PUBLIC HOUSING UNITS VS. HOUSING VOUCHERS: ACCESSIBILITY, LOCAL PUBLIC GOODS, AND WELFARE

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Public Housing Units vs. Housing Vouchers:

Accessibility, Local Public Goods, and Welfare*

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Abstract

A perennial debate worldwide over housing aid policy focuses on whether the government should provide housing vouchers or subsidized public housing units. To complement the empirically-dominated literature, this paper builds a general equilibrium model that merges urban land use (monocentric city) and Tiebout frameworks. In our model, public housing units or housing vouchers are rationed and some lower-income people have to compete with those with higher incomes in the private rental market. We discuss how location of public housing units is an essential policy variable in addition to the numbers and sizes of units, and argue why housing vouchers may be preferable to public housing.

Keywords: Public housing, housing vouchers, housing policy, welfare, residential location choice, local public goods, endogenous sorting.
JEL Classification: H40, D60, H82, R13

"Yet there are interesting problems that a theory of urban land must consider. There is, for instance, a paradox in American cities: the poor live near the center, on expensive land, and the rich on the periphery, on cheap land. On the logical side, there are also aspects of great interest, but which increase the difficulty of the analysis. When a purchaser acquires land, he acquires two goods (land and location) in only one transaction, and only one payment is made for the combination. He could buy the same quantity of land at another location, or he could buy more, or less land at the same location." (Alonso, 1960, p.149)

"The public housing units themselves have frequently become slums and hotbeds of crime, especially juvenile delinquency. The most dramatic case was the Pruitt-Igoe public housing project in St. Louis-a massive apartment complex covering fifty-three acres that won an architectural prize for design. It deteriorated to such an extent that part of it had to be blown up. At that point only 600 of 2,000 units were occupied and the project was said to look like an urban battleground. We well remember an episode that occurred when we toured the Watts area of Los Angeles in 1968. We were being shown the area by the man who was in charge of a well-run self-help project sponsored by a trade union. When we commented on the attractiveness of some apartment houses in the area, he broke out angrily: "That's the worst thing that ever happened in Watts. That's public housing." He went on to say, "How do you expect youngsters to develop good character and values when they live in a development consisting entirely of broken families, almost all on welfare?" (Friedman and Friedman, 1990, p.110)

1 Introduction

Housing markets are typically intervened by governments all around the world. The forms of these interventions differ significantly both across and within economies.[1] For instance, while the United States tends to provide cash subsidies, European countries are more inclined to directly provide physical structures, even when the spending are comparable:[2] In 2001, the United States spent slightly above 1.5% of its GDP on public aid on housing, while the counterpart in France is similar, slightly above 1.7% of the GDP.[3] Yet the construction-subsidized rental sector (mainly the habitation à loyer modéré, or HLM in short) accommodates 17% of households in France, with less than 2% for the U.S. counterpart. Even a simple policy like a cash subsidy can lead to very different outcomes when institutional details differ. For instance, Priemus (2001) finds that, while in the USA, 100% of the additional rent is paid by the tenants; the Netherlands tenants will only pay 25%. The government will be responsible for the rest. Moreover, in the Netherlands, there is no waiting list and the rent subsidy is perceived as a "right." As a result, the number of applicants

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[1]See Smeeding et al. (1993) for cross country comparisons. Olsen (2003) comments on the large number of programs in the U.S.

[2]Priemus (2000).

[3]Laferrère and Le Blanc (2006).

rose from 348,000 in 1976, to 922,000 in 1996. In 1998, there were more than a million households receiving a housing allowance, out of more than 6 million households in Netherlands.

Housing aid policies are among most expensive welfare programs in many countries. Olsen (2003) comments on the amount of research on the effects of housing assistance programs in U.S. as "shockingly small." The existing research on public housing is mainly empirical, evaluating the effects of certain programs. Endogenous variables and program specific details can burden such analyses at times. In addition, housing assistance programs may have a larger impact on the economy than a few policy outcome variables of direct interest. For instance, Banerjee (1997) discusses how government interventions, in general, may introduce new distortions (e.g., rationing) which may outweigh the original benefits of interventions. Documenting such general equilibrium effects is certainly interesting. Given the enormous costs of "experiments" that will provide clear answers to such questions, there are large potential benefits from studying a formal model that enables a thorough comparison of the effects of alternative policy proposals. Identifying the general equilibrium effects, endogeneity problems, etc., can also help with improving empirical research strategies on housing aid policies.

We study the effects of two of the most common housing aid policies, subsidized units and housing vouchers, using a general equilibrium model that merges urban land use (monocentric city) and Tiebout frameworks. The land is differentiated by both distance and local public goods, and the housing aid policies are financed by general income taxes. Households differ in their incomes and preferences for the local public good, education. The quality of education in a neighborhood depends on peer quality and educational expenditures. The expenditures are determined by property taxes, rates of which are chosen by majority voting. We pay particular attention to equilibrium outcomes such as rents, spatial distribution of households, neighborhood compositions, school qualities, and welfare.

Two attributes households care about when making residential choices, accessibility and local public goods and services, have traditionally been studied separately. Urban land use theory, based

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on pioneering works of Alonso (1964), Muth (1969), and Mills (1972), focused on the implications of the trade-off between accessibility and space, while Tiebout models considered the implications of local public goods and services (Tiebout (1956); Ellickson (1971); Epple, Filimon, and Romer (1984, 1993); Nechyba (1999, 2000); Fernandez and Rogerson (1996)).[4] Urban land use frameworks predict households locating in rings around the center according to their types, whereas Tiebout models predict strong stratification based on income and preferences for the local public goods. The documented household sorting is stronger than that predicted by urban models, and weaker than the sorting predicted by Tiebout models (Glaeser, Kahn, and Rappaport (2000)). There are a few recent attempts to merge the two lines of research and obtain a more realistic description of urban location patterns (de Bartolome and Ross (2003), Hanushek and Yilmaz (2007)). Our approach, based on Hanushek and Yilmaz (2007), allows us to account for both key issues in residential decision making -accessibility and local public goods- simultaneously, providing us with a rich model that captures many essential features of urban spatial structure.

We start by discussing the benchmark equilibrium, with no government intervention in the housing market. We then introduce public housing into the model and study the effects both for the hosting and neighboring communities. Public housing programs exhibit large variations from one market to another, and sometimes even within the same market. We combine the most common elements of the widespread applications to construct our program, and investigate the effects on rents, sorting of households, school qualities, and welfare of different types of households as well as overall welfare. Building public housing in a neighborhood causes a fiscal burden problem and leads households to sort stronger across neighborhoods. Rents increase in general, and so does the education quality gap between neighborhoods, decreasing overall welfare. Then we investigate the effects of location by relocating the public housing units. We find that in fact the location matters. Our results suggest that household sorting increases as public housing units move further from the city center, because the fiscal burden problem created by public housing gets more serious as the spatial distribution and of households deviates further from that of the "no-intervention"

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[4]See Straszheim (1987) and Fujita (1999) for a review of urban literature.

equilibrium. This further increases the school quality gap across neighborhoods. As an alternative, we consider providing housing vouchers to the same participants instead of subsidized units and provide comparisons. We find housing vouchers to be superior to subsidized units in the presence of peer group externalities as well as the spatial characteristics (hence with a non-convex consumption set). Thus, the intuition behind basic microeconomic textbook discussion for "in-cash versus in-kind" may apply to a more general environment than it seems.

The organization of the rest of this paper is as follows: Section 2 develops the theoretical framework, and discusses the calibration and results of the computable model without government intervention. We discuss our models of the two housing assistance programs and their results in Sections 3 and 4. Section 5 discusses the validity of our findings under alternative specifications and formulations. Section 6 concludes.

2   The Model

We incorporate a Tiebout economy into Alonso's (1964) basic land use framework. It may be useful to provide an informal description of the model before going into its details: Households in a monocentric city work at the Central Business District (CBD hereafter) and reside in the surrounding area. The distance of home to the workplace matters because of pecuniary and time costs of commuting to work. A straight line that goes through the CBD (e.g., a river) divides the city into two jurisdictions, East (e) and West (w). Each jurisdiction provides its residents a local public good, education. The provision level (quality) of education in a district is endogenous, and depends on both the composition of the households and local spending - property tax revenues, rates of which are determined by majority voting. The jurisdictions may differ in their tax and expenditure policies, and agents are allowed to move across jurisdictions. This creates a form of competition à la Tiebout (1956).

2.1 Households

Households choose a neighborhood (East or West, which also determines the school that the off-spring can attend), a location in that neighborhood (r, distance to the CBD), amount of land to reside on $s > 0$, leisure $l \in [0, 24]$, and consumption level of a composite consumer good z. Each household has one school-age child. The preferences of households can be represented by the utility function $U(q, s, z, l) = q^\alpha s^\eta z^\gamma l^\delta$, with q representing the quality of education at the school the offspring attends. A member of every household supplies labor to earn an exogenously determined hourly wage W. Households differ according to the wages they earn and in their preferences for education. We name the higher income types *skilled* workers (earning W_S), and the lower income types *unskilled* workers (earning $W_U < W_S$). Skilled workers value education more than unskilled do ($\alpha_S > \alpha_U$).

The city has a dense radial transportation system. The further one lives away from the CBD, the higher commuting costs he/she will face. In particular, if a household lives r miles away from the CBD, the cost of daily roundtrip commute will be ar dollars (pecuniary cost, $a > 0$) and br hours (time cost, $b > 0$), which converts to bWr dollars given the opportunity cost of time. We normalize the price of the composite consumption good to one and denote the unit rent of land r miles away from the CBD by $R(r)$. Households pay a property tax on value of land. Let τ denote the property tax rate as a proportion of daily rent.[5] The budget constraint of a household can be written as:

$$z + (1 + \tau)R(r)s + Wl = Y(r) \equiv 24W - (a + bW)r. \tag{1}$$

The term on the RHS of the above equation is household income net of transportation costs.[6]

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[5] The conversion can be done as follows: The total annual rent is 365*R*s , and the property taxes paid in a year amount to τ*365*R*s. With an annual interest rate r, the property value (the present value of the perpetual rent stream) is (365*R*s)/r. The annual property tax rate is then the ratio of annual tax paid τ*365*R*S to the property value, i.e., τ*r.

[6] The number of schools in a typical city exceeds the number of employment centers, so the average distance to a school is considerably less than the average distance to the downtown area. Second, we measure the time cost of commuting via workers' foregone earnings. Schoolchildren do not have foregone earnings. Third, we measure the money cost of commuting by the cost of operating a car. Schoolchildren are typically walked to school or take the school bus, which would cost much less than operating a car. Also, travel within the downtown area is very quick

Given the market rent curves $\{R_e(r), R_w(r)\}$ and quality-tax packages $\{(q_e, \tau_e), (q_w, \tau_w)\}$ for each jurisdiction, type $i \in \{S, U\}$ household solves the problem:

$$\max_{s, z, l, j \in \{e, w\}} U(q, s, z, l) = q_j^{\alpha_i} s^{\eta} z^{\gamma} l^{\delta} \tag{2}$$

$$s.t. \ z + (1 + \tau_j) R_j(r) s + W_i l = Y_i(r).$$

2.2 Market Rent Curves and Allocation of Land

Land is owned by absentee landlords and auctioned off to the highest bidders. The reservation price of the landlord, R_a, is determined by an alternative use of land, such as agriculture, and is independent of the location. For a given utility level $\bar{u}$ we can find the maximum rent a household is willing to pay per unit of land and optimal lot size r miles away from the CBD by solving the problem $\Psi(r, \bar{u}, q, \tau) = \max_{s, z, l} \left\{ (Y(r) - z - Wl)/((1 + \tau)s) \middle| U(q, s, z, l) = \bar{u} \right\}$ to obtain the *bid rent function*:

$$\Psi_i(r, \bar{u}, q_j, \tau_j) = \frac{k^{1/\eta}}{(1 + \tau_j) W_i^{\delta/\eta}} \ q_j^{\alpha_i/\eta} Y_i(r)^{(\eta + \gamma + \delta)/\eta} \bar{u}^{-1/\eta} \tag{3}$$

and the *bid-max lot size* function:

$$s(r, \bar{u}, q_j, \tau_j) = \frac{\eta}{(\eta + \gamma + \delta)(1 + \tau_j)} \ \frac{Y_i(r)}{\Psi_i(r, \bar{u}, q_j, \tau_j)} \tag{4}$$

where $k = \frac{\eta^{\eta} \gamma^{\gamma} \delta^{\delta}}{(\eta + \gamma + \delta)^{\eta + \gamma + \delta}}$, $i \in \{S, U\}$, and $j \in \{e, w\}$.[7] At an auction for a particular location r^*, the winner is the type with the highest bid rent curve at that location. Given the two types in the model, in each jurisdiction there are two bid rent curves.[8] The *equilibrium rent curve* $R_j(r)$ is the upper envelope of the bid rent curves of two types and the agricultural rent R_a. Because all bid rent curves are convex and decreasing, the equilibrium rent curve $R_j(r)$ is also decreasing up to a

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and inexpensive compared to the average daily commute. So we ignore non-commute transportation costs, and travel within the CBD.

[7]For derivations and a detailed discussion of the properties of these bid-rent functions see Hanushek and Yilmaz (2007).

[8]If as a result of a policy, the number of types increases, so does the number of bid-rent curves.

distance r_{jf}^*, the *fringe distance*, and is constant from that point on. Households with steeper bid rent curves locate closer to the CBD. Higher income increases the demand for land consumption and attracts households further away from the CBD, but it also increases the opportunity cost of commuting time.

Our city is a *closed city*, population is given exogenously. Let $L(r)$ represent the land density r miles away from the CBD, and $n_j(r)$ the equilibrium density function of the household population in jurisdiction $j \in \{e, w\}$. Suppose in equilibrium the residents of the land at distance r in jurisdiction j are type i households. If the equilibrium level of utility of the type i agent, $i \in \{S, U\}$ is u_i^*, then $n_j(r) = \frac{L(r)}{s(r, u_i^*, .)}$. Let $\bar{N}_S$, $\bar{N}_U$ denote the populations of the respective types. The population constraint for each type can then be stated as:

$$\int_0^\infty \frac{L(r)}{s_w(r)} I[t_w^*(r) = i] dr + \int_0^\infty \frac{L(r)}{s_e(r)} I[t_e^*(r) = i] dr = \bar{N}_i \qquad (5)$$

where $t_j^*(r)$ is a function showing the type of the occupant at distance r in jurisdiction j, and $I[.]$ is an indicator function that takes the value 1 when the condition in brackets is satisfied, and 0 otherwise. The population constraint implicitly assumes the land market clears in each jurisdiction:

$$\forall r \le r_{fj}^*, \qquad s_j(r) n_j(r) = L(r). \qquad (6)$$

2.3   Neighborhoods

The two neighborhoods differ only in the *quality of education* and *property tax rate* (q_j, τ_j) packages they provide. There is one public school in each jurisdiction. Public schools are neighborhood schools, enrollment is open to residents of the community only. Admission is free, schools are financed by property taxes on residential land. The quality of education $q(\Pi, E)$ in a school is determined by (per-student) instructional expenditures E and the peer quality Π. For a given group of students, an increase in the instructional expenditures increases the quality of education

$(\partial q/\partial E \geq 0)$.[9] An equilibrium property of our model is that in each jurisdiction there is a distance r_{fj}^* called the *fringe distance* beyond which no households reside. In each community entire revenue from property taxes is spent on education. Given the equilibrium rent function $R_j(r)$, and equilibrium tax rate τ_j, we can calculate the tax base, and total tax revenues to find the per-student expenditure in the public school system:

$$E_j = \frac{1}{N_j} \quad \tau_j \int_0^{r_{fj}^*} R_j(r)L(r)dr \tag{7}$$

for $j \in \{e, w\}$ where N_j denotes the number of students in neighborhood j.

Different groups of students may benefit differently from a given amount of instructional expenditures. That is what the peer quality (or efficiency) component captures $(\partial q/\partial \Pi \geq 0)$. Some parents value education more than others, and as a result they may spend more time helping with the kid's homework, provide a nicer study environment at home, be more involved in how schools operate, etc. Recall that type S agents value education more than type U counterparts, and as a result having more students from type S families may bring in a higher level of positive externality through the peer group effect. The following formulation has been proved to be very tractable and captures the idea that the peer quality is increasing in the proportion of S types:[10]

$$\Pi = c_0 + c_1 \ \exp\left(-c_2 \frac{N_U}{N_S}\right), \quad c_0, c_1, c_2 \geq 0. \tag{8}$$

The timing of events is as follows: At the beginning of each period, households make residential choice decisions, expecting last period's quality-tax rate packages to prevail. They move in and vote for the tax rate. The households are myopic when voting; they do not consider the implications on migration patterns and the composition of neighborhoods. The quality tax rate package may

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[9]There is a debate on the effectiveness of monetary inputs on student's achievements (see Burtless (1996)), however, it is reasonable to assume that households would value an increase in educational expenditures, and in that case the equilibrium implications would be identical.

[10]Alternative specifications give similar results. See, for example, Hanushek and Yilmaz (2007) and Hanushek, Sarpça, and Yilmaz (2007).

be different from what they expected, however they are stuck until the beginning of next period. Then they update their expectations and the events start over again. For a type i household the most preferred tax rate $\tau_i^* = \frac{\alpha_i}{\eta - \alpha_i}$ is the solution to the indirect utility maximization problem:

$$\tau_i^* = \underset{\tau}{\operatorname{argmax}} \ V(.) = \frac{k}{(1+\tau_j)^\eta R(r)^\eta W_i^\delta} q_j^{\alpha_i} Y_i(r)^{\eta+\gamma+\delta} \qquad \text{s.t.} \ \ q_j = \Pi_j E_j \tag{9}$$

$$\text{and} \ \ E_j = \tau_j \bar{R}_j.$$

We study the *stationary equilibrium*, which is attained when no one has an incentive to relocate in response to the voting results.

Definition: *An **equilibrium** is a set of utility levels $\{u_S^*, u_U^*\}$, market rent curves $\{R_e(r), R_w(r)\}$, quality of education and property tax rate pairs $\{(q_e, \tau_e), (q_w, \tau_w)\}$, household population distribution functions $\{n_e(r), n_w(r)\}$, and location-type functions $\{t_e^*(r), t_w^*(r)\}$ that show the equilibrium occupant of the location at distance r in community j such that:*

- *Households' choices are determined by solving (2),*
- *The market rent function $R_j(r)$ in each jurisdiction is determined through a bidding process among different types of households,*
- *Same types of households obtain the same level of utility regardless of their choices,*
- *The tax rates in each jurisdiction are determined by majority voting by myopic voters,*
- *Local governments' budgets balance in each jurisdiction, (7),*
- *Labor and land markets clear,*
- *The population constraints (5) hold.*

2.4   Calibration

The equilibrium of our model can be calculated only numerically.[11] We specify parameter values to match certain statistics from mid-size U.S. cities in 2005. Normalizing the sum $\eta + \gamma + \delta$ to 1,

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[11]The algorithm used to solve the models in this paper can be found in the Appendix.

the solution to the household problem gives the optimal budget shares for leisure, consumption, and lot size as δ, γ, and η, respectively. In the U.S. average hours of work per week in full time jobs is 40 hours, and average annual earnings of workers are \$30,104 for high school graduates and \$58,114 for college graduates.[12] Accordingly, we set the hourly wages for unskilled and skilled types as $W_U = 14$ and $W_S = 27$. In a 168 $(= 24 * 7)$ hour week, 40 hours of work implies a 0.762 budget share for leisure. The data on household expenditures suggest that expenditures on shelter constitute about 20% of the budget of an average household.[13] This implies budget shares of 4.76% for housing and 19% for consumption. There are two possibilities for the most preferred property tax rate according to (9). We set these most preferred tax rates equal to 1.97% (1.04%) for the high (low) valuation types. The average population density in a city with population 1 to 2.5 million is 2901 people per square mile.[14] The utility function parameters consistent with all these are $\alpha_L = 0.014$, $\alpha_H = 0.021$, $\delta = 0.762$, and $\gamma = 0.19$.[15]

We calculate the commuting costs assuming the households drive to work. The pecuniary cost can be calculated based on the cost of owning and operating an automobile. In 2004 pecuniary cost per mile was \$0.56, and we set $a = 1.1$. Assuming the commuting speed in the city is 20 miles per hour, we set $b = 0.13$. We assume 1.5 million households populate the city. When computing the equilibrium, we target for a (endogenous) fringe distance (city radius) around 15 miles in each jurisdiction. The proportion of college graduates in U.S. is about 30%. We expect this proportion to be slightly higher in a city. Hence, we set the proportion of skilled households to 40%. We set the parameters of the school quality function to $c_0 = 0.1, c_1 = 1.3677$, and $c_2 = 0.05743$ to match some related empirical observations.

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[12] Current Population Reports 2005, US Census Bureau.

[13] U.S. Statistical Abstract.

[14] US Census Bureau, Census 2000 Summary File 1.

[15] Property taxes are paid over property value (the present value of the perpetual rent stream), whereas the model is written for a day. See footnote 5 for conversion of tax rates.

2.5 Equilibrium

Figure 1 provides a visualization -a map- of the city, and Table 1 provides some summary statistics. Several observations are immediate:[16] (1) Neighborhoods are heterogeneous, both household types exist in each neighborhood; (2) There is (partial) Tiebout sorting across neighborhoods; (3) There is income-sorting with respect to distance within neighborhoods: Low income types choose smaller lots closer to the CBD, while high income households live on larger lots away from the CBD.

(Figure 1, Table 1 about here)

The results are intuitive. Costly commuting causes the market rents in each jurisdiction to decrease as one moves away from the CBD. Higher income increases the land demand and attracts households further away from the CBD where the land is cheaper. High income also increases the opportunity cost of commuting time, but our calibration suggests that this effect is dominated by the former, consistent with residential patterns in the U.S. As a result, in each community lower income U types occupy a semi circle around to the CBD. The S types reside in a semi-ring that surrounds the semi-circle of U types. The outer end of the S semi-ring is the fringe distance, the land beyond which is left for agricultural use.[17] The intuition behind heterogenous communities is also straightforward: Households choose the lot size, distance to the CBD, and the community (with a given quality of education and a property tax rate) simultaneously. So lower taxes or lower rents in community (hence larger lots) can compensate for lower quality of education and vice versa.

Notice that about seventy percent of high income types live in the same neighborhood, constituting a fifty five percent majority of the population there. As a result, the voting outcome is the higher tax rate. Without loss of generality, we refer to the higher tax neighborhood as *West*

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[16]The first two observations highlight an advantage of our approach: The partial sorting across heterogeneous communities is consistent with empirical findings on household sorting (e.g., Davidoff (2005))

[17]This "rings" structure dates back to von Thunen's model of land use (1826), and is one of the building blocks of Urban Land Use Theory. In our models we have semi-rings instead of full ones because of the jurisdiction boundaries, or the local public goods problem. As a result of this, the widths of rings may differ between neighborhoods. The ordering of households around the CBD, however, does not change.

school district throughout this paper. Because the two neighborhoods have comparable populations, higher taxes and rents mean higher educational expenditures per student in West ($3653 vs $2027). Also, the peer quality is higher in the West neighborhood, thus quality of education exceeds that in East.[18] This difference in quality of education is capitalized into rents; Rents in West are about twenty-five percent higher on average.

3 Public Housing

We study two housing aid policies in this and the following sections: government provided subsidized units and housing vouchers. These programs are financed by uniform income taxes with rate θ on earnings of all city residents (and participant contributions in the case of public housing units). Only low-income (U-type) households are eligible. The government sets the maximum number of participants N_P exogenously. If the number of program applicants exceed N_P, participants are selected by a lottery. For comparison purposes, we keep the sizes of two programs same.

3.1 Public Housing Model

The government announces that public housing units will locate between r_P and $r_P + w_P$ miles from the CBD in one of the neighborhoods.[19] All U-type households that find this beneficial apply, and in the case of excess demand, a lottery selects N_P of them. The rents are determined by the same auction process described above, with bidders excluding the N_P program participants and including the government. For any location in the public housing band, the government pays the

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[18]The higher tax/expenditure community providing a higher quality of education with higher per-student-expenditure could easily mislead one to overemphasize the role of expenditures on school quality. Hanushek, Sarpça, and Yilmaz (2007) show the existence of a private sector for education breaks the link between expenditures and school quality.

[19]It is possible to build public housing in both neighborhoods, such as a full-ring on the map, and guarantee certain outcomes such as desegregation exogenously. In the equilibrium of such a model we would obtain results similar to the benchmark equilibrium with slightly weaker sorting, and slightly less inequality in quality of education across jurisdictions. Typically, however, public housing is not built in the best neighborhoods, because of both costs and political reasons. Moreover, building public housing in a neighborhood is likely to cause some residents to relocate, altering the composition and the desirability of the neighborhood. So keeping one of the neighborhoods free from public housing gives us a chance to observe the interesting effects of such a program on its hosting neighborhood, resulting inbound-outbound migration, and the effects of this migration on neighborhood composition/characteristics as well as on local public good provision in both hosting and neighboring localities.

maximum rent households are willing to pay, had this land been available to them. This land is then divided into lots with equal size s_P and allocated to N_P participants with subsidized rents. A program participant household pays a fixed price $\overline{R_P}$ as its program contribution, independent of location of its lot in the band. This policy reduces a public housing unit resident's problem to leisure-consumption choice only:

$$\max_{z,l} \ U_P(q_e, s_P, z, l) = q_e^{\alpha_U} s_P^\eta z^\gamma l^\delta \ \ s.t. \tag{10}$$

$$z + (1+\tau)\overline{R_P} + (1-\theta)W_U l = Y_P(r) = 24(1-\theta)W_U - (a + b(1-\theta)W_U)r,$$

whereas others solve the original problem (2) in the presence of income taxes:

$$\max_{s,z,l,j\in\{e,w\}} \ U(q, s, z, l) = q_j^{\alpha_i} s^\eta z^\gamma l^\delta \ \ s.t. \tag{11}$$

$$z + (1+\tau_j)R_j(r)s + (1-\theta)W_i l = Y_i(r) = 24(1-\theta)W_i - (a + b(1-\theta)W_i)r,$$

$$i \in \{S, U\}.$$

taking rents and neighborhood quality-tax pairs as given.

The cost of public housing is financed by income tax revenues and participant contributions. The time constraint of a household gives the labor supply as all the time except leisure and commute time: $n = 24 - l - br$. The solution to (11) gives the optimal leisure of household residing at distance r as $l_i^*(r) = \delta \frac{Y_i(r)}{(1-\theta)W_i}$ for $i \in \{S, U\}$. For a household that lives in public housing, optimal leisure is $l_P^*(r) = \frac{\delta}{\delta+\gamma} \frac{Y_P(r)-(1+\tau)\overline{R_P}}{(1-\theta)W_U}$. We can define a $t_j^*(r)$ function that shows the type of the occupant at distance r in jurisdiction j (whether S, or U, or P, i.e., U in public housing). Let $I[.]$ be an indicator function that takes the value 1 when the condition in brackets is satisfied, and 0

otherwise. The public housing program budget constraint is:

$$\int_{r_P}^{r_P+w_P} R_e(r)L(r)dr - N_P\overline{R_P} = \theta \sum_{j\in\{e,w\}}\left[\int_0^{r_{fj}^*} \frac{L(r)}{s_j(r)}\left(\sum_{i\in\{S,U,P\}} I[t_j^*(r) = i](24 - l_i^*(r) - br)W_i\right)dr\right]$$

(12)

The inside summation in the RHS identifies which household type resides at a particular location and calculates their labor income at equilibrium. The integral calculates the total labor income using the households' density at that location. The outside summation adds labor income of two communities. A fraction θ of this gives us the income tax revenues. Program's total cost (LHS) is the price of land minus the contributions. Equilibrium now also requires program budget constraint (12) holds.

3.2 Equilibrium

We first study a model in which the government provides public housing on the land that is between 4 and 6 miles away from the CBD in the East neighborhood.[20] This causes most high income types to reside in west, which then becomes the higher rent/tax/school quality neighborhood. In the benchmark model U types lived in this band in east, and the lot sizes ranged from 4,739 to 6,691 sq feet (at 4 and 6 miles away from the CBD) with an average lot size of 5,575 sq feet. The average monthly rent was \$3,520 per mile square. Each public housing unit measures 6,970 sq feet now, regardless of its location within the band. This is about twenty-five percent larger than the average unit within that band in benchmark equilibrium.[21] This band then accommodates about fifteen percent of all low income types.[22] Some lots may be closer to city center than others, so the equilibrium utility of public housing residents may vary depending on their lot location, an outcome of the lottery. We set the monthly rent of a subsidized unit to \$239 and the income tax

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[20]The choice of neighborhood does not matter. Knowing where public housing units will locate, households adjust their bids and sort accordingly, and the neighborhood with public housing becomes the lower-tax/quality/rent neighborhood.

[21]Alternative specifications for public housing lot size yield similar results. We present results from additional analysis in Section 5.1.

[22]In France, HLM accommodates about 17% of all households.

rate to 0.57%, so that utility increase from public housing is equivalent to that from a 10% income subsidy in equilibrium and (12) holds.

We present the welfare effects of this policy in the third row (4-6.00 miles) of Table 2. Our measure is the change in rents necessary to provide households with their utility level in benchmark equilibrium. A negative number means the household type is worse off, since rents need to be decreased to make the households indifferent to the benchmark allocation. The welfare gain to public housing residents is equivalent to a gain from an eighty-six percent decrease in rents. The welfare loss to the rest of the households is equivalent to about a 5.5 percent increase in rents. As a measure of the change in overall welfare, we calculate the change in rents necessary to keep aggregate utility at the benchmark equilibrium level and present it in column 4 (AU). Overall welfare is lower than that in benchmark equilibrium.

(Table 2 about here)

Table 3 presents some highlights of the new equilibrium, and Figure 2 displays a map of the city. Sorting is stronger compared to the benchmark: Seventy-five percent of high income types live in West (up from 69% in benchmark) and constitute the majority there.

(Figure 2, Table 3 about here)

Rents are higher than those in benchmark equilibrium in both neighborhoods. The intuition is straightforward: The public housing policy removes a substantial amount of land from the private market. The non-recipients, who are either the skilled or the less lucky unskilled workers, compete for the land remaining. However, the composition of the demand changes, and so does the equilibrium price. To see this, assume that originally the total land supply is $\mathcal{L}$. The amount of skilled and unskilled workers are N_S and N_U respectively. Without any public housing policy, the ratio of skilled relative to the unskilled is N_S/N_U. Now the government removes an amount of land $\mathcal{L}_P > 0$, and the remaining amount of land available for the market has decreased to $\mathcal{L} - \mathcal{L}_P$. At the same time, a portion $N_P > 0$ of unskilled workers receive public housing units and leave the

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market. Thus, among those remaining in the market, the ratio of skilled relative to the unskilled $N_S/(N_U - N_P)$ is larger than the original ratio N_S/N_U. Other things being equal, as the skilled naturally demand more land than the unskilled, such a change in the composition of demand is likely to generate higher rents. But other things are not equal. First, there is a fiscal burden issue here. The public housing residents' effective rents are below market values, and they pay property taxes as a proportion of these subsidized rents. This hurts the neighborhood's property tax revenues. To finance the public housing project, the government needs to impose income taxes, and this weakens the incentives to work. Moreover, the skilled workers may "vote on foot" and move to another neighborhood, because of their stronger preferences for education. They would prefer to live in West where the marginal effect of a tax dollar on per-student expenditures are higher. As a result, the difference in quality of education is higher between two neighborhoods compared to the benchmark, since both the spending and the peer quality in West (East) are higher (lower) than before. Our numerical implementation of the model simply attempts to capture this chain of effects quantitatively.

Public housing residents decrease their labor supply by about 6.5% on average, to 38.6 hours per week down from 41.3 hours in benchmark. This is a result of both income (the decrease in housing expenditures) and substitution (the decrease in the relative price of leisure) effects. Additional analysis -summarized in Section 5.4- confirms the presence of such a strong effect under several alternative formulations. These results are consistent with the empirical findings on housing subsidies and labor supply.[23]

3.3 Does Within-district Location of Public Housing Units Matter?

We further exploit the spatial features of our model by conducting additional analysis that compares the economic outcomes with public housing units provided at different locations. To isolate the effects of location, we keep the sizes and the number of subsidized units constant, while varying

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[23]Clearly, it is beyond the scope of this paper to review this literature. Among others, see Bingley and Walker (2001), Hulse and Randolph (2005), Olsen et al (2005), and the references therein.

their distance to the CBD. Recall that in the above model the public housing units are built in the belt that is between 4 and 6 miles from the CBD. Table 3 summarizes welfare effects of public housing units provided at 0, 2, and 6 miles away from the CBD.

The overall welfare decreases as public housing band moves away from its original location (4 miles) in each direction (See Table 3). At 4 miles, the public housing unit size is slightly larger than a typical lot at the same location in benchmark model. When we move the public housing units, we keep their sizes same. Now as we move towards the CBD, the public housing unit becomes larger and larger compared to a typical lot around that location. Similarly, as we move away from the CBD, the size of public housing units relative to its neighboring locations continues to decrease. These deviations from market lot sizes affect community compositions and local public finance in a way that decreases overall welfare.

As public housing moves away from the CBD, we see a stronger sorting. More and more S types choose to reside in West, obtaining larger lots and decreasing population density. This has significant implications on quality of education and welfare. As public housing moves away from the CBD, the quality gap increases: The quality keeps on increasing in West, and decreasing in East. Both per student expenditure and peer quality affect this, but the role of expenditures is larger. Government provided band houses a fixed number of households (N_P) that pay less than full taxes over subsidized rents, but are fully entitled to educational expenditures. Notice that the area of the band stays constant as we move it around. As this band moves out, and since lot sizes increase in distance from the CBD, the band replaces less and less people with a constant number of public housing residents, increasing the population density in the neighborhood. Rents also decrease in distance to the CBD, so the tax revenue from public housing residents decrease too, increasing the magnitude of the fiscal burden problem.

Our analysis suggests that location of public housing units is an important policy variable in addition to the size and the number of units. Location matters through its distortion on population density and neighborhood compositions, which in turn affect the magnitude of the fiscal burden

problem described in Section 3.2.

4 Housing Vouchers

4.1 Voucher Model

Instead of providing physical units with subsidized rents, the government can simply redistribute the income tax revenues to the low income households in the form of housing vouchers. Under this scheme, each of the N_P program participants gets a voucher towards rent with amount ν_P. For government budget to balance, income tax revenues must equal the total amount of these vouchers:

$$N_P\nu_P = \theta \sum_{j\in\{e,w\}} \left[\int_0^{r^*_{fj}} \frac{L(r)}{s_j(r)} \left(\sum_{i\in\{S,U,P\}} I[t^*_j(r) = i](24 - l^*_i(r) - br)W_i \right) dr \right] \quad (13)$$

A household's problem is same as the one in (11), but a voucher recipient's housing expenditures are those exceeding ν_P only. This is equivalent to creating a third household type (say type P) with the same preferences as type U, and with the kinked budget constraint:

$$z + max\{0, (1 + \tau_j)R_j(r)s - \nu_P\} + W_U l = Y_P(r) = 24(1 - \theta)W_U - (a + b(1 - \theta)W_U)r. \quad (14)$$

Since household utility increases in lot size, no household will spend less on housing than the voucher amount. Whether the household will spend more depends on the voucher amount and some model parameters. The leisure choice of a voucher recipient is $l^*_P(r) = \delta\frac{Y_P(r)+\nu_P}{(1-\theta)W_U}$ if the household spends on top of the voucher amount, and $l^*_P(r) = \frac{\delta}{\gamma+\delta}\frac{Y_P(r)}{(1-\theta)W_U}$ otherwise. The land is allocated according to the competitive auction mechanism described in Section 2. An additional equilibrium condition is that the program budget (13) holds.

4.2 Equilibrium

For comparison, we keep the (number of) recipients and the tax rate same as the public housing policy. This implies a housing voucher with amount $227 per month. Vouchers shift land demand, increasing rents in both neighborhoods. The equilibrium rents, however, are lower than those under public housing policy, since more land is available in the private market now. The fiscal burden problem we described in public housing units section is no concern here, since voucher recipients still pay property taxes at the market rent level. However, households without vouchers are hurt by the higher rents and income taxes. The equilibrium utility levels of non-recipients (both S and U types) are higher than those under public housing, but lower than benchmark levels. The (average) utility level of voucher recipients falls below that of public housing recipients. We present the change in household welfare according to their types in the last row of Table 3. The welfare gain to voucher recipients is equivalent to a gain from a fifty-nine percent decrease in rents. The welfare loss to the rest is about 3.3 percent increase in rents, about half of that under public housing policy. Unlike public housing, the change in total welfare (as measured by AU) is positive under this policy.

Sorting of households is stronger than benchmark, equivalent to the public housing policy levels: Seventy-four percent of high income types live in West (as opposed to 69% in benchmark) and constitute a fifty-six percent majority there. The major cause of this is the increase in land demand in East: All voucher recipients reside in East (where low income types are a majority) because of their weaker preferences for education.[24] Figure 3 gives a map of the city, and Table 4 gives some statistics. The quality of education in West is slightly higher than both the benchmark and public housing models, because of both the sorting and higher expenditures. A policy maker concerned with education of the poor may prefer vouchers over public housing: The quality in East, the poorer neighborhood, is higher than that in public housing model.

(Figure 3, Table 4 about here)

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[24]This is not an intrinsic feature of the model, and is mainly due the budget and scale of the voucher program. We show in Section 5.5 that voucher recipients are observed to reside in both neighborhoods under a voucher program with a different budget and size.

Income taxes hurt work incentives by decreasing the relative price of leisure. Also, the income effect of vouchers allow recipients to increase their leisure consumption. But voucher recipients work more than public housing recipients (42 vs. 38.6 hours). This should not be puzzling: The public housing program also creates an income effect, but by restricting households' location-lot size choices and their housing expenditures, it also causes a disproportionate increase in consumption of the other two commodities available for purchase, composite good and leisure. Housing voucher recipients are not restricted in terms of their residential choices, and they move further away from U types, obtaining larger lots and increasing their leisure proportionally. The increase in distance and lot size increases voucher recipients labor supply, but as a result of the income effect they work slightly less than U types living at the same distance in benchmark.

5 Robustness and Extensions

In this section we present summary results from additional analysis conducted with alternative formulations and specifications. Our analysis so far suggests that vouchers not only cause less distortion on social welfare compared to public housing, but they also improve overall welfare. Additional analysis, results of which are summarized below, reveals that our findings are robust to changes in the size of public housing units and in the levels for property tax rates, as well as to incorporation of a housing industry. We also verify that public housing recipients work less than housing voucher recipients under alternative specifications. We conclude this section with an example illustrating that it is possible for housing vouchers to reside in both neighborhoods under an alternative specification for the budget and size of the housing vouchers program. Tables 5 through 8 allow us to compare welfare implications of public housing and housing voucher models under different scenarios. Our measure is the change in rents necessary to provide households with their utility level in benchmark equilibrium. A negative number means the household type is worse off, since rents need to be decreased to keep the households indifferent to the benchmark allocation. As a measure of the change in overall welfare, we calculate the change in rents necessary to keep

aggregate utility (AU) at the benchmark equilibrium level.

5.1 Size of Public Housing Units

In the analysis presented in Sections 3.2 and 3.3, the public housing units are 25% larger than the average unit in the same area in benchmark equilibrium. Table 5 presents results from alternative specifications. Rows 1 through 5 summarize welfare results with the size of a public housing unit ranging from -25% to 75% of benchmark size, and the bottom row displays welfare results of the housing vouchers model of Section 4 for reference.

(Table 5 about here)

With a given level of income tax revenues (that constitute total government subsidy), an increase in public housing unit size increases the cost of the program on public housing residents, explaining the decrease in P-types' welfare in size. Public housing programs with all sizes considered in this paper decrease overall welfare, whereas housing vouchers program improves the overall welfare.

5.2 Utility Parameters and Tax Rates

The most desired property tax rates for the two household types are chosen to be slightly higher/lower than the U.S. average of 1.40%, and the utility parameters α_H and α_L are calibrated according to equation (9). We conduct additional analysis with alternative specifications for the levels of α_i's, which affect desired tax rates by about $\pm 20\%$ or higher. Table 6 reports results from these alternative specifications along with the original specification given in the middle row of each panel.

(Table 6 about here)

An inspection of the columns reporting the changes in overall welfare (AU) under two policies shows our findings are robust to such changes in property tax rates.

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5.3 Housing Construction

Our framework is based on Alonso's model (1964) which assumes that each household manages the construction of its house by itself. An alternative approach is Muth's model (1969) in which households derive utility from consuming housing space H, produced by competitive firms using Λ units of land and K units of capital with the production function:

$$H = AK^a \Lambda^{1-a} \tag{15}$$

for $a \in (0,1)$ and $A > 0$. Then as land gets more expensive closer to the CBD, the share of capital to land in the construction of housing space increases, i.e., taller/multi-unit buildings are observed.

We incorporate this housing industry to our framework, and repeat the analysis presented so far in this paper. This change in the formulation does not alter our qualitative findings regarding welfare comparisons, while adding considerable complexity to the analysis. We solve the equilibrium without government intervention (benchmark), with public housing, and with housing vouchers. We calibrate the parameters of the housing production function so that the ratio of housing space to land is about 16 near the CBD and 1 near fringe in the benchmark model.[25]

We initially locate the public housing units at 4 miles away from the CBD, keeping the ratio of housing space to land at the benchmark level at this distance (about 4). We set the public housing unit size same as the average unit in the same area in benchmark. We have also solved alternative models with: 1. The public housing unit size 25% smaller and 25% larger than the average unit in the same area in benchmark, 2. The ratio of housing space to land is 3 and 5, 3. The location of public housing units at 3 and 5 miles away from the CBD. We summarize results from this analysis in Table 7 below. The middle rows in top three panels (titled as "same," "4," "4 miles") represent the main public housing model, and the very bottom row in the table presents

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[25]These can be interpreted as the number of floors of buildings, but one should also keep in mind that it is a continuous variable. The implied parameters for the production function are a=0.70 and A=0.00149. Given the change in tax base, we also recalibrate the parameters of the education production function to facilitate comparisons with the models in Sections 3 and 4. Further details are available from the authors.

the welfare implications of the voucher model with housing industry.

(Table 7 about here)

An inspection of Table 7 reveals that housing vouchers are preferable to public housing units in this extended model too. We also solve this extended model with alternative utility-tax parameters. Table 8 presents the counterpart to Table 6 with housing industry.

(Table 8 about here)

In the first and last rows of the table, where the utility parameters of the two types get close to each other, the overall welfare under public housing is slightly greater than that under vouchers. However, a closer inspection of the Table reveals that households that do not participate in the program still prefer housing vouchers to public housing units. The higher overall welfare results from the large difference from program participants' utility levels.

5.4 Labor Supply

One of our findings is that households decrease their labor supply when provided with public housing units. Table 9 summarizes labor supply implications from additional analysis with: 1. different sizes for public housing units; 2. different locations for the public housing units; 3. different utility (and tax rate) parameters. One row in each panel (+25%, 2-4.90, and (.014, .021)) represents the original model for reference. Public housing recipients work less than voucher recipients. The only exception arises when public housing is built at the CBD, since public housing residents' travel time becomes very small, contributing to their labor supply.

(Table 9 about here)

The variations in the first column of top two panels are caused by the changes in the reference band. We compare the labor supply of public housing residents to labor supply of the (same type)

households that live on the same exact land in benchmark equilibrium. Then, for example, going from 0 to +25%, the area occupied by public housing expands, and so does our reference group, explaining the change in their average. The third column presents the same equilibrium in the top panels, since the changes apply to the public housing model only. With the change in utility parameters in the bottom panel, equilibrium of the voucher model changes too.

5.5   Vouchers and Desegregation

We have discussed several reasons why a housing voucher program may be preferred over a public housing program, with a given program size and an income tax rate. An additional benefit of housing vouchers over public housing is that the vouchers do not impose restrictions on location choice of households. Hence, a policy-maker with concern over the extent of household sorting across neighborhoods may be particularly interested in how the housing vouchers can influence this sorting.

The equilibrium neighborhood compositions under the two programs are, however, almost identical in the above analysis. West community provides higher quality of education at the cost of higher taxes on land consumption. The S type households have stronger preferences (and willingness to pay) for education compared to voucher recipients, so they outbid the voucher recipients on West land away from the CBD. On the other hand, U type households without vouchers value proximity to the CBD and outbid voucher recipients on West land close to the CBD. The U type households demand smaller lots compared to voucher recipients, and therefore are not affected by larger taxes as much as voucher recipients who demand relatively larger lots. As a result, voucher recipients are not observed residing in West.

These observations suggest that whether a voucher program and a public housing program with the same size and income tax rates will have the same impact on household sorting may depend on the combinations of some parameter values. This encourages us to further explore other parameterizations. First, we study a set of parameterizations in which policy maker increases the

number of recipients without changing the income tax rates. This of course means a lower voucher amount for every recipient. Second, we study the effects of increasing tax rates while keeping the number of recipients the same as in the previous section. Neither attempt causes enough increase in bids of voucher recipients in West to overcome the effects summarized in the previous paragraph.[26] The intuition is straightforward: Increasing number of recipients decreases the number of U types without vouchers, weakening their competition. It also lowers the bids by voucher recipients since the amount of the voucher decreases in the number of recipients. On the other hand, increasing voucher amount for the same number of recipients just help them afford larger lots in East instead of relocating, pushing some U types to West instead. However, it is possible to induce voucher recipients residing in West with both a larger program size and a larger budget, increasing their land bids in both neighborhoods and allowing them to outbid some households in West. We present the equilibrium of one such model in Figure 4 in which 25% of unskilled types receive housing vouchers with amount $537 per month. This program is financed by an income tax rate of 1.5%, about twice the income tax rate in the earlier sections. In the equilibrium of this model, about one fifth of the voucher recipients live in West, occupying a semi ring between the U types' semi-circle around the CBD and the S types' semi-ring. Other aspects of the equilibria remain qualitatively same for our purposes, so we skip a detailed discussion here.

(Figure 4 about here)

6   Concluding Remarks

After surveying a vast literature on the housing market and housing policies in the U.S., Green and Malpezzi (2003, p.94) argue that "Most economists like vouchers because they are generally more efficient than other programs. (...) But in the United States, political support is generally stronger for programs tied more closely to the consumption of specific goods (housing, food, and medical care) than for income support." This paper attempts to contribute to the related debate.

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[26]Detailed results are available from the authors.

In particular, this paper explicitly highlights the importance of location of public housing on equilibrium outcomes such as rents, neighborhood compositions, schooling opportunities, labor supply decisions, and social welfare. We explain the channels through which such location effects work. Using a rich general equilibrium model that combines land use theory with Tiebout framework, we provide a comparison of public housing and housing vouchers policies, and discuss several reasons why vouchers may be preferred over subsidized units. The results of our analysis are consistent with findings from previous studies that compare in kind versus in cash welfare programs, verifying their validity in a richer framework with spatial elements, local public goods, and peer group externalities. In addition, we also find that public housing policy tends to discourage labor supply, especially for the unskilled workers who reside in public housing, as some empirical literature has suggested. This seems to strengthen the in-cash rather than in-kind arguments even further.

The framework we present in this paper can be adapted to compare public housing and/or vouchers to other housing aid programs, or to compare the outcomes of any single program under different institutional details.

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Appendix: The Algorithm Used for Solving the Models

The following algorithm is based on the sequence of events described in the paper. Also see Figure A1 below. The school district j could be East or West. If we know the bid-rent curve of a household type in one district, we can draw the bid-rent curve for the same type in the other district as well, since identical households obtain the same level of utility regardless of where they live.

1. Define model parameters and discretize the space.

2. Equation 3 (and some algebra) suggests that bid-rent curve Ψ_1 of a household is steeper than that Ψ_2 of another if and only if for all r, $(a + bW_1)Y_2(r)/(a + bW_2)Y_1(r) > 1$. Using this, check if single crossing property holds and determine the spatial order of households. This ratio is always larger than 1 for any r^* and the spatial order happens to be first U-types followed by S-types as r increases.

3. Randomize the initial tax rate/quality of education package in each district.

4. Initialize fringe distance, r_{fj}^*. Find u_S^* by using equation 3 in the paper and the fact that at r_{fj}^*, the rent is R_a. Use this information to calculate bid-rent and lot sizes for S-type households in both districts.

5. Calculate the rent at r_{SUj}^* by using the bid-rent function of U-type households. Then find u_U^* by using equation 3. Calculate bid-rent and lot sizes for U-type households in both districts.

6. Determine the land area that S-type households outbid U-type households. By using lot sizes, calculate the population of U-type households.

7. If it is larger (smaller) than the target value, go back to 6 and increase (lower) r_{SUj}^*. If it equal to the target value, the land occupied by either household type are determined. Move to step 8.

8. Find majority winner property tax rates, tax bases, and quality of education in each school district.

9. Go back to step 3 and update tax rate/education package. Repeat until the current period tax rate/education packages are equal to those in the last period.

(Figure A1 about here)

Table 1: Benchmark Equilibrium

	West	East
Average Monthly Rent (per Acre)	$2729	$2194
Average Rent in S area	2204	1433
Average Rent in U area	5416	3368
Average Lot Size S	0.791	1.209
Average Lot Size U	0.173	0.275
Property Tax Rate	1.97%	1.04%
Neighborhood Quality	14	7
Distribution Across Neighborhoods		
S	69%	31%
U	42%	58%
Neighborhood Population Breakdown		
S	53%	26%
U	47%	74%

Table 2: Welfare I

	S	U	P	**AU**
Location of Public Housing				
0-4.47 miles	-9.83	-9.20	33.53	**-6.71**
2-4.90 miles	-9.61	-8.41	52.72	**-5.23**
4-6.00 miles	**-5.64**	**-5.39**	**85.67**	-0.35
6-7.48 miles	-9.29	-6.20	88.37	**-2.37**
Vouchers				
	-3.29	-3.31	59.01	**0.54**

Table 3: Public Housing

	West	East
Income Tax Rate: 0.57%		
	West	East
Average Monthly Rent (per Acre)	$2764	$1955
Average Lot Size S	0.770	1.261
Average Lot Size U	0.165	0.301
Average Lot Size P	-	0.320
Property Tax Rate	1.97%	1.04%
Neighborhood Quality	14.15	6.02
Distribution Across Neighborhoods		
S	75%	25%
U	41%	59%
		(14% P)
Neighborhood Population Breakdown		
S	55%	22%
U	45%	78%
		(19% P)

Table 4: Housing Vouchers

	West	East
Income Tax Rate: 0.57%		
	West	East
Average Monthly Rent (per Acre)	$2741	$2186
Average Lot Size S	0.771	1.246
Average Lot Size U	0.166	0.247
Average Lot Size V	-	0.423
Property Tax Rate	1.97%	1.04%
Neighborhood Quality	14.25	6.56
Distribution Across Neighborhoods		
S	74%	26%
U	39%	61%
		(14% V)
Neighborhood Population Breakdown		
S	56%	22%
U	44%	78%
		(18% V)

Table 5: Welfare II - Size of Public Housing

	S	U	P	**AU**
Size				
-25%	-9.24	-5.71	81.16	**-2.41**
0	-9.28	-6.45	86.63	**-2.57**
+25%	-5.64	-5.39	85.67	**-0.35**
+50%	-9.55	-8.04	63.51	**-4.49**
+75%	-9.72	-8.81	44.80	**-5.87**
Vouchers				
	-3.29	-3.31	59.01	**0.54**

Table 6: Welfare III - Property Tax Rates

	Public Housing				Vouchers			
$(\alpha_H, \alpha_L; \tau_H^*, \tau_L^*)$	S	U	P	**AU**	S	U	P	**AU**
(**.018**, .014; 1.52, 1.04)	-4.85	-5.18	81.74	**-0.34**	-2.93	-2.72	59.42	**1.03**
(**.021**, .014; 1.97, 1.04)	-5.64	-5.39	85.67	**-0.35**	-3.29	-3.31	59.01	**0.54**
(**.024**, .014; 2.54, 1.04)	-5.26	-5.20	82.04	**-0.25**	-3.01	-2.72	59.42	**0.97**
(.021, **.012**; 1.97, 0.84)	-4.89	-4.90	76.36	**-0.17**	-2.99	-2.73	59.50	**0.99**
(.021, **.014**; 1.97, 1.04)	-5.64	-5.39	85.67	**-0.35**	-3.29	-3.31	59.01	**0.54**
(.021, **.016**; 1.97, 1.26)	-5.20	-5.49	88.16	**-0.08**	-3.00	-2.78	59.23	**0.96**

Table 7: Models with Housing Industry I - Size, Share of Capital, and Location

	S	U	P	AU
Size				
-25%	-5.23	-4.67	70.39	-0.47
same	-5.13	-4.64	81.96	0.11
+25%	-5.06	-4.63	81.93	0.15
Housing Space/Land				
3	-6.01	-5.47	83.09	-0.67
4	-5.13	-4.64	81.96	0.11
5	-4.61	-4.13	77.99	0.44
Location				
3 miles	-5.08	-4.78	86.45	0.25
4 miles	-5.13	-4.64	81.96	0.11
5 miles	-5.04	-4.46	68.04	-0.39
Vouchers				
	-2.84	-3.34	56.22	0.60

Table 8: Models with Housing Industry II - Utility Parameters

$(\alpha_H, \alpha_L; \tau_H, \tau_L)$	Public Housing				Vouchers			
	S	U	P	**AU**	S	U	P	**AU**
(**.018**, .014; 1.52, 1.04)	-4.60	-3.29	79.97	**0.97**	-3.03	-2.94	56.16	**0.73**
(**.021**, .014; 1.97, 1.04)	-5.13	-4.64	81.96	**0.11**	-2.84	-3.34	56.22	**0.60**
(**.024**, .014; 2.54, 1.04)	-8.79	-17.81	46.27	**-9.94**	-3.12	-2.95	56.01	**0.65**
(.021, **.012**; 1.97, 0.84)	-3.79	-3.66	67.26	**0.54**	-2.49	-3.16	56.69	**0.87**
(.021, **.014**; 1.97, 1.04)	-5.13	-4.64	81.96	**0.11**	-2.84	-3.34	56.22	**0.60**
(.021, **.016**; 1.97, 1.26)	-5.17	-4.02	89.61	**0.75**	-2.96	-3.00	56.16	**0.72**

Table 9: Labor Supply

	Benchmark	Public Housing	Housing Vouchers
Size			
-25%	41.2	35.6	42.0
0	41.2	37.3	42.0
+25%	41.3	38.6	42.0
+50%	41.3	40.6	42.0
+75%	41.3	42.3	42.0
Location			
0-4.47	40.7	42.5	42.0
2-4.90	40.9	41.1	42.0
4-6.00	41.3	38.6	42.0
6-7.48	41.7	37.2	42.0
(α_L, α_H)			
(.012,.021)	41.3	39.1	41.7
(.014,.018)	41.3	39.0	41.7
(.014,.021)	41.3	38.6	42.0
(.014,.024)	41.3	38.9	41.7
(.016,.021)	41.3	38.8	41.7

Figure A1:
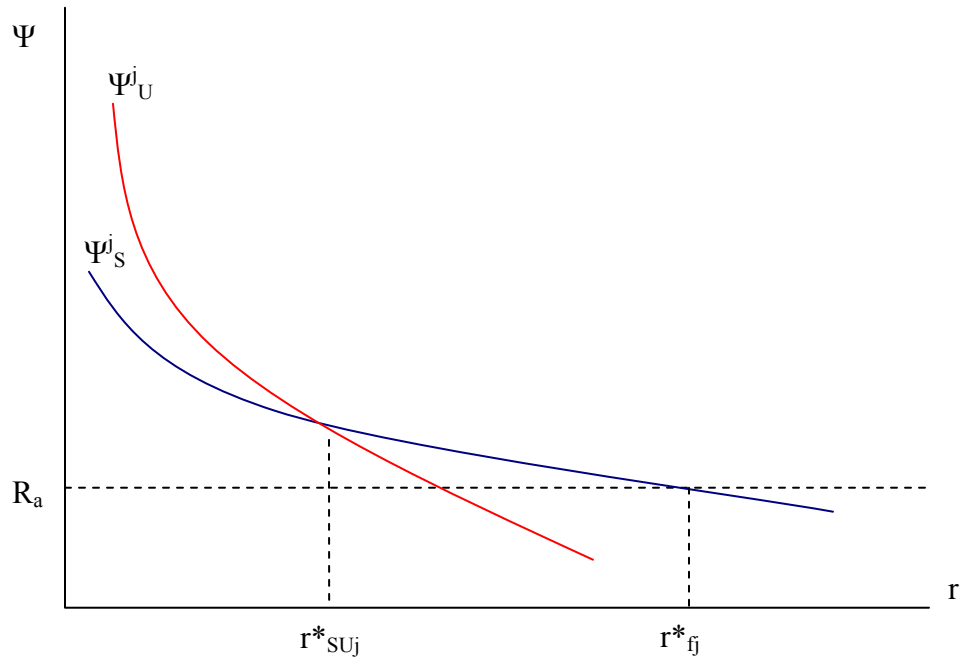
Figure 1: Benchmark Model

WEST

EAST

S types

S types

U types

U types

CBD

14.01

5.68

8.47

13.49
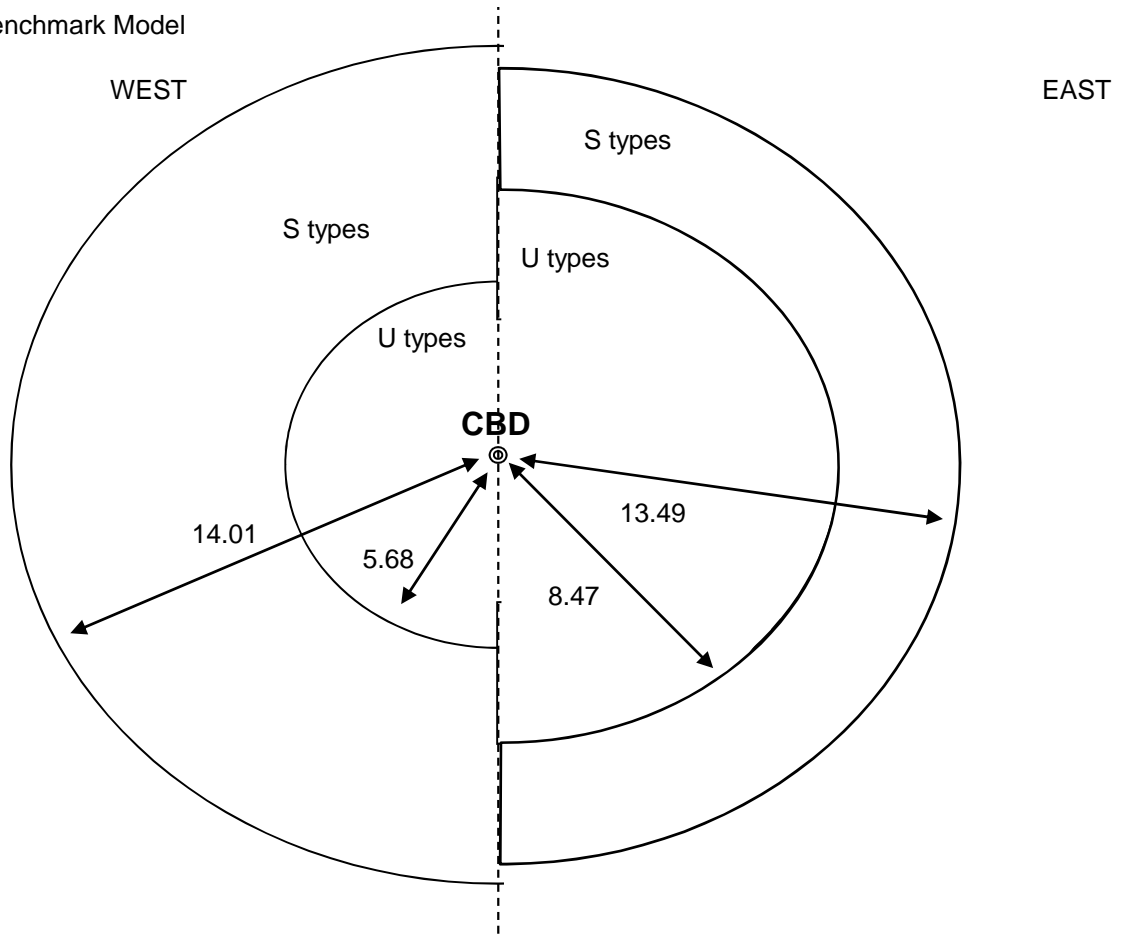
Figure 2: Public Housing Model

WEST

EAST

S types

S types

U types

13.18 mi

P types

U types

U types

8.95 mi

CBD

6 mi

14.25 mi

5.54 mi

4 mi

Figure 3: Voucher Model

WEST

EAST

S types
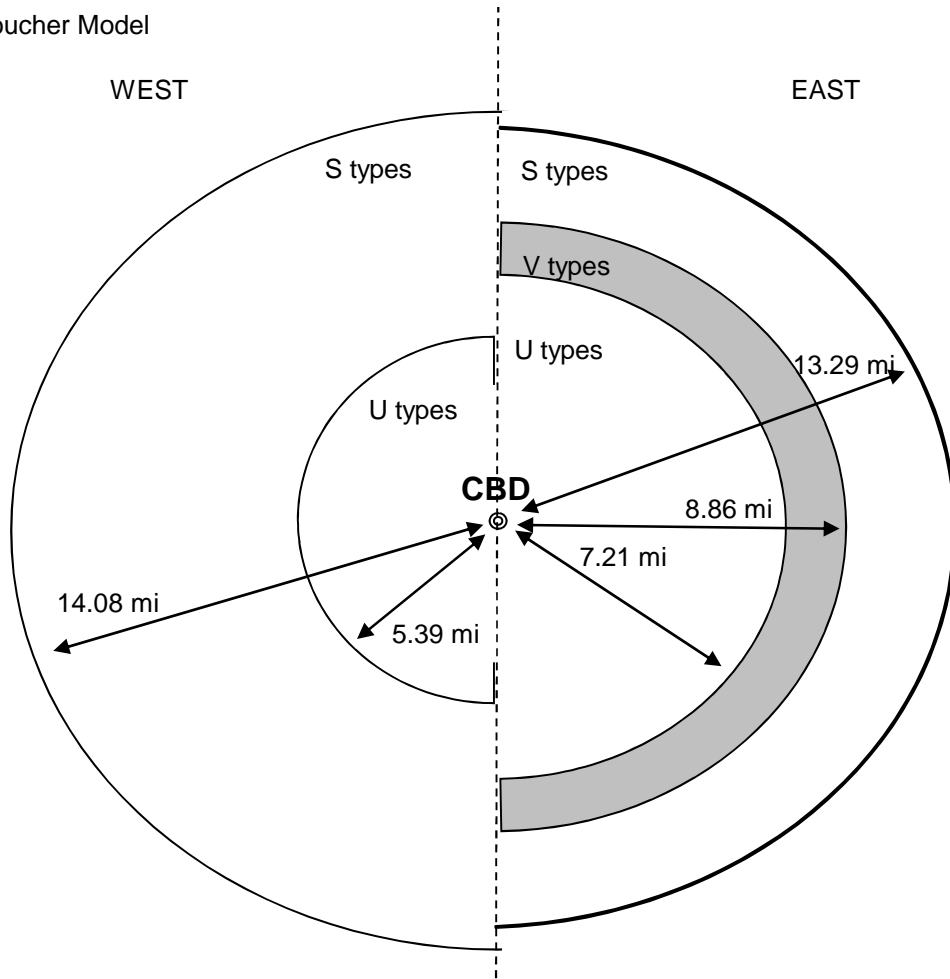
S types

V types

U types

U types

U types

CBD

13.29 mi

8.86 mi

7.21 mi

14.08 mi

5.39 mi
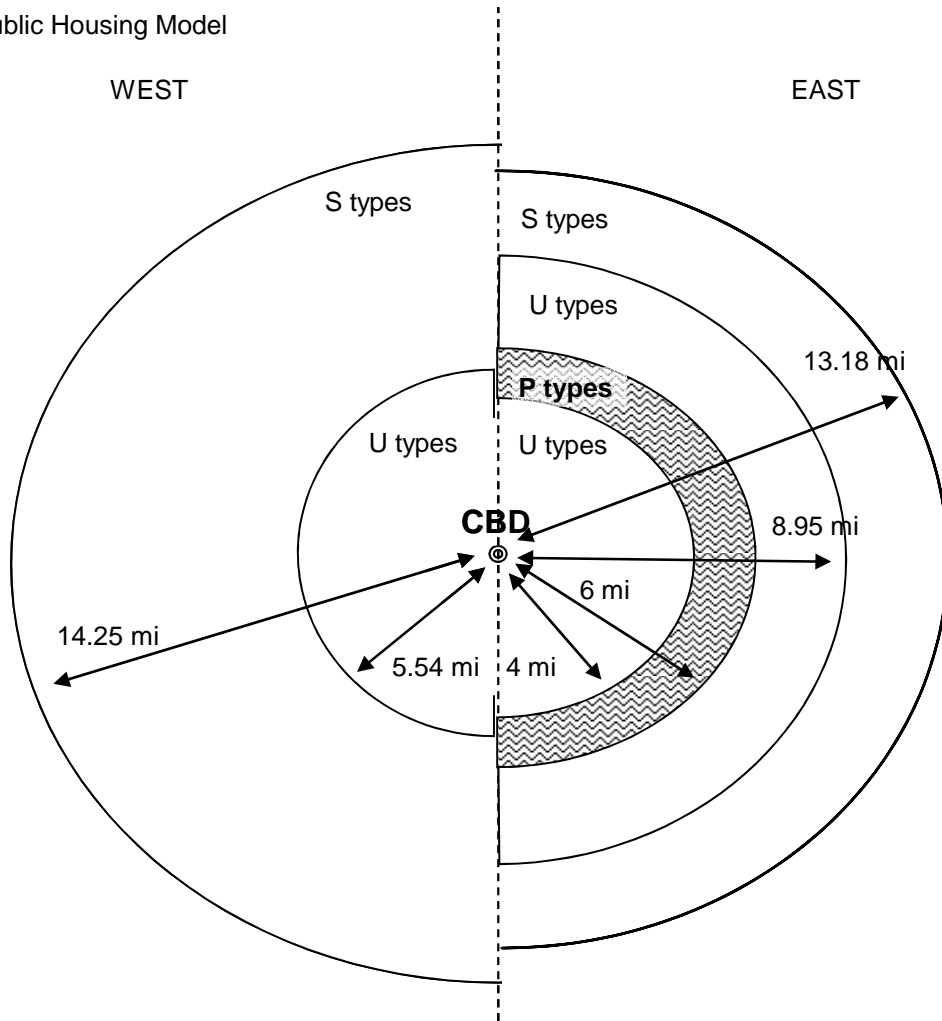
Figure 4: Desegregation via Vouchers

WEST   EAST

S types   S types

V types

V types   U types

U types

U types

CBD

12.72 mi

9.11 mi

6.67 mi

12.59 mi

6.08 mi   6.08 mi