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Market connectedness: spillovers, information flow, and relative market entropy*

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Abstract

The degree of connectedness of equity markets on a given day can be assessed by decomposing the forecast error variance resulting from a vector autoregressive model, applied to daily returns on stock indices. This well-known procedure leads, for each day, to a spillover matrix which can be readily interpreted as a network structure and summarized into the so-called spillover index, to which much recent research work has been devoted.

Taking a sequence of spillover matrices as starting point, we show how the scope of this concept can be broadened in several ways. Firstly, we develop a concept to quantify a market's potential to spread information, which is related to the eigenvector structure of spillover matrices. Secondly, a Markov chain approach allows the definition of relative market entropy, quantifying the amount of information gained from day to day. A further entropy concept can be related to the speed of shock digestion and network stability. As an empirical example, we analyze a system of five markets represented by stock indices Dow Jones Industrial Average (USA), FTSE (UK), Euro Stoxx 50 (euro area), Nikkei 225 (Japan), and SSE Composite (China). It is demonstrated that increasing trends in the spillover index as well as in speed of information digestion are an empirical fact but no logical necessity — theoretical examples show that there can be opposite trends in these series.

Key words: Market connectedness; spillover matrix; propagation values; news spreader; informational divide; relative market entropy; network stability

1 Introduction

With the achievements of what came to be called the "Great Moderation"[1] in mind, many experts in the 1990s and early 2000s agreed that contagion is an issue of the past, or, at best,

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[1]The term "Great Moderation" was coined by Stock and Watson [38] to refer to the decreased level of volatility in GDP growth rates in the US from the mid-1980s onwards.

a matter of emerging markets.[2] Indeed, the last global crisis sparked by the developed world had been the Great Depression of 1929–1932, originating in the US, while recent crises with tangible international repercussions had started in emerging markets: the 1997 Asian financial crisis, starting in Thailand (see Didier, Mauro and Schmukler [9]); the 1998 Russian financial crisis (see Pinto and Ulatov [29]); the 1999 Brazilian financial crisis (see Rigobon [32]); the 2001 Turkish financial crisis (see Didier, Mauro and Schmukler [9], Forbes [13]); the 2001 Argentine financial crisis (see Forbes [13]).

This perception has been overtaken by reality with the advent of the financial crisis 2007–2009: Sparked from issues in the US sub-prime mortgage market, "…it rapidly spread across virtually all economies, both advanced and emerging, as well as across economic sectors. It also affected equity markets worldwide, with many countries experiencing even sharper equity market crashes than the United States" (see Bekaert et al. [3]).

In the wake of Thailand's 1997 and Russia's 1998 currency devaluations affecting global financial markets, the notion of contagion entered mainstream economic terminology, prompting a series of academic investigations in the early 2000s (see Forbes [13]) as well as sparking concerns among policy makers. In particular, the IMF (see IMF [16, 17]) has underlined the importance of investigating financial sector spillovers, encouraging that "work on spillovers should continue, with modalities (e.g., frequency, coverage, and context) that could evolve as experience is gained with the exercise" (IMF [17]), and recent IMF Spillover Reports (e.g., IMF [18, 19]) assess the external effects of policies in five systemic economies: China, the euro area, Japan, the United Kingdom, and the United States of America.

There is a consensus neither on the exact definition of contagion nor on the methodology to quantify it (for surveys of definitions, classifications and transmission channels of contagion, see Karolyi [20] and Dungey et al. [12]). An explanation by the World Bank[3] covers many aspects of the phenomenon:

> "Contagion is the cross-country transmission of shocks or the general cross-country spillover effects. Contagion can take place both during 'good' times and 'bad' times. Then, contagion does not need to be related to crises. However, contagion has been emphasized during crisis times."

The scope of what is understood by contagion depends, among others, on the intended purpose of an investigation: strict definitions are more useful for understanding how crises are transmitted, whereas broad definitions allow for a broader set of options for policy makers to support an economy in crisis irrespective of the origin of the problem, according to Forbes [13]. The common ground is that understanding contagion will allow us "to ascertain the causes of global market volatility in order, ultimately, to elaborate strategies for containing crises and minimizing susceptibility to cross-border shocks", see Peckham [27]. Furthermore, from an investment point of view, "contagion reduces benefits from portfolio diversification and raises issues for risk management", see Kyle and Xiong [25].

Shifting its focal point away from shocks and negative events, the notion of interdependence (or interconnectedness) covers a broader scope than contagion (Forbes [13]), thus paving the way for the application of a wide variety of stochastic models. Two principal methods of approach can be distinguished in quantitative models of interconnectedness: (i) those trying to identify spillover channels; (ii) those which are "agnostic" in the sense that spillover channels are not explicitly considered. As an example of the former, IMF [16] states that "potential spillover

[2] See, for example, Bloomberg, 2013-01-25: "When Kristin Forbes [a well-known researcher in the field] sought tenure at the MIT early last decade, some colleagues said her research focus on financial contagion led to a dead end."

[3] Worldbank: http://go.worldbank.org/JIBDRK3YC0, retrieved on 2013-06-02.

channels [were] identified (e.g., the importance of outward financial spillovers from the U.S.)"; this perspective may also focus on balance-sheet information and the Global Systemically Important Bank (G-SIB) identification methodology of the Basel Committee on Banking Supervision. Approach (ii) uses mainly market data and can be justified on grounds of financial market data mapping expectations about future economic activity (see Rigobon [31], Forbes [13]). The "conditional value-at-risk" (CoVaR) approach utilizing the Adrian and Brunnermeier [2] methodology, the Chan-Lau/Mitra/Ong methodology [5] estimating extreme contributions to outward spillovers as well as the Diebold/Yilmaz methodology [10, 11] also belong to this class.

One way to quantify the risk associated with investing in an asset is to estimate the variance of the error when forecasting a future return on the asset price. Multiple return series can be analyzed in a vector autoregressive (VAR) model, and the forecast error variance of each asset can then be decomposed with respect to its origin within the set of assets included in the analysis. Diebold and Yilmaz [10, 11] use this idea to develop a network view of a set of assets (or markets, each represented by a stock index), with assets (or markets) as nodes and edge weights determined by variance shares. This approach provides a framework to discuss pairwise spillovers, and Diebold and Yilmaz go one step further and suggest a summary measure for the degree of market connectedness which they call the spillover index.

Our methodological efforts in the present paper extend the spillover perspective, as elaborated by Diebold and Yilmaz [10, 11]. We show that the extant spillover perspective has shortcomings insofar as it confines the focus to either pairwise spillovers or the summary spillover index, not exploiting additional information contained in spillover tables which can provide further insight into the fashion markets interact. Our approach provides two perspectives, one with a focus on the current state of a system of markets (the network), capable of identifying a market's potential to act as a news spreader through what we call propagation values, and the other with a focus on market dynamics. In this context, we introduce several versions of relative market entropy (defined in terms of the Kullback-Leibler divergence of two probability distributions), which is a measure of the amount of information created from day to day, sensitive to different modalities in the injection of news into the system of markets. A further entropy measure (the Kolmogorov-Sinai entropy) can quantify the speed of information digestion in the system and can be interpreted as a measure of system stability. It is shown that a growing spillover index does not necessarily imply a growing speed of information digestion, although this is what we observe in the empirical example used as an illustration. Our methodology is in line with the requirements stated by the IMF, and some of our empirical findings mirror IMF findings, the latter requiring, supposedly, a more intricate array of research methods and resources.

Research on the extent and role of shock propagation in a network — with a focus on its dependence on the structure of interactions in the network — has recently been undertaken by Acemoglu et al. [1]. In this context, they provide a framework for the assessment of aggregate effects of microeconomic shocks, depending on intersectoral input-output channels. They attribute to the network a "defining effect" on the speed of decay of aggregate output fluctuations, "even at high levels of disaggregation", thus refuting the diversification argument. They address the potential of different sectors to induce "cascade effects", to which our propagation values constitute a quantitative analogue. Among their further findings is that asymmetry (for example, with respect to sectoral output shares in the input supply of the economy) makes the network more vulnerable to shocks. The present paper reaches similar conclusions (for example, asymmetry reducing network stability), even though the target of the analysis is different: While Acemoglu et al. [1] focus on the aggregate output, and its volatility, of an economy, we investigate the quality of the interplay of network components and the network's internal structure. Building on the Diebold/Yilmaz methodology [10, 11], and using a very simple data structure (series of daily returns on assets), our results extend into the area investigated by Acemoglu et al. [1].

This paper is organized as follows. An overview of the daily return data used to illustrate the methodology is provided in Section 2. Section 3 briefly reviews, and comments on, the Diebold/Yilmaz approach. Basic assumptions for further methodological advances are discussed in Section 4, and the foundation of how to interpret a spillover table in terms of information propagation in the network of markets is laid, with a focus on the traces a hypothetical shock leaves in the network. The perspective is shifted to the location of a shock in Section 5, leading to several probability distributions for each day. This allows the definition of entropy-based measures (relative market entropy) related to news injection in Section 6, and the Kolmogorov-Sinai entropy measuring speed of news digestion and network stability in Section 7. A method to construct a VAR model when a spillover table is given is presented in Section 8 and used to obtain insight into the fashion of market interplay. Results of the empirical example are reported in Section 9. Finally, Section 10 concludes the paper. — All computations were carried out with R [30] in a GNU/Linux environment. An R package with functionality covering the methods on which the present paper is based is available upon request from the authors.

2 Data

The empirical starting point of the present study consists of five time series of daily closing quotations of the stock indices Dow Jones Industrial Average (USA; in the following called dji), FTSE (UK; ftse), Euro Stoxx 50 (euro area; sx5e), Nikkei 225 (Japan; n225), and SSE Composite (China; ssec) in the time period beginning with 1997-07-03 (the first day for which all five series were available) and ending 2013-05-31 (4140 observations). Series of daily simple returns in percent are plotted in Figure 1.

A visual inspection of the return series in Figure 1 suggests a simultaneous occurrence of periods of high volatility in the five markets considered, and the impression that returns are somehow "connected" is reinforced in Figure 2, which shows scatterplots of pairwise daily returns below and the corresponding correlations above the diagonal. The weakest correlations are observed in the case of pairs with ssec as one component. — The main tool to measure the amount of connectedness is based on a decomposition of the forecast error variance, which will be briefly described next.

3 Measuring spillovers based on forecast error variance decomposition (fevd)

3.1 Vector autoregressive models and fevd

The approach of this paper is developed on the basis of a vector autoregressive (VAR) model with N return series as components. For ease of exposition, an order (lag) of 1 is assumed here. A structural VAR model in the form $\mathtt{B}x_t = \Gamma_0 + \Gamma_1 x_{t-1} + \epsilon_t$ (where (ϵ_t) is white noise with uncorrelated components) can also be written in terms of the MA representation:

$$x_t = \mu + \sum_{i=0}^{\infty} (\mathtt{B}^{-1}\Gamma_1)^i \, \mathtt{B}^{-1}\epsilon_{t-i} = \mu + \sum_{i=0}^{\infty} \Phi(i)\epsilon_{t-i} \tag{1}$$

with impulse response functions $i \mapsto \Phi(i) \equiv (\Phi_l^k(i))_{l,k}$, where $\Phi_l^k(i)$ quantifies the response of x_{lt} to a shock in $\epsilon_{k,t-i}$, happening i time units earlier. Impulse response analysis is often performed in terms of an MA representation (1) with unit variance shocks (which can be enforced by

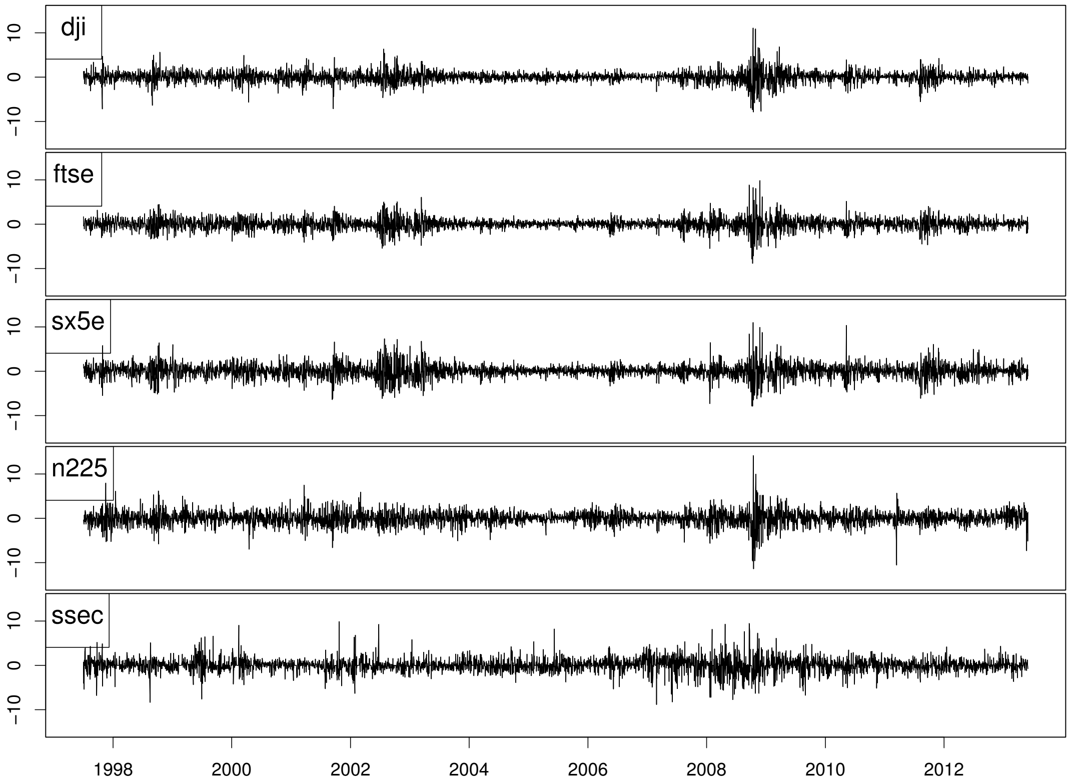


Figure 1: The series of daily returns

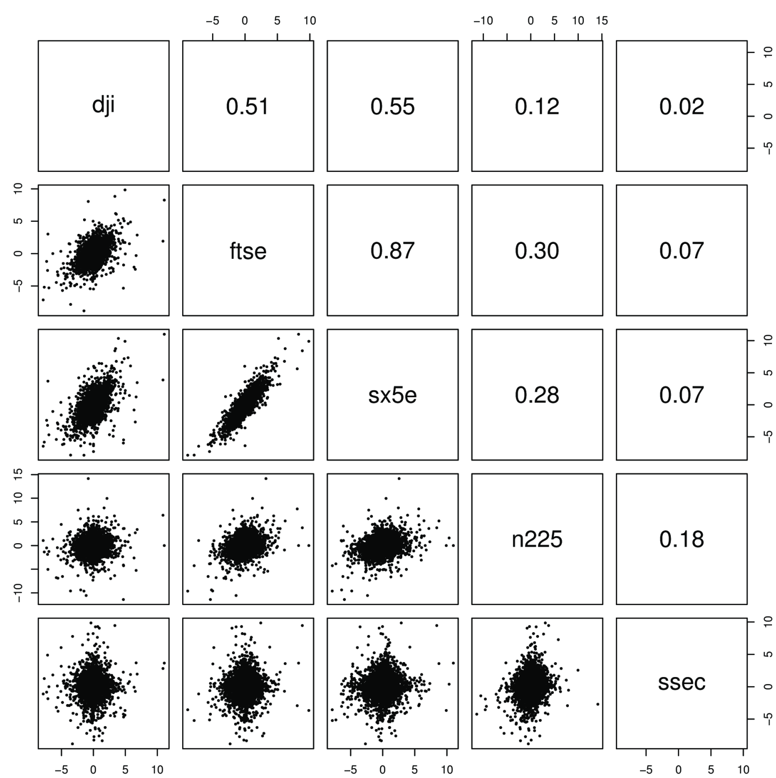


Figure 2: Scatterplots of daily returns

pre-multiplying the above structural equation by the diagonal matrix of standard deviation reciprocals).[4] Thus, Φ quantifies responses to shocks of size one standard deviation.

The variance of the n-period-ahead forecast of x_l ($l = 1, \ldots, N$) can then be written as a sum:

$$\mathrm{var}(x_{l,t+n} - \hat{x}_{l,t+n}) = \sum_{k=1}^{N} \sum_{i=0}^{n-1} (\Phi_l^k)^2(i) \tag{2}$$

The fevd is then expressed in terms of the ratios

$$\frac{\sum_{i=0}^{n-1} (\Phi_l^k)^2(i)}{\sum_{k=1}^{N} \sum_{i=0}^{n-1} (\Phi_l^k)^2(i)}, \quad l = 1, \ldots, N. \tag{3}$$

These expressions give the share of forecast variability in x_l due to shocks in x_k, or, in other words, the return spillovers to volatility. The shares are independent of relative volatility levels. Spillovers can be arranged in a spillover table (or matrix); for example with $N = 3$ return series:

$$\begin{array}{rr|ccc} & & \multicolumn{3}{c}{\text{from return } (x_k)} \\ & & x_1 & x_2 & x_3 \\ \hline & x_1 & \square & \blacksquare & \blacksquare \\ \text{to forecast error variability share in } (x_l) & x_2 & \blacksquare & \square & \blacksquare \\ & x_3 & \blacksquare & \blacksquare & \square \end{array} \tag{4}$$

Each row thus sums up to 1 (or 100%) and provides a breakdown of the forecast error variance of the corresponding stock index return with respect to its origin. Each entry in the spillover table is called a directional spillover.

Only a standard VAR model, without explicit contemporaneous interaction between variables but correlated errors, can be fitted to data. Assumptions about priorities concerning the path of contemporaneous influence lead to the identification of the structural VAR model, which yields the impulse response functions Φ. The shape of Φ, especially for small values of i, depends on these assumptions. In the applications below, we follow Diebold and Yilmaz [11] and use an approach suggested by Pesaran and Shin [28], namely: to identify the impulse response function of a component, give highest priority to that component. In other words: For the identification of Φ_l^k, use a Cholesky decomposition which allows x_k to have a contemporaneous impact on *all* other components $x_1, \ldots, x_N$. The fevd (3) converges rapidly as n grows; using an $n > 1$ can therefore be seen as a mitigation of the identification problem which is in line with the Pesaran/Shin approach.

For each day considered, the spillover table will consist of N^2 numbers, inflated from the originally given N returns for that day — there is clearly a need for summary. Diebold and Yilmaz [10] introduced the spillover index, defined as

$$S = \frac{\sum \blacksquare}{\sum \blacksquare + \sum \square} = 1 - \frac{1}{N} \sum \square. \tag{5}$$

In view of the parameterization (1), fevd and spillover index are (i) independent of the general volatility level, (ii) independent of the relative volatility levels (for example, multiplying one return series by a positive constant will affect neither fevd nor spillover index).

It is illuminating to compare the MGARCH and the fevd approach (as applied to returns), because both share the idea that returns in one market can directly influence aspects of volatility across markets. However, the models differ with respect to their goals. In the case of MGARCH,

[4] Following Lütkepohl [26], the R package "vars" provides an extensive set of VAR estimation tools which build on orthogonalized unit variance residuals.

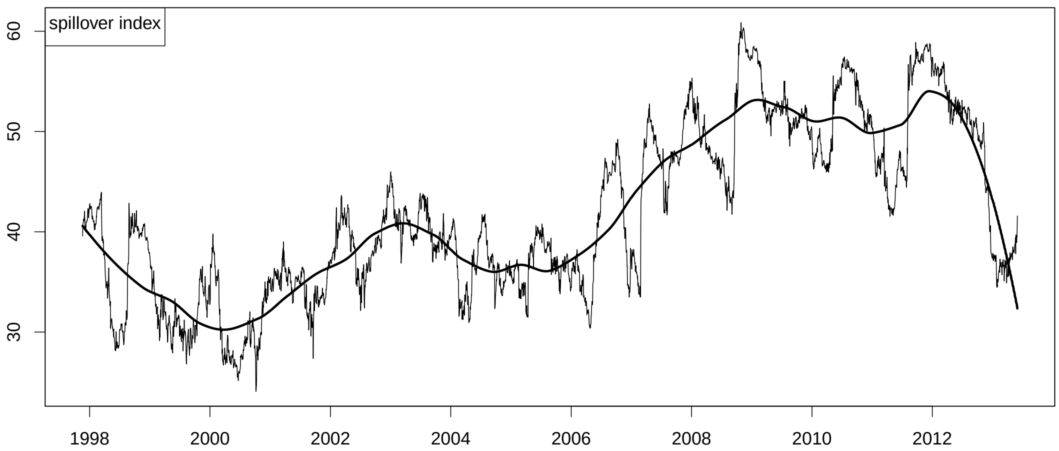


Figure 3: The spillover index series, the case dji/ftse/sx5e/n225/ssec

the goal is to forecast conditional (co-) variances, using a genuinely dynamic volatility model which specifies volatility as standard deviation of returns. In contrast, the fevd is based on a homoskedastic model, and volatility is an average over the period considered, specified as forecast error variance, the goal being to decompose volatility with respect to its origin across the markets. Furthermore, fitting an MGARCH usually requires series with well over 100 data points, which amounts to an implicit assumption that there is no structural break over a long period. On the other hand, a VAR model can be fitted to a shorter series, making the assumption of no structural break less restrictive, and in turn a sequence of fevds can pick up a change in circumstances much more readily.

Simulating a VAR model yields an empirical spillover table which will be more or less close to the theoretical spillover table. For purposes of illustrating our arguments below, the reverse question is also of relevance: Given a spillover table, how can we retrieve a VAR process such that the given table is its spillover table? This topic will be taken up again in Section 8 below.

3.2 Empirical spillovers

For the data set described in Section 2, a spillover index series was obtained by proceeding along the steps outlined above, where a moving window of 100 days (and $n = 5$) was used for fitting a sequence of VAR models of order 1, resulting in a spillover table, and hence spillover index, for every day. The result of this operation is shown in Figure 3, together with a smoothed version (obtained by local polynomial regression with a span of 10%) of the series. The smoothed line is a concession to the need for a more concise summary, and we shall see in Sections 8 and 9 that joint monotonicity of series can provide insight into the market mechanism. This series in Figure 3, and therefore market connectedness, has been strongly increasing since 2000, with intermittent shorter periods of decline. Similar findings were made by Diebold and Yilmaz [10]. It has dropped sharply since early 2012.

3.3 Limitations of the spillover perspective

A methodological focus on spillover index and directional spillovers in the investigation of market interaction can be called a *spillover perspective*. This perspective has limitations, though. For example, the spillover patterns in Table 1 differ widely, in particular with respect to the involvement of market 1: It is interacting neither with market 2 nor with market 3 in case (a); a shock to market 1 will have repercussions across other markets in cases (b), (c) and (d); and these repercussions will be most severe in case (d), the "from x_1" column containing the largest entries. However, the spillover index equals 40% in all four cases.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.4 & 0.6 \\ 0 & 0.6 & 0.4 \end{pmatrix} \qquad \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{pmatrix} \qquad \begin{pmatrix} 0.4 & 0.3 & 0.3 \\ 0.1 & 0.7 & 0.2 \\ 0.2 & 0.1 & 0.7 \end{pmatrix} \qquad \begin{pmatrix} 0.8 & 0.1 & 0.1 \\ 0.4 & 0.5 & 0.1 \\ 0.3 & 0.2 & 0.5 \end{pmatrix}$$

(a) (b) (c) (d)

Table 1: Four hypothetical examples: different spillover patterns, but equal spillover index values

Although it provides a summary of *aggregated* spillovers to and from markets (it can be expressed in terms of the white squares in (4), see Equation (5)), the spillover index is able to capture only a limited aspect of spillovers across markets. Of course, it would be possible to keep track of directional spillovers, in addition to the summary measure. We shall see below, however, that adopting a dynamic view which incorporates the entire spillover matrix at once leads to deeper insight into the interplay of markets. This is the program of the remainder of this study.

4 Shock propagation: the size of a shock

4.1 Assumptions

How are future volatilities across markets affected by a hypothetical shock hitting x_k on day t? The goal of this section is to discuss this question in terms of the spillover matrix for day t, which is assumed to summarize all relevant, and in particular the most recent available, information. While the rows of a spillover matrix characterize a market's exposure to shocks, broken down by origin of shock, the propagation of a shock needs to be read and tracked column-wise.

Consider, for example, a shock to market 2 in case (b) of Table 1. This can be read as resulting in volatility shares of 20%, 60% and 30% in markets 1, 2 and 3, respectively, or, in matrix notation: $\mathbf{M}_b \cdot (0,1,0)' = (0.2, 0.6, 0.3)'$. Interpreting a volatility share as a *potential* for further shocks, we can think of another round of shock propagation where $(0.2, 0.6, 0.3)'$ — instead of $(0,1,0)'$ — is fed into the system,[5] yielding the new shock potentials, and so on.

This reasoning is similar to a GARCH(1,1), where a shock to a return will impact the next period's volatility forecast (now in absolute terms, as the variance), and if a forecast for a later period is desired, one could substitute the first variance forecast (quantifying the "shock potential") for the squared (and mean-corrected) return.

These ideas can be condensed into the following set of **assumptions**:

> The propagation of a shock (or "information") across markets can be modeled by the transmission equation
>
> $$\mathbf{n}_{s+1} = \mathbf{M} \cdot \mathbf{n}_s, \quad s = 0, 1, 2, \ldots, \tag{6}$$
>
> where $\mathbf{M}$ denotes the spillover matrix of a certain day and $\mathbf{n}_0$ is a unit vector indicating the market from which the shock originates. The further propagation of the initial shock (that is, $s \geq 1$) can take place in a short time interval of unspecified length according to Equation (6). The index s therefore denotes a hypothetical step

[5] We use the terms *system of markets* and *network of markets* synonymously. Markets constitute the nodes, directional spillovers (or the transition probabilities developed in Section 5.2 below) the weights of edges; see also Diebold and Yilmaz [11].

in information flow, $\mathbf{n}_s$ characterizing what remains of the initial shock $\mathbf{n}_0$ across markets after s steps. Moreover, assuming that information flow across markets can proceed instantly, with spillover conditions persisting throughout the day in question, it makes sense to investigate steady-state properties (as $s \to \infty$) of the model defined by Equation (6).

The four hypothetical examples from Table 1 will serve to illustrate the dynamics of shock propagation, and its relation to where a shock comes from; see Table 2. Under spillover con-

	step number			$s=1$			$s=2$			$s=\infty$		
	origin of shock			1	2	3	1	2	3	1	2	3
(a)	1	0	0	1	0	0	1	0	0	1	0	0
	0	0.4	0.6	0	0.4	0.6	0	0.52	0.48	0	0.5	0.5
	0	0.6	0.4	0	0.6	0.4	0	0.48	0.52	0	0.5	0.5
(b)	0.6	0.2	0.2	0.6	0.2	0.2	0.40	0.30	0.30	0.20	0.40	0.40
	0.1	0.6	0.3	0.1	0.6	0.3	0.15	0.47	0.38	0.20	0.40	0.40
	0.1	0.3	0.6	0.1	0.3	0.6	0.15	0.38	0.47	0.20	0.40	0.40
(c)	0.4	0.3	0.3	0.4	0.3	0.3	0.25	0.36	0.39	0.21	0.35	0.44
	0.1	0.7	0.2	0.1	0.7	0.2	0.15	0.54	0.31	0.21	0.35	0.44
	0.2	0.1	0.7	0.2	0.1	0.7	0.23	0.20	0.57	0.21	0.35	0.44
(d)	0.8	0.1	0.1	0.8	0.1	0.1	0.71	0.15	0.14	0.64	0.19	0.17
	0.4	0.5	0.1	0.4	0.5	0.1	0.55	0.31	0.14	0.64	0.19	0.17
	0.3	0.2	0.5	0.3	0.2	0.5	0.47	0.23	0.30	0.64	0.19	0.17

Table 2: Four hypothetical examples: courses of shock propagation

ditions (a), the shock propagation ultimately (as $s \to \infty$) depends heavily on where the shock originates. This peculiarity is due to the limited connectedness of markets in this example. In examples (b), (c) and (d), however, the shock distribution will become stationary in the long run, with shock repercussions ultimately settling equally among the three markets, irrespective of the initial shock's origin. Here, all markets in the system are connected. However, the ultimate relative *impact level* does depend on the shock's origin: In examples (b) and (c), a shock coming from market 1 produces the lowest impact level as compared to shocks from other origins, while example (d) differs in this respect. The ratios of impact levels ("propagation values", see below) resulting from the spillover matrices (b), (c) and (d) in Table 1 are: 1: 2 : 2, 1 : 1.71 : 2.14, and 1 : 0.30 : 0.26, respectively. Tracing impact level ratios can provide valuable insight into the relative time-dependent dynamics of shock propagation, and hence vulnerability, in real markets; see Section 9 below. We shall see in the following how these ratios can be related to the eigenvalue structure of the spillover matrix.

4.2 The structure of shock propagation; propagation values

If the network of markets (the graph) is strongly connected in the sense that each pair of markets is connected within a finite number of transitions (case (a) above is not strongly connected), Perron-Frobenius theory carries over (see e.g. Seneta [35] and Caswell [4]). Then, the spillover matrix is irreducible and primitive, and its right and left eigenvectors $\mathbf{u}$ and $\mathbf{v}$ corresponding to the dominant eigenvalue 1 (a consequence of the row-stochasticity of the matrix), satisfying $\mathbf{u} = \mathbf{M} \cdot \mathbf{u}$ and $\mathbf{v}' = \mathbf{v}' \cdot \mathbf{M}$, respectively, are unique up to a scalar, and describe the stable relative impact structure in the long run.

In the cases (b), (c) and (d) of Table 2, right eigenvectors are the *column* vectors under $s = \infty$ (all equal to $(1/3, 1/3, 1/3)'$ when normed such that the sum of their components equals 1). Left eigenvectors can be found when reading the entries under $s = \infty$ *row-wise*, which coincide with the vectors in Table 3 after scaling so that the first entry is equal to 1. That right eigenvectors have equal components reflects the fact that a shock ultimately distributes evenly among strongly connected markets. The components of the left eigenvector constitute impact level ratios. In example (b), a shock coming from market 2 (or 3) has twice the power of a shock from market 1, while market 1 stands out as the most powerful one with respect to shock propagation in example (d). — Remember that the spillover index equals 40% for all examples (a) through (d).

From now onwards, we will use the following terminology: The k-th entry of the (normed, unless clear from the context) left eigenvector of the spillover matrix is called the **propagation value** of market k. This value can be interpreted as the relative value of a shock to market k (that is, to x_k) as seed for future variability in the markets.

left	right
n.a.	not
n.a.	unique
n.a.	

(a)

left	right
1	1/3
2	1/3
2	1/3

(b)

left	right
1	1/3
1.71	1/3
2.14	1/3

(c)

left	right
1	1/3
0.30	1/3
0.26	1/3

(d)

Table 3: Four hypothetical examples: eigenvectors (corresponding to eigenvalue 1)

Two further examples, shown in Table 4, may illustrate that the notion of a market's propagation value does not simply mirror its *direct* spillovers to other markets. These examples

$$\begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{pmatrix} \qquad \begin{pmatrix} 0.7 & 0.1 & 0.2 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}$$

left	right
1	1/3
1	1/3
1	1/3

left	right
1	1/3
0.71	1/3
0.86	1/3

Table 4: Two examples with equal aggregate spillovers (and equal spillover index values)

again share a spillover index of 40%, as well as aggregate spillovers to other markets of 0.4 (all off-diagonal column sums equal 0.4). However, there is a fundamental difference in the system's behavior which would be disregarded when confining the analysis to only one propagation step. This difference is — to some extent — captured by means of propagation values.

Concepts similar to those presented in this section have been used in matrix-based population theory, see Caswell [4], the fundamental structure being a Leslie-matrix. A life cycle (reproductive value) in the population context translates into a "shock cycle" (propagation value, respectively) in our context.

5 Shock propagation: the location of a shock

5.1 A shock traveling through the network

The focal point of Section 4 was the relative size of repercussions when a shock hits one market in the network. We will now switch to a different (and in a certain sense dual) perspective, namely: a shock of unit size traveling randomly through the network according to a Markov chain, where transition probabilities are determined by the spillover matrix. It is again assumed that a spillover matrix is given for each day, which rules the information propagation — the latter now in the sense of the shock moving from node to node (market to market) within that day. In light of this, a variety of phenomena can be studied:

- the "information equilibrium" or "news balance", that is, the stationary distribution of shock location (Section 5.2),
- the disturbance of the news balance, when a shock hits the network (Section 6),
- the information gain from day to day (Section 6),
- the speed of convergence back to news balance after a shock has hit the network (Section 7).

5.2 Spillover matrices and transition matrices

The most important ingredient of a discrete-time Markov chain, the transition matrix, is essentially a collection of conditional distributions in the form of a row- or column-stochastic matrix, the layout depending on the formalism adopted to update state probabilities. The condition is expressed in the current state of the process. It is thus clear that a spillover table, as given in (4), does not constitute the transition matrix of a Markov chain: Column sums of a spillover table need not equal 1. (Columns constitute the "from" part, i.e. they specify the condition.) Coerced into the Markov chain scheme, spillover tables would make the Markov chain run in reverse time ("from" followed by "to"). However, it is again a concept first developed in the context of population dynamics that provides the necessary means to define a suitable Markov chain.

The shock propagation equation (6) transforms into a Markov chain running forward in time along the following steps (cf. Tuljapurkar [39]): First, it holds that

$$\mathbf{n}'_{s+1} \cdot \mathbf{V} \;=\; \mathbf{n}'_s \cdot \mathbf{V} \cdot \mathbf{V}^{-1} \cdot \mathbf{M}' \cdot \mathbf{V}, \quad s = 0, 1, 2, \ldots,$$

where the diagonal matrix $\mathbf{V}$ is made up by the left eigenvector $\mathbf{v}$ of matrix $\mathbf{M}$ (corresponding to eigenvalue 1). Now, the Markov transition matrix can be identified (we use the notation of Tuljapurkar [39]; subscript "F" means "forward"):

$$\mathbf{P}_F \;=\; \mathbf{V}^{-1} \cdot \mathbf{M}' \cdot \mathbf{V}, \tag{7}$$

and after re-scaling

$$\pi'_s \;=\; \frac{\mathbf{n}'_s \cdot \mathbf{V}}{\mathbf{n}'_0 \cdot \mathbf{v}},$$

the Markov chain equation emerges:

$$\pi'_{s+1} \;=\; \pi'_s \cdot \mathbf{P}_F, \quad s = 0, 1, 2, \ldots, \tag{8}$$

with probability vectors π_s.[6] In the case of stationarity (if markets are strongly connected), $\pi_s \to \pi_\infty$ as $s \to \infty$, where π_∞ satisfies

$$\pi'_\infty \;=\; \pi'_\infty \cdot \mathbf{P}_F. \tag{9}$$

Table 5 shows the Markov chain transition matrices obtained for the hypothetical examples of Table 1 (with case (a) excluded), together with state probabilities π_1, π_2, and π_∞, when π_0 is one of $(1,0,0)'$, $(0,1,0)'$, or $(0,0,1)'$. Two observations are remarkable: (i) the shock origin is irrelevant in the long run; (ii) the stationary distributions π_∞ equal the propagation values.

[6]The Markov chain transformation running backward in time would employ the matrix $\mathbf{M}$ itself as transition matrix, cf. Tuljapurkar [39], and our introductory remarks.

step number / origin of shock		$s=1$: 1	$s=1$: 2	$s=1$: 3	$s=2$: 1	$s=2$: 2	$s=2$: 3	$s=\infty$: 1	$s=\infty$: 2	$s=\infty$: 3
(b)	$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.6 \\ 0.2 \\ 0.2 \end{bmatrix}$	$\begin{bmatrix} 0.1 \\ 0.6 \\ 0.3 \end{bmatrix}$	$\begin{bmatrix} 0.1 \\ 0.3 \\ 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.4 \\ 0.3 \\ 0.3 \end{bmatrix}$	$\begin{bmatrix} 0.15 \\ 0.47 \\ 0.38 \end{bmatrix}$	$\begin{bmatrix} 0.15 \\ 0.38 \\ 0.47 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$
(c)	$\begin{bmatrix} 0.4 & 0.17 & 0.43 \\ 0.18 & 0.7 & 0.13 \\ 0.14 & 0.16 & 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.4 \\ 0.17 \\ 0.43 \end{bmatrix}$	$\begin{bmatrix} 0.18 \\ 0.7 \\ 0.13 \end{bmatrix}$	$\begin{bmatrix} 0.14 \\ 0.16 \\ 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.25 \\ 0.26 \\ 0.49 \end{bmatrix}$	$\begin{bmatrix} 0.21 \\ 0.54 \\ 0.25 \end{bmatrix}$	$\begin{bmatrix} 0.18 \\ 0.25 \\ 0.57 \end{bmatrix}$	$\begin{bmatrix} 0.21 \\ 0.35 \\ 0.44 \end{bmatrix}$	$\begin{bmatrix} 0.21 \\ 0.35 \\ 0.44 \end{bmatrix}$	$\begin{bmatrix} 0.21 \\ 0.35 \\ 0.44 \end{bmatrix}$
(d)	$\begin{bmatrix} 0.8 & 0.12 & 0.08 \\ 0.33 & 0.5 & 0.17 \\ 0.38 & 0.12 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.8 \\ 0.12 \\ 0.08 \end{bmatrix}$	$\begin{bmatrix} 0.33 \\ 0.5 \\ 0.17 \end{bmatrix}$	$\begin{bmatrix} 0.38 \\ 0.12 \\ 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.71 \\ 0.17 \\ 0.12 \end{bmatrix}$	$\begin{bmatrix} 0.49 \\ 0.31 \\ 0.20 \end{bmatrix}$	$\begin{bmatrix} 0.54 \\ 0.16 \\ 0.30 \end{bmatrix}$	$\begin{bmatrix} 0.64 \\ 0.19 \\ 0.17 \end{bmatrix}$	$\begin{bmatrix} 0.64 \\ 0.19 \\ 0.17 \end{bmatrix}$	$\begin{bmatrix} 0.64 \\ 0.19 \\ 0.17 \end{bmatrix}$

Table 5: Four hypothetical examples: transient and stationary distributions of the Markov chain

This is no coincidence. In fact, the Markov chain in Equation (8) becomes stationary in the long run if the spillover matrix $\mathbf{M}$ is irreducible and primitive; cf. Tuljapurkar [39]. According to arguments in Section 4.2, this translates into the condition of strongly connected markets. Then, from Equation (8), it is obvious that the stationary distribution of the Markov chain must equal the left eigenvector (normed to 1) of the Markov transition matrix $\mathbf{P}_F$ (corresponding to eigenvalue 1). Indeed, what turns out in particular is that the stationary distribution and the — normed — vector of propagation values, introduced as left eigenvector $\mathbf{v}$ of $\mathbf{M}$ in Section 4.2, are the same:

$$\pi_\infty \;=\; \frac{\mathbf{V}\cdot\mathbf{u}}{\mathbf{v}'\cdot\mathbf{u}} \;=\; \frac{\mathbf{v}}{|\mathbf{v}|}. \tag{10}$$

(Recall from Section 4.2 that the vector $\mathbf{u}$ denoting the right eigenvector of $\mathbf{M}$ corresponding to eigenvalue 1, has equal components if $\mathbf{M}$ is irreducible and primitive.)

5.3 The within-day perspective

This perspective picks up directly on the model dynamics presented in Section 4.1: a hypothetical shock hits the network on day t and travels through the network, from node to node, according to matrix P_F. The stationary distribution of shock location is given by the normed left eigenvector of $\mathbf{M}$, that is, the propagation value. Whichever market generates the shock initially, if the system is strongly connected, the Markov chain will move towards the equilibrium which is governed by the markets' propagation values. The vector of propagation values represents the "news balance" of markets in the long run. The origin of shocks is "forgotten" as shocks are "digested" by the system. The speed of digestion will be discussed in Section 7 below.

The analogy to Tuljapurkar's [39] interpretation of the Markov chain transformation as a sequence space description of genealogies is that the Markov chain in (8) describes the "ancestry" of a shock. An "ancestor" (here: market) with a high propagation value is a "super-spreader" (to use the terminology of Haldane and May [15] or Scott [34]). In Table 5, market 1 of case (d), with a propagation value of 0.64 (compared to 0.19 and 0.17 of the other markets), is a "super-spreader" in this sense.

5.4 The day-to-day perspective

A non-homogeneous Markov chain — with time-dependent transition matrices — can be defined on the basis of the sequence $(\mathbf{M}_t)_t$, where $\mathbf{M}_t$ is the spillover table for day t. Here, the idea is that the initial shock took place at the beginning ($t = 0$), so that news balance is not attained within a day. If $P_{F,t}$ designates the transition matrix obtained from $\mathbf{M}_t$, as outlined in Section 5.2, the distribution of shock location on day t is given by

$$\pi_t' = \pi_0' \cdot \prod_{r=1}^{t} P_{F,r}, \tag{11}$$

and this distribution will (in real-world markets) be independent of π_0. This perspective will be useful for measuring the amount of information injected into the system of markets on a daily basis.

6 Shocks, equilibrium, and relative market entropy

The spillover table is updated every day, reacting to new information provided in the form of price changes. How much information is created every day? How much information would a shock inject into the system? The importance of a shock in this sense will depend on the time and location of its appearance: it may sometimes disturb the equilibrium heavily and go almost unnoticed at other times. These questions can be discussed in terms of information entropy. Entropy measures are functionals of one or more probability distributions. Several distributions are associated with each day:

- the initial shock location distribution (a unit vector),
- the stationary distribution of shock location (Section 5.3),
- the non-stationary distribution of shock location (Section 5.4),

and within-day as well as between-day comparisons, using the concept of relative entropy, can make sense. The relative entropy of probability distribution π_a with respect to distribution π_b is defined as

$$\text{KLIC} \;=\; \sum_i \pi_a(i) \cdot \log_2 \left(\frac{\pi_a(i)}{\pi_b(i)} \right); \tag{12}$$

KLIC stands for Kullback-Leibler information criterion.[7] If, for example, π_a denotes any initial distribution of a Markov chain and π_b its stationary distribution ("news balance" in our interpretation), KLIC provides a measure of how distant the initial distribution is from equilibrium. Translated into the context of shock propagation, it measures the initial information content of a shock (news) with respect to the news balance between markets in the long run. In cases like this (when π_b characterizes the system of markets), we call KLIC the **relative market entropy**. The corresponding market entropy (not used here) would be $-\sum_i \pi_b(i) \log_2 \pi_b(i)$.

The examples in Table 6 illustrate the relative entropy of the initial shock distribution (a unit vector) with respect to the stationary distribution (see also Table 5). With the convention "$0 \cdot \log_2 0 = 0$", the relation between relative market entropy and propagation value is KLIC $= -\log_2 \pi_b(i)$, where i indicates the shock origin: information gain of a shock is the largest when it comes from the market where it is the least expected.

This concept of relative market entropy will be applied in Section 9 below to three cases:

[7] It is also called Kullback-Leibler divergence of π_b from π_a or information gain of π_a with respect to π_b. The original term used by Kullback and Leibler is "discrimination information", cf. [22], [23], [24], as it was conceived as the average information for discrimination between two statistical populations a and b per observation from a. Therefore, KLIC provides a measure of "information gain" of observing distribution π_a as compared to distribution π_b. Intuitively, the relative entropy is a lower bound for the number of additional symbols needed for identifying the outcome of a random experiment governed by π_a when using a code which is optimal for identifying outcomes of experiments governed by π_b; see Cover and Thomas [6].

	transition matrix	π_∞	relative entropy origin of shock 1	2	3	KS entropy
(b)	$\begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.6 & 0.3 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$	$\begin{bmatrix} 0.2 \\ 0.4 \\ 0.4 \end{bmatrix}$	2.322	1.322	1.322	1.311
(c)	$\begin{bmatrix} 0.4 & 0.17 & 0.43 \\ 0.18 & 0.7 & 0.13 \\ 0.14 & 0.16 & 0.7 \end{bmatrix}$	$\begin{bmatrix} 0.21 \\ 0.35 \\ 0.44 \end{bmatrix}$	2.280	1.503	1.181	1.242
(d)	$\begin{bmatrix} 0.8 & 0.12 & 0.08 \\ 0.33 & 0.5 & 0.17 \\ 0.38 & 0.12 & 0.5 \end{bmatrix}$	$\begin{bmatrix} 0.64 \\ 0.19 \\ 0.17 \end{bmatrix}$	0.646	2.363	2.585	1.101

Table 6: Relative entropy (information gain) and KS entropy

1. π_a is a unit vector, specifying the initial location of the shock, and π_b is the stationary distribution (news balance; see Equation (9)) of the day in question,

2. π_a is today's stationary distribution, π_b is yesterday's stationary distribution, thus adopting the perspective of Section 5.3 to two subsequent days,

3. π_a is today's non-stationary distribution (Equation (11)), π_b is yesterday's non-stationary distribution, thus adopting the perspective of Section 5.4 to two subsequent days.

Case 2 compares intrinsic properties of spillover tables, completely ignoring the previous history of the process. Case 3 allows a somewhat smoother transition from past to present, possibly leaving traces of earlier state probabilities. This makes the comparison of non-stationary distributions more sensitive to *abrupt* changes, while the comparison of stationary distributions is sensitive to *any* change, with an emphasis on measuring its magnitude.

Schoen and Kim [33], in a population theoretical context, relate the Kullback-Leibler distance to the momentum of a population. This idea suggests an analogy to the momentum of the "injection" of news to the market.

7 Speed of shock digestion

News balance, as defined in Section 5.1, is an intrinsic equilibrium property of a system of markets which is determined by the spillover table. A hypothetical shock to a market will disturb this equilibrium, but the markets will, under the assumptions made in Section 4.1, ultimately "digest" the shock and reach equilibrium again. How fast can the system converge back to equilibrium after being hit by a shock? Technically speaking, the question here is: Given a unit vector as initial distribution (specifying shock origin), how fast will the transient distribution of shock location converge to the stationary distribution?

An appropriate measure of convergence speed, and hence the speed of shock "digestion" of the system, is the Kolmogorov-Sinai (KS) entropy, which characterizes the dynamical randomness that accompanies the paths of a stochastic process. Given a set of initial states of the process, it is the exponential rate at which the number of most probable paths will increase with time, thus measures the rate of convergence to equilibrium.[8] It is defined as

$$\mathrm{KS} \;=\; -\sum_{i,j} \pi(i) \cdot \log_2 \left(\mathbf{p}_{ij}^{\mathbf{p}_{ij}} \right). \tag{13}$$

Here, $\mathbf{p}_{ij}$ denote the entries in the transition matrix of the Markov chain (Equation (8)) and $\pi(i)$ are the stationary probabilities.

Table 6 also gives KS entropy values for cases (b), (c) and (d) of Table 1. Case (b) provides the highest and case (d) the lowest speed of convergence, with case (c) in between. The main difference between cases (c) and (d) is that there is no "super-spreader" in the former, making the news digestion faster. Intuitively, the large diagonal elements of (c) and (d) make the initial distribution stickier than the smaller ones in (b). Equivalently, the rows of (b) are "more similar", enabling the system to stir more forcefully.

Demetrius [8] introduced this entropy measure to population theory as "population entropy"; Tuljapurkar [39] relates it to the rate of convergence of a population, respectively its corresponding Markov chain, to its equilibrium.

In Section 8, further examples illustrate that the concept of KS entropy in our context of a system of markets can be used as a "barometer" to measure the network's "stability".

8 Construction of processes with time-varying characteristics

8.1 Motivation

A spillover table results from a VAR process; it is at the center of the present investigation. This section will tackle the reverse problem: Given a spillover table, how can we retrieve a VAR process such that the table at hand is its spillover table? This is an interesting question in its own right, but actually not treated here as an end in itself since it serves a specific purpose in our line of argument.

It was shown in the previous sections how several market characteristics can be derived from a spillover table. Conversely, having the means to construct a process with predetermined features can be an important tool in studying the joint behavior of, for example, spillover index and KS entropy, especially by constructing a process with time-varying properties. This helps evaluate and interpret empirical findings in Section 9 below.

An algorithm for process construction will be outlined in Section 8.2; it will be applied to obtain examples of processes with time-varying properties in Section 8.3.

8.2 Process construction algorithm, spillover table given

Retrieving a VAR model on the basis of a given spillover table clearly creates an identification problem. It is not the mere mathematical task of finding a matrix function Φ of time such that sums of squared entries across time periods can reproduce the corresponding spillover table entries up to a constant factor and within a sufficient tolerance. An adequate answer has to take the impulse response character of Φ into account and ensure that matrix components jointly specify a stable, therefore stationary and well-defined structural VAR model. An underdetermined, though constrained optimization problem arises. In an effort to find (and simulate) first-order VAR processes yielding the example spillover patterns in Table 1, the sum of squared distances between given spillover entries and those resulting from a structural VAR specification was minimized with respect to VAR coefficient matrices B and Γ_1. Parameter values were searched within box constraints of ± 1 and subject to the following conditions:

[8] In information theory, it is known under the name "Shannon entropy", in recognition of the work by Shannon [36]; cf. Kolmogorov [21], and Sinai [37].

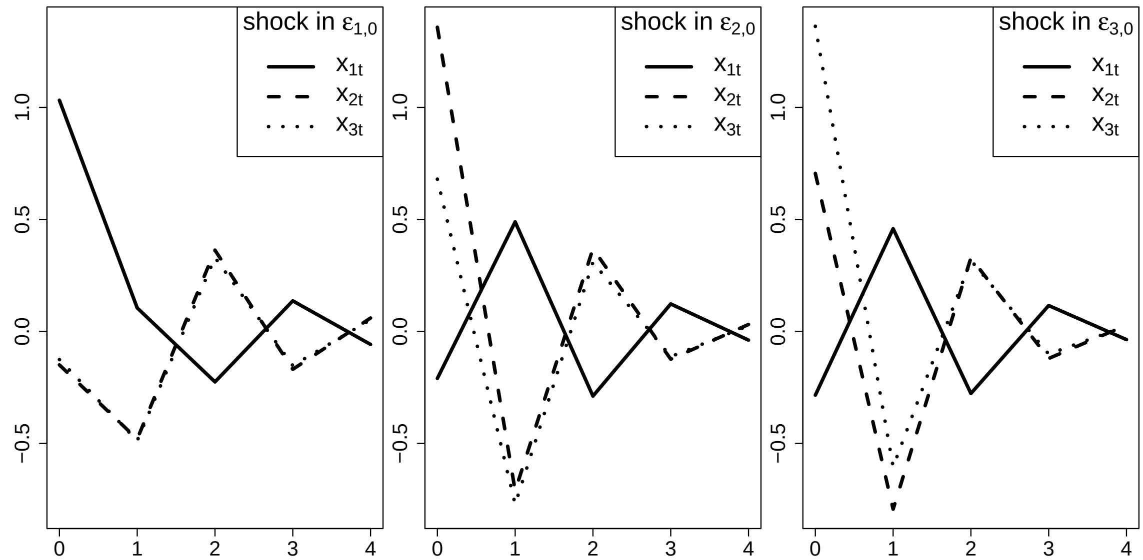


Figure 4: An impulse response function reproducing the spillover table in example (b)

- All eigenvalues of $\mathtt{B}^{-1}\,\Gamma_1$ have modulus less than 1 (process stability).
- The n-period-ahead fevd is sufficiently settled for $n = 5$ (fevd convergence).
- All elements in the main diagonal of $\mathtt{B}$ equal 1 (normalization).

(It would be possible to include a Cholesky-style condition, that is: "$\mathtt{B}$ is an upper, or lower, triangular matrix", as an additional constraint; however this is not necessary and was not done in the examples below.)

A typical resulting impulse response function for the case of example (b) in Table 1 is illustrated in Figure 4. Time series realizations can be generated based on a simulation of a white noise process (with uncorrelated, unit variance components) and the VAR coefficients provided. (No such simulation is shown here.) The role of markets 2 and 3 as equally effective news disseminators is clearly visible in the impulse response functions, which is in line with their propagation values of 40% each. Market 1 is different, with its damped impact mirroring its lower propagation level of 20%.

8.3 Time-varying spillover patterns

A VAR process with time-varying properties can be constructed, for example, by starting with the VAR model from Section 8.2 and then switching to a new set of parameters, each obtained from a new given spillover table, every 50 steps (time units), until step 1000 is reached. Two scenarios have been created in this manner, both with spillovers increasing from 40% to about 50%, but with their KS entropy series behaving differently:

- Scenario 1: KS entropy increasing, the final parametrization of the process is such that propagation values are more or less balanced, namely (34% : 35% : 31%);
- Scenario 2: KS entropy decreasing, with final parametrizatition such that propagation values are more distinct, namely (8% : 43% : 49%).

Their theoretical spillover index series and KS entropy series are displayed in Figure 5, simulations of both scenarios are plotted in Figure 6. The empirical spillover index series and KS entropy series are similar to their theoretical analogues of Figure 5.

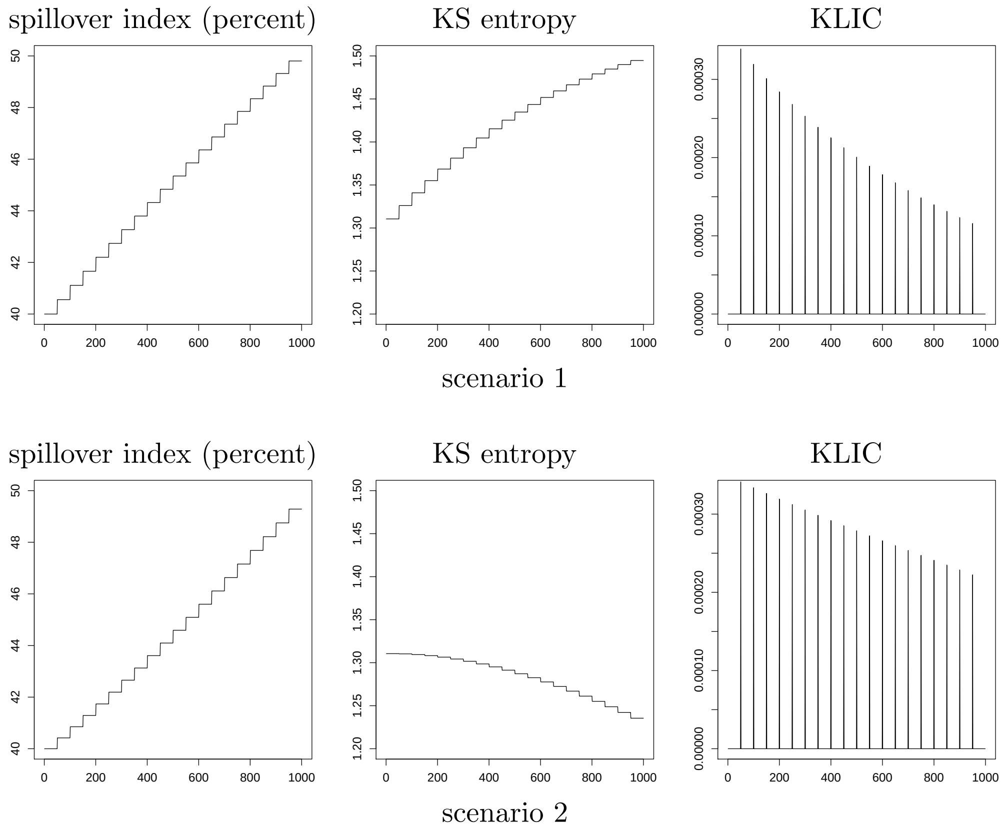


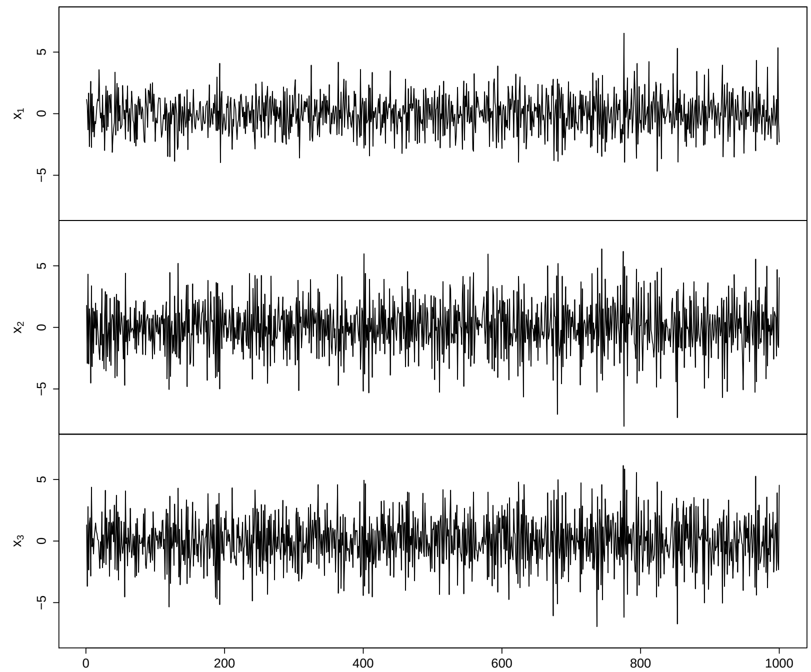
Figure 5: Time-varying spillover patterns: two scenarios

It is thus demonstrated that different qualitative patterns of shock propagation, or information transmission, can be consistent with an increasing spillover index series — information transmission is speeding up in scenario 1, while it is slowing down in scenario 2.

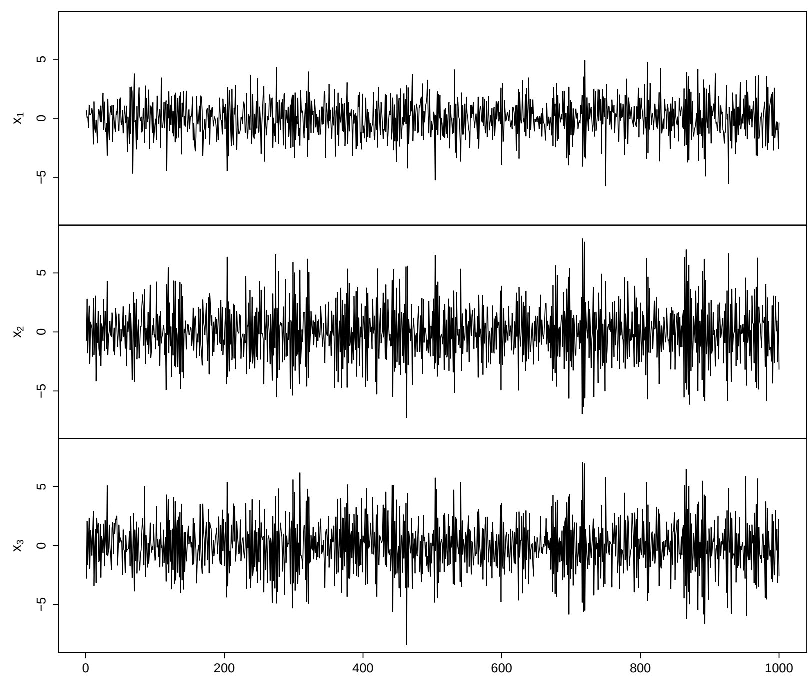
As was pointed out in Section 7, a high value of KS entropy relates to fast "digestion" of a hypothetical shock hitting a system of markets, in terms of fast convergence towards the system's news balance. In scenario 1, the system's potential of fast "digestion" strengthens across time which is accompanied by a leveling of propagation values. There is no "super-spreader" in the end. A shock is neither sticky in one market nor bounced off by another market. Scenario 2, on the other hand, manifests increasing spillover asymmetries, to the effect that two "super-spreaders" emerge.

In network theory, the speed of "relaxation" after perturbations is a measure of a network's "stability"; see Csermely [7] characterizes network stability as "robust behavior gravitating towards certain parameter sets, or attractors of the network", which represent equilibrium conditions. Fast relaxation occurs when perturbations "dissipate locally", distributed over the network's elements, which requires power-law distributions ("scale-freeness") and efficient connectivity ("small-worldness" and "nestedness"), while stickiness of perturbations in one of the network's elements accompanies lack of stability.

Therefore, from the network point of view, the course of KS entropy in scenario 1 suggests a strengthening of stability, while stability is lost in scenario 2. In this respect, the KS entropy of spillover dynamics is a "barometer" of network stability.



scenario 1



scenario 2

Figure 6: Time-varying spillover patterns: simulation results

9 Empirical findings

The empirical basis of our study of interaction between the stock indices Dow Jones Industrial Average (dji), FTSE (ftse), Euro Stoxx 50 (sx5e), Nikkei 225 (n225) and SSE Composite (ssec) is a daily sequence of return spillover matrices which resulted from fitting a sequence of VAR models along the steps outlined in Section 3. It was collapsed into the return spillover index series in Figure 3, which essentially indicates an upward trend in market integration since 2000, showing a downward movement only recently. The spillover index is an appealing summary measure; spillover patterns, however, may differ widely given one spillover index value, as was pointed out in Section 3. We will now proceed and apply the theoretical framework established in Sections 4 through 8 to obtain further insight into the markets' interplay, and how it evolved across time.

Figure 7 displays the sequence of propagation values for each of the five markets.

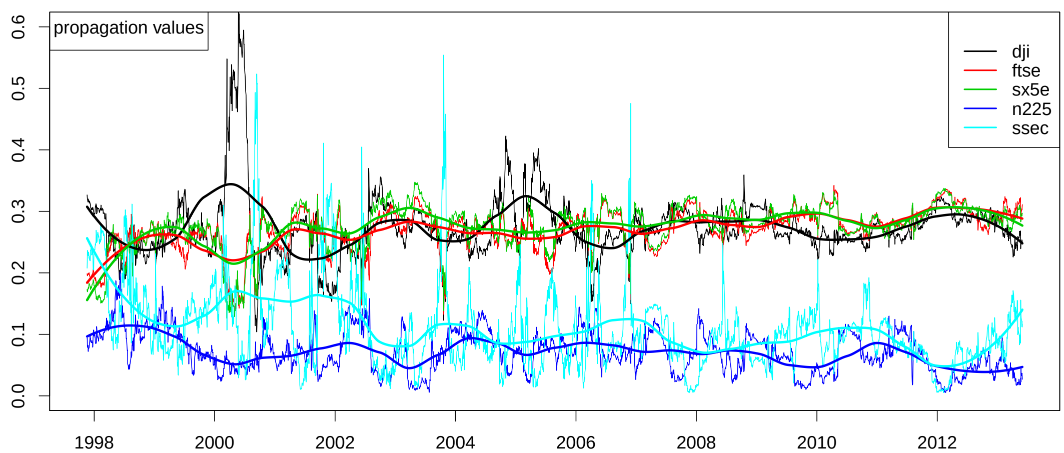


Figure 7: Propagation values

As a matter of fact, news balance between the five economies considered in this study is in a continuous process of relocation in response to economic and geopolitical events. However, there is evidence for rather synchronous movements between the European markets (ftse, sx5e). From 2006 onward, dji appears to have taken over a less prominent role as news spreader and to have approached the level of European markets, after earlier distinct peaks of propagation potential, which can be related to the crash of the "dot-com bubble" in early 2000[9], the FED's reversal of its loose-interest stance in June 2004, and the "dollar crisis" of 2005[10]. The amount of news produced by the n225 and ssec is much less uniform. Wild swings of propagation values can be observed for ssec, while n225 shows rather damped propagation values at the lowest level among the five markets.

Different versions of relative market entropy are plotted in Figures 8, 9 and 10; they were introduced in Section 6 as the Kullback-Leibler divergence of distribution π_b from distribution π_a,

[9] "...The technology-heavy Nasdaq reached its pinnacle of 5,048.62 on March 10, [2000]. Then the Internet bubble burst and the index plummeted nearly 40 percent, dropping below 3,000 in December [2000] in its worst annual loss", The New York Times, 2012-03-13.

[10] Gourinchas [14] argues that "...As deflationary risks started to fade and the economy's recovery took hold, the FOMC [Federal Open Market Committee] shifted towards tightening its monetary stance. Starting in June 2004, the FOMC increased interest rates gradually and measurably until June 29, 2006 when they reached a plateau of 5.25%." Diebold and Yilmaz [10] refer to the "dollar crisis" of March 2005 as "...associated with remarks from policy makers in several emerging and industrialized countries (South Korea, Russia, China, India and Japan) indicating that they were considering central bank reserve diversification away from the US dollar."

both distributions either characterizing the initial distribution of a hypothetical shock or aspects of news dispersion.

The relative market entropy in terms of market repercussions of a hypothetical shock with respect to news balance (as given by the stationary distribution) can be traced in Figure 8. It suggests the existence of an "informational divide" between the US (dji) and European markets (ftse, sx5e) on the one hand and the Asian markets (n225, ssec) on the other: the magnitude of market repercussions of a hypothetical shock still depends on whether the shock comes from the US/European or the Asian markets. No informational divide in this sense exists between dji, ftse and sx5e.

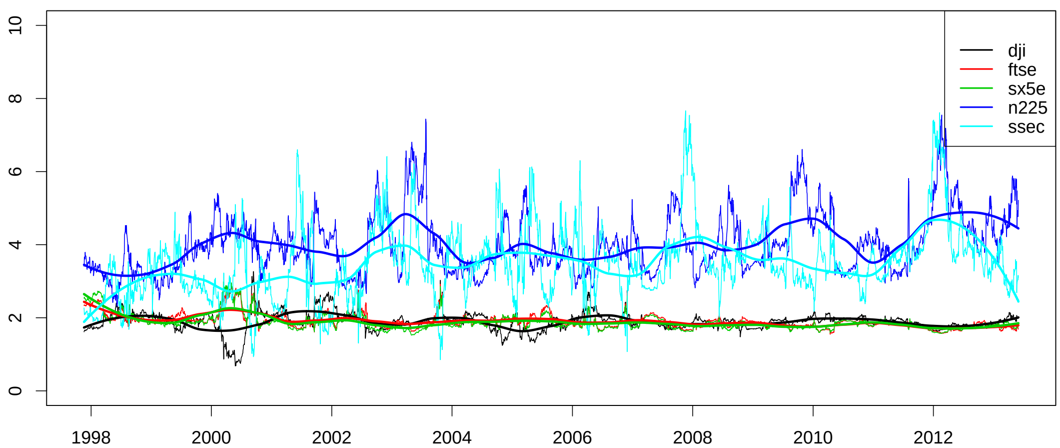


Figure 8: Relative entropy, different shock origins

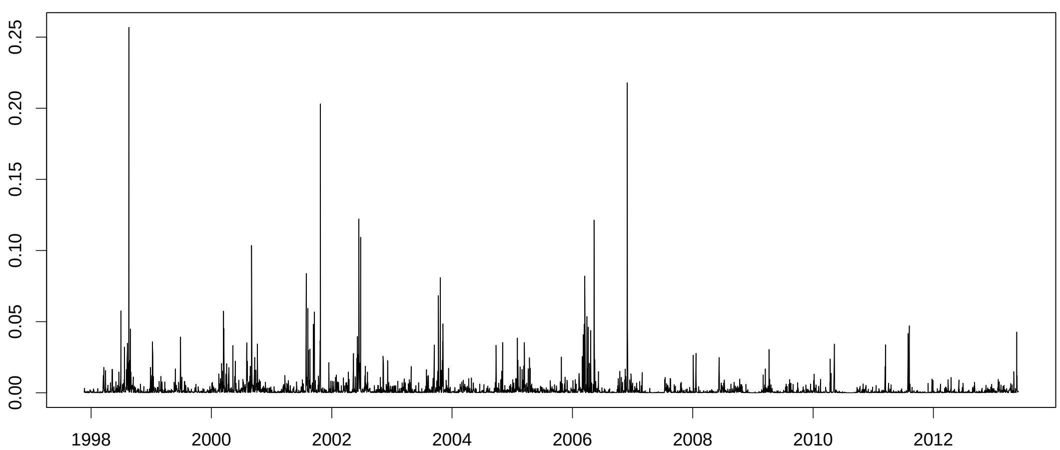


Figure 9: Relative entropy, day by day, stationary case

Figure 9 shows the relative market entropy, with an emphasis on measuring the amount of information created from day to day (disregarding its character: more gradual or more abrupt). The magnitude of daily information gain has decreased since about 2007. On the other hand, relative market entropy, when measured in terms of non-stationary distributions, has increased (Figure 10): daily information gain has become more abrupt.

Finally, the speed of information digestion (defined via the Kolmogorov-Sinai entropy, see Section 7) has been increasing over the larger part of the time period considered, according to Figure 11. A similar observation was made for the spillover index series (Figure 3), and it is

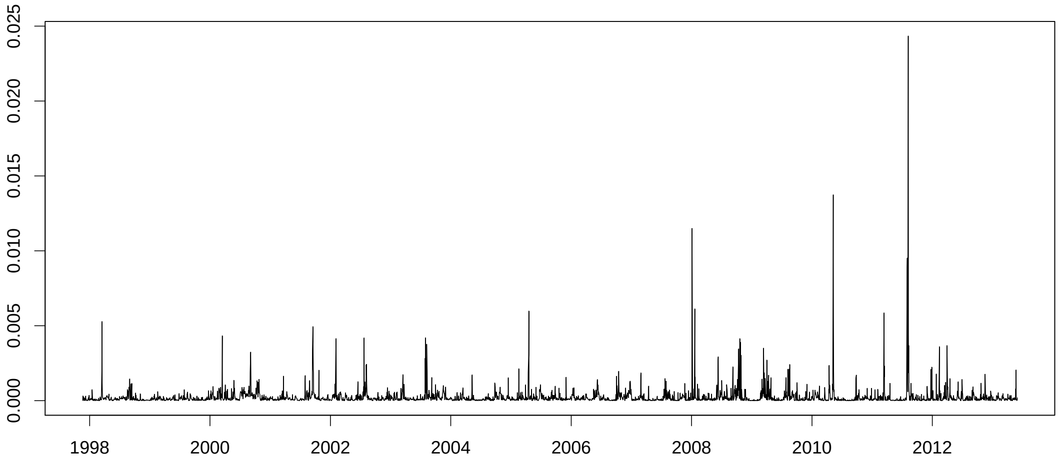


Figure 10: Relative entropy, day by day, non-stationary case

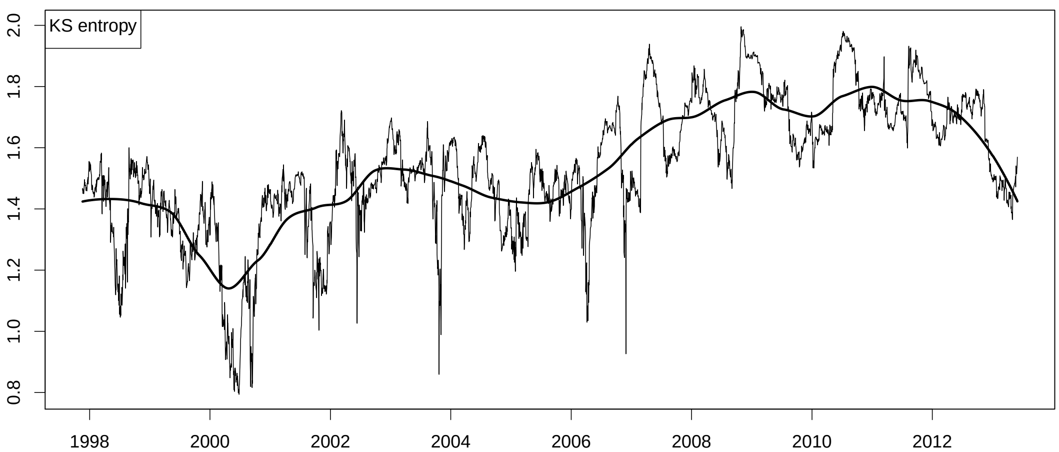


Figure 11: KS entropy (speed of convergence)

revealing to investigate the *joint* monotonicity behavior of both series. It can be shown that the series are both increasing or both decreasing on about 75% of all 4140 days covered here; on 25% of the days, they move in opposite directions. When smoothed, these shares will change, depending on the amount of smoothing; the share of days when both (smoothed) series move in the same direction does not drop below 75%, no matter how strongly the series are smoothed. It was shown in Section 8.3 that moving into the same direction is not a logical necessity: spillover index and KS entropy could move in opposite directions; a decrease in KS entropy would have been compatible with an increase in spillover index. It can therefore be said that this system of markets (dji/ftse/sx5e/n225/ssec) has become more stable, but this increase in stability should not be seen as a consequence of the higher degree of connectedness.

10 Summary and conclusions

The goal of the present study was to add to the methodology concerned with operationalizing and measuring contagion and interdependence with application to equity markets. Our efforts extend the spillover perspective developed by Diebold and Yilmaz [10, 11], which, building on forecast error variance decomposition in a vector autoregressive model framework, focuses on directional spillovers and the spillover index as a summary measure in the investigation of market connectedness. We show that adopting a dynamic view which incorporates the entire spillover matrix at once leads to deeper insight into the interplay of markets.

Our reading of the spillover matrix suggests two perspectives: firstly, a focus on the current state of a system of markets, permitting an assessment of the vulnerability of the system to unforeseen shocks and the identification of a market's potential as news spreader, and secondly, a focus on the system's interday dynamics and the process of news creation. Applying tools and concepts from population dynamics and Markov chain theory, we defined an eigenvector-based measure which proved to be of dual character: the markets' news propagation values (the normed left eigenvector of the spillover matrix), and the current "news balance" within the system of markets (the stationary distribution of a corresponding Markov chain). The Markov chain perspective permitted us to develop further, entropy-based measures derived from information theory: the system's intrinsic speed of news digestion pointing to the system's stability, and several versions of relative market entropy which are capable to track and quantify the dynamics of information transmission among markets across time, but also to quantify the potential disturbance of news balance in case of a hypothetical shock.

Among our empirical findings, using daily data from five major economies (United States of America, United Kingdom, the euro area, Japan, and China) from 1997 through 2013, is that Euro Stoxx 50, FTSE and Dow Jones Industrial Average have converged with respect to their importance in passing on information, and that the Dow Jones Industrial Average is playing a less prominent role as news disseminator than it did before. An "informational divide" becomes visible when comparing two groups of economies, namely the United States of America, the United Kingdom, and the euro area on the one hand and Japan and China on the other. Furthermore, the power of daily news to disturb the "news balance" has been waning since about 2007, going along with an increase in system stability, though the character of information has changed — it has become more abrupt.

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