{\delta}{6(1-\delta)}c_L - \frac{\delta}{6(1-\delta)}c_H$, that is, $z \leq \frac{2-\delta}{6(1-\delta)}c_H$.

Hence, if $z \leq \frac{2-\delta}{6(1-\delta)}c_H$, then $EV_B^*(x_L, 1) = \frac{1}{2}[-\frac{1}{3}c_L + \delta EV_B^*(x_L, 1)] + \frac{1}{2}[-z - \frac{\delta}{6(1-\delta)}c_L]$, which implies that $EV_B^*(x_L, 1) = -\frac{z}{2-\delta} - \frac{1}{6(1-\delta)}c_L$. And if $z > \frac{2-\delta}{6(1-\delta)}c_H$, then $EV_B^*(x_L, 1) = \frac{1}{2}[-\frac{1}{3}c_L + \delta EV_B^*(x_L, 1)] + \frac{1}{2}[-\frac{1}{3}c_H + \delta EV_B^*(x_L, 1)]$, which implies that $EV_B^*(x_L, 1) = -\frac{1}{6(1-\delta)}c_L - \frac{1}{6(1-\delta)}c_H$.

Substituting $EV_B^*(x_L, 1)$ in the condition $-z \geq -\frac{1}{3}c_L + \delta EV_B^*(x_L, 1)$, we find that if $z \leq \frac{2-\delta}{6(1-\delta)}c_H$, then under the initial precedent, x_L triggers investigation if $-z \geq -\frac{1}{3}c_L + \delta[-\frac{z}{2-\delta} - \frac{1}{6(1-\delta)}c_L]$, that is, if $z \leq \frac{(2-\delta)^2}{12(1-\delta)^2}c_L$. And if $z > \frac{2-\delta}{6(1-\delta)}c_H$, then under the initial precedent, x_L triggers investigation iff $-z \geq -\frac{1}{3}c_L + \delta[-\frac{1}{6(1-\delta)}c_L - \frac{1}{6(1-\delta)}c_H]$, that is, if $z \leq \frac{2-\delta}{6(1-\delta)}c_L + \frac{\delta}{6(1-\delta)}c_H$.

Note that $\frac{2-\delta}{6(1-\delta)}c_H > \frac{(2-\delta)^2}{12(1-\delta)^2}c_L$ if and only if $\frac{c_H}{c_L} > \frac{2-\delta}{2-2\delta}$ and $\frac{2-\delta}{6(1-\delta)}c_H > \frac{2-\delta}{6(1-\delta)}c_L + \frac{\delta}{6(1-\delta)}c_H$ if and only if $\frac{c_H}{c_L} > \frac{2-\delta}{2-2\delta}$.

Suppose $\frac{c_H}{c_L} > \frac{2-\delta}{2-2\delta}$. Then $\frac{2-\delta}{6(1-\delta)}c_H > \frac{(2-\delta)^2}{12(1-\delta)^2}c_L$ and $\frac{2-\delta}{6(1-\delta)}c_H > \frac{2-\delta}{6(1-\delta)}c_L + \frac{\delta}{6(1-\delta)}c_H$. It follows that x_L triggers investigation under the initial precedent iff $z \leq \frac{(2-\delta)^2}{12(1-\delta)^2}c_L$.

Suppose $\frac{c_H}{c_L} \leq \frac{2-\delta}{2-2\delta}$. Then $\frac{2-\delta}{6(1-\delta)}c_H \leq \frac{(2-\delta)^2}{12(1-\delta)^2}c_L$ and $\frac{2-\delta}{6(1-\delta)}c_H \leq \frac{2-\delta}{6(1-\delta)}c_L + \frac{\delta}{6(1-\delta)}c_H$. It follows that x_L triggers investigation under the initial precedent if $z \leq \frac{2-\delta}{6(1-\delta)}c_H$ or if $\frac{2-\delta}{6(1-\delta)}c_H < z \leq \frac{2-\delta}{6(1-\delta)}c_L + \frac{\delta}{6(1-\delta)}c_H$, that is, if $z \leq \frac{2-\delta}{6(1-\delta)}c_L + \frac{\delta}{6(1-\delta)}c_H$.

Next consider $x_1 = x_H$. An investigation takes place if and only if $-z \geq -\frac{1}{3}c_H + \delta EV_B^*(0, x_H)$. Straightforward calculation similar to the above shows that if $z \leq \frac{2-\delta}{6(1-\delta)}c_L$, then $EV_B^*(0, x_H) = -\frac{z}{2-\delta} - \frac{1}{6(1-\delta)}c_H$, and in this case, x_H triggers an investigation if $z \leq \frac{(2-\delta)^2}{12(1-\delta)^2}c_H$. And if $z > \frac{2-\delta}{6(1-\delta)}c_L$, then $EV_B^*(0, x_H) = -\frac{1}{6(1-\delta)}c_L - \frac{1}{6(1-\delta)}c_H$, and in this case, x_H triggers an investigation if $z \leq \frac{2-\delta}{6(1-\delta)}c_H + \frac{\delta}{6(1-\delta)}c_L$. Since $\frac{2-\delta}{6(1-\delta)}c_H \leq \frac{(2-\delta)^2}{12(1-\delta)^2}c_L$ and $\frac{2-\delta}{6(1-\delta)}c_H \leq \frac{2-\delta}{6(1-\delta)}c_L + \frac{\delta}{6(1-\delta)}c_H$, it follows that x_H triggers an investigation if $z \leq \frac{2-\delta}{6(1-\delta)}c_H + \frac{\delta}{6(1-\delta)}c_L$.

Let $\bar{z}_B = \frac{2-\delta}{6(1-\delta)}c_H + \frac{\delta}{6(1-\delta)}c_L$. And let $\underline{z}_B = \frac{(2-\delta)^2}{12(1-\delta)^2}c_L$ if $\frac{c_H}{c_L} \geq \frac{2-\delta}{2-2\delta}$ and $\underline{z}_B = \frac{2-\delta}{6(1-\delta)}c_L + \frac{\delta}{6(1-\delta)}c_H$ if $\frac{c_H}{c_L} < \frac{2-\delta}{2-2\delta}$. Note that $\underline{z}_B < \bar{z}_B$. It follows that under the initial precedent, if $z \leq \underline{z}_B$, then both x_L and x_H trigger an investigation; If $\underline{z}_B < z \leq \bar{z}_B$, then only x_H triggers an investigation; if $z > \bar{z}_B$, then neither triggers an investigation. Recall that $\underline{z}_N = \frac{c_L}{3(1-\delta)}$ and $\bar{z}_N = \frac{(2-\delta)c_H + \delta c_L}{6(1-\delta)}$. Hence, $\underline{z}_N < \underline{z}_B$ and $\bar{z}_N = \bar{z}_B$. ∎

Appendix B: Partial learning

Let θ_{LM} denote the informational state such that the DM believes that $\theta = \theta_L$ and θ_M with equal probability. Define θ_{MH} and θ_{LMH} similarly. In what follows, we assume that $\ell(x_L, \theta_L) < \ell(x_L, \theta_M)$ and $\ell(x_H, \theta_H) < \ell(x_H, \theta_M)$, which imply that under nonbinding precedent, the DM finds it optimal to permit x_L in state θ_{LM} and to ban x_H in state θ_{MH}.

Consider the initial informational state θ_{LMH}. As the next two lemmas show, the characterization and the comparison of the optimal investigation policies with partial learning are qualitatively the same as those in Lemma 5 and 6.

Lemma B.1. *There exist $\underline{z}'_N$ and $\bar{z}'_N$ such that under nonbinding precedent in state θ_{LMH}, if $z \leq \underline{z}'_N$, both x_L and x_H trigger an investigation; if $\underline{z}'_N < z \leq \bar{z}'_N$, only x_H triggers an investigation; if $z > \bar{z}'_N$, there is no investigation.*

Proof: For any informational state s_θ, let $EV^*_N(s_\theta)$ denote the DM's equilibrium payoff in state s_θ. Consider informational state θ_{LM}. Note that x_H does not trigger an investigation, and if x_L triggers an investigation, then $EV^*_N(\theta_{LM}) = \frac{1}{2}(0+\delta EV^*_N(\theta_{LM})) - \frac{1}{2}z$, which implies that $EV^*_N(\theta_{LM}) = -\frac{z}{2-\delta}$. Since it is optimal to investigate in state θ_{LM} if $-z \geq -\frac{1}{2}c_L + EV^*_N(\theta_{LM})$, it follows that x_L triggers an investigation if $z \leq \frac{c_L(2-\delta)}{4(1-\delta)}$. Hence, $EV^*_N(\theta_{LM}) = -\frac{z}{2-\delta}$ if $z \leq \frac{c_L(2-\delta)}{4(1-\delta)}$ and $EV_N(\theta_{LM}) = -\frac{c_L}{4(1-\delta)}$ if $z \geq \frac{c_L(2-\delta)}{4(1-\delta)}$.

Similarly, in informational state θ_{MH}, x_H triggers an investigation iff $z \leq \frac{c_H(2-\delta)}{4(1-\delta)}$. Hence, $EV^*_N(\theta_{MH}) = -\frac{z}{2-\delta}$ if $z \leq \frac{c_H(2-\delta)}{4(1-\delta)}$ and $EV^*_N(\theta_{MH}) = -\frac{c_H}{4(1-\delta)}$ if $z > \frac{c_H(2-\delta)}{4(1-\delta)}$. In what follows, we focus on the case in which $z \leq \frac{c_H(2-\delta)}{4(1-\delta)}$ for simplicity. (Similar results can be derived when $z > \frac{c_H(2-\delta)}{4(1-\delta)}$.)

Consider state θ_{LMH}. We first show that if $z \leq \frac{2-\delta}{(1-\delta)(6-\delta)}c_L$, then both x_L and x_H trigger an investigation. Note that if both x_L and x_H trigger an investigation, then $EV^*_N(\theta_{LMH}) = -z + \frac{\delta}{3}(EV^*_N(\theta_{MH}) + EV^*_N(\theta_{LM})) = -\frac{6-\delta}{3(2-\delta)}z$. Since x_L triggers an investigation iff $-z + \frac{2}{3}\delta EV^*_N(\theta_{MH}) \geq -\frac{1}{3}c_L + \delta EV^*_N(\theta_{LMH})$ and x_H triggers an investigation iff $-z + \frac{2}{3}\delta EV^*_N(\theta_{LM}) \geq -\frac{1}{3}c_H + \delta EV^*_N(\theta_{LMH})$, it follows that both trigger an investigation iff $z \leq \frac{2-\delta}{(1-\delta)(6-\delta)}c_L$.

We next show that if $\frac{2-\delta}{(1-\delta)(6-\delta)}c_L < z \leq \frac{(2-\delta)[(2-\delta)c_H+\delta c_L]}{2(1-\delta)(6-\delta)}$, then only x_H triggers an investigation. Note that if only x_H triggers an investigation, then $EV^*_N(\theta_{LMH}) = \frac{1}{2}[-\frac{1}{3}c_L + \delta EV^*_N(\theta_{LMH})] + \frac{1}{2}[-z + \frac{2}{3}\delta EV^*_N(\theta_{LM})]$, which implies that $EV^*_N(\theta_{LMH}) = \frac{1}{2-\delta}[-\frac{1}{3}c_L - z + \frac{2}{3}\delta EV^*_N(\theta_{LM})] = \frac{1}{2-\delta}[-\frac{1}{3}c_L - z + \frac{2\delta z}{3(2-\delta)}]$. Since x_H triggers an investigation iff $-z+\frac{2}{3}\delta EV_N^*(\theta_{LM}) \geq -\frac{1}{3}c_H+\delta EV_N^*(\theta_{LMH})$ and x_L does not trigger an investigation iff $-z+\frac{2}{3}\delta EV_N^*(\theta_{MH}) < -\frac{1}{3}c_L+\delta EV_N^*(\theta_{LMH})$, it follows that if $\frac{2-\delta}{(1-\delta)(6-\delta)}c_L < z \leq \frac{(2-\delta)[(2-\delta)c_H+\delta c_L]}{2(1-\delta)(6-\delta)}$, then only x_H triggers an investigation.

If neither x_L or x_H triggers an investigation, then $EV_N^*(\theta_{LMH}) = -\frac{c_L+c_H}{6(1-\delta)}$. Since neither case triggers an investigation if $-z+\frac{2}{3}\delta EV_N^*(\theta_{LM}) < -\frac{1}{3}c_H+\delta EV_N^*(\theta_{LMH})$ and if $-z+\frac{2}{3}\delta EV_N^*(\theta_{MH}) < -\frac{1}{3}c_L+\delta EV_N^*(\theta_{LMH})$, it follows that if $z > \frac{(2-\delta)[(2-\delta)c_H+\delta c_L]}{2(1-\delta)(6-\delta)}$, then no investigation takes place in state θ_{LMH}.

The lemma follows by letting $\underline{z}'_N = \frac{2-\delta}{(1-\delta)(6-\delta)}c_L$ and $\bar{z}'_N = \frac{(2-\delta)[(2-\delta)c_H+\delta c_L]}{2(1-\delta)(6-\delta)}$. ■

Lemma B.2. *There exists $\underline{z}'_B$ and $\bar{z}'_B$ such that in the first period under binding precedent, if $z \leq \underline{z}'_B$, both x_L and x_H trigger investigation; if $\underline{z}'_B < z \leq \bar{z}'_B$, only x_H triggers investigation; if $z > \bar{z}'_B$, there is no investigation. Furthermore, $\bar{z}'_N = \bar{z}'_B$ and $\underline{z}'_N < \underline{z}'_B$.*

Proof: Under binding precedent and with partial learning, a state s consists of the precedent (L, R) and the informational state s_θ. Let $EV_B^*(L, R, s_\theta)$ denote the DM's equilibrium payoff in state s. Consider the state in the first period, (L_1, R_1, θ_{LMH}). The case x_L triggers an investigation iff $-z+\frac{2}{3}\delta EV_B^*(x_L, R_1, \theta_{MH}) \geq -\frac{1}{3}c_L+\delta EV_B^*(x_L, R_1, \theta_{LMH})$.

In state (x_L, R_1, θ_{LMH}), the DM's problem is the same as in perfect learning. To see this, note that given the precedent (x_L, R_1), the DM must permit x_L. Therefore x_L does not trigger an investigation and the value of information from perfect learning or partial learning when x_H triggers an investigation is the same. As shown in the proof of Lemma 6, if $z \leq \frac{2-\delta}{6(1-\delta)}c_H$, then x_H triggers an investigation and $EV_B^*(x_L, R_1, \theta_{LMH}) = -\frac{z}{2-\delta} - \frac{1}{6(1-\delta)}c_L$; if $z > \frac{2-\delta}{6(1-\delta)}c_H$, then x_H does not trigger an investigation and $EV_B^*(x_L, R_1, \theta_{LMH}) = -\frac{1}{6(1-\delta)}c_H - \frac{1}{6(1-\delta)}c_L$.

In state (x_L, R_1, θ_{MH}), the DM's problem is the same as in nonbinding precedent since in the informational state θ_{MH}, it is optimal to permit x_L. It follows that $EV_B^*(x_L, R_1, \theta_{MH}) = -\frac{z}{2-\delta}$.

Substituting for $EV_B^*(x_L, R_1, \theta_{MH})$ and $EV_B^*(x_L, R_1, \theta_{LMH})$, we find that the condition for x_L to trigger an investigation in state (L_1, R_1, θ_{LMH}) is $z \leq \frac{(2-\delta)^2}{4(1-\delta)(3-2\delta)}c_L$ for $z \leq \frac{2-\delta}{6(1-\delta)}c_H$ and is $z \leq \frac{(2-\delta)^2}{2(1-\delta)(6-\delta)}c_L + \frac{\delta(2-\delta)}{2(1-\delta)(6-\delta)}c_H$ for $z > \frac{2-\delta}{6(1-\delta)}c_H$.

Note that $\frac{2-\delta}{6(1-\delta)}c_H \geq \frac{(2-\delta)^2}{4(1-\delta)(3-2\delta)}c_L$ and $\frac{2-\delta}{6(1-\delta)}c_H \geq \frac{(2-\delta)^2}{2(1-\delta)(6-\delta)}c_L + \frac{\delta(2-\delta)}{2(1-\delta)(6-\delta)}c_H$ are both equivalent to $\frac{c_H}{c_L} \geq \frac{6-3\delta}{6-4\delta}$. Hence, if $\frac{c_H}{c_L} \geq \frac{6-3\delta}{6-4\delta}$, then x_L triggers an investigation in state (L_1, R_1, θ_{LMH}) iff $z \leq \frac{(2-\delta)^2}{4(1-\delta)(3-2\delta)}c_L$; if $\frac{c_H}{c_L} < \frac{6-3\delta}{6-4\delta}$, then x_L triggers an investigation in state (L_1, R_1, θ_{LMH}) iff $z \leq \frac{(2-\delta)^2}{2(1-\delta)(6-\delta)}c_L + \frac{\delta(2-\delta)}{2(1-\delta)(6-\delta)}c_H$.

We can derive the conditions for x_H to trigger an investigation in a similar way. Since $\frac{c_L}{c_H} < \frac{6-3\delta}{6-4\delta}$, x_H triggers an investigation in state (L_1, R_1, θ_{LMH}) iff $z \leq \frac{(2-\delta)^2}{2(1-\delta)(6-\delta)}c_H + \frac{\delta(2-\delta)}{2(1-\delta)(6-\delta)}c_L$.

Note that if $\frac{c_H}{c_L} \geq \frac{6-3\delta}{6-4\delta}$, then $\frac{(2-\delta)^2}{4(1-\delta)(3-2\delta)}c_L < \frac{(2-\delta)^2}{2(1-\delta)(6-\delta)}c_H + \frac{\delta(2-\delta)}{2(1-\delta)(6-\delta)}c_L$. If $\frac{c_H}{c_L} \geq \frac{6-3\delta}{6-4\delta}$, let $\underline{z}'_B = \frac{(2-\delta)^2}{4(1-\delta)(3-2\delta)}c_L$; if $\frac{c_H}{c_L} < \frac{6-3\delta}{6-4\delta}$, let $\underline{z}'_B = \frac{(2-\delta)^2}{2(1-\delta)(6-\delta)}c_L + \frac{\delta(2-\delta)}{2(1-\delta)(6-\delta)}c_H$. Also, let $\bar{z}'_B = \frac{(2-\delta)^2}{2(1-\delta)(6-\delta)}c_H + \frac{\delta(2-\delta)}{2(1-\delta)(6-\delta)}c_L$. Note that $\bar{z}'_N = \bar{z}'_B$ and $\underline{z}'_N < \underline{z}'_B$. Lemma B.2 follows. ∎

Hence, if $z \leq \underline{z}'_N$, then both cases trigger an investigation in the first period under either institution; if $z > \bar{z}'_N = \bar{z}'_B$, then neither case trigger an investigation in the first period under either institution; if $\underline{z}'_N < z \leq \underline{z}'_B$, then under binding precedent, both cases trigger an investigation in the first period whereas under nonbinding precedent, only x_H triggers an investigation in the first period; if $z < \underline{z}'_B < \bar{z}'_N$, then only x_H triggers an investigation in the first period under either institution. Accordingly, we have the following comparison of social welfare analogous to Proposition 7.

Proposition B.1. *If the cost of information acquisition is sufficiently low or sufficiently high, then welfare is the same under two institutions with partial learning, otherwise, it is different. Formally, (i) If $z \leq \underline{z}'_N$ or $z > \bar{z}'_N = \bar{z}'_B$, then welfare is the same under nonbinding precedent and binding precedent. (ii) If $\underline{z}'_N < z \leq \underline{z}'_B$, then welfare is higher under binding precedent. (iii) If $\underline{z}'_B < z \leq \bar{z}'_N = \bar{z}'_B$, then welfare is higher under nonbinding precedent.*

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